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# A Quantum Relationship between Degree and Radian to find the Accurate Value of the Pi *(π)*

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***Abstract:*** *The aim of this research paper is to derive an accurate relationship between degree and radian in very small level of angle. This relationship is called as "Quantum Relationship". The Value of Pi (π) has been calculated in degree as well as radian (number) using this relationship.*

***Keywords:*** *Degree, Radian, Linear, Nonlinear, Pi (π)*

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## I. Introduction

*Pi (π)* is the important mathematical constant in all the domains of the Sciences and technologies. First, the value of *Pi (π*) was roughly estimated by The Greek mathematician Archimedes in around 250 BC. The Value of *Pi (π)* is defined by the ratio between the circumference and two times of radius (i.e. diameter) of a circle. Mainly, different geometrical shapes are used to measure the value of *Pi (π)* using angle in degree method. Since, angle in degree method is always represented by the linear ratio of any two arms of a geometrical object and it measure linear length instead of curve. As a result, a small difference of length is present between line *(BC)* and curve *(BSC)* as shown in Fig. 1. So, there is limitation to derive the value of *Pi (π)* due to error in measurement of circumference of a circle .Use of radian would be the more accurate in measurement of circumference of a circle because it is directly related to curvature of any geometrical shape. Therefore, we need an accurate relationship between degree and radian to get perfect value of circumference of a circle. In this paper, an accurate relationship has been proposed and the value of the *Pi (π)* has been calculated in degree as well as radian based on this.

## II. Derivation of Proposed Equation

Consider a circle with radius *r* and a triangle *ΔABC* on this circle as shown in Fig. 1.

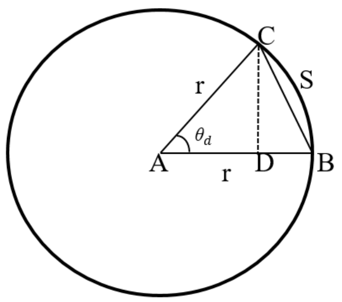


**Figure 1: A Circle with Triangle**

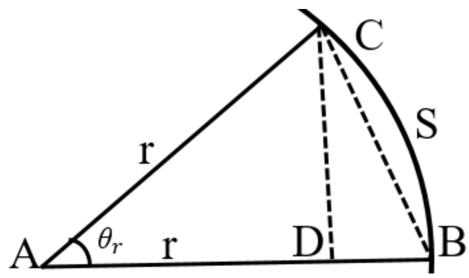


**Figure.2. Associating Curvature with Sampled Triangle**

Assume that the angle between *AB* and *AC* is $\theta_d$ in degree, and *AB=AC=r.* Where *CD* is the perpendicular on the *AB* ($CD \perp AB$).
Therefore,

$$AD = r\cos\theta_d \quad \text{and}$$
$$CD = r\sin\theta_d$$

So,

$$DB = AB - r\cos\theta_d$$

Since *AB=r,* therefore

$$DB = r - r\cos\theta_d$$

Now, the value of *CB* using the Pythagoras formula on the triangle *ΔDBC* is

$$CB = \sqrt{DB^2 + CD^2}$$
$$CB = \sqrt{(r - r\cos\theta_d)^2 + (r\sin\theta_d)^2}$$
$$= \sqrt{(r^2 - 2r^2\cos\theta_d + (r\cos\theta_d)^2 + (r\sin\theta_d)^2}$$
$$= \sqrt{(2r^2 - 2r^2\cos\theta_d}$$
$$= r\sqrt{2(1 - \cos\theta_d)} \tag{1}$$

We have to apply the definition of radian to get how much curvature or circumference *BSC* (*S*) of that circle associated with the triangle *ΔABC* as shown in Fig. 2. So, angle in radian ($\theta_r$) is

$$\theta_r = \frac{Circumfernace\,(S)}{Radius\,(r)}$$

Therefore, Circumference

$$S = r\theta_r \tag{2}$$

So, the error in length between curve *BSC* and linear *BC* is

$$\varepsilon = r\theta_r - r\sqrt{2(1 - \cos\theta_d)} \tag{3}$$

This equation (3) shows that error depends on the radius *(r)* of the circle and size ($\theta_d$) of the sampled geometrical shape. Therefore, the error would be considerable in measurement of circumference of a large circular objects like Earth, Sun.

Therefore, we need an accurate relationship between degree and radian to get perfect value of circumference of a circle. To get that relationship, the derivation will be related to the Fig.2 and derived equation (3). Now, from Fig. 2, we can see that the error (ε) would be zero when $\theta_d$ is closed to zero, i.e.

$$CB \cong BSC \quad \text{When } \theta_d \cong 0$$

Therefore,

$$\theta_r = \sqrt{2(1 - \cos\theta_d)} \tag{4}$$

Equation (4) represents a very important relationship between the radian and degree when angle is very small. We would like to call it as "*quantum relationship*" since it is valid only in quantum level. Therefore, this equation is capable to give the more accurate value of radian and used to calculate the circumference, where value of $\cos\theta_d$ is simply calculated by the Taylor Series expansion.

A part of circumference (*BSC*) of a circle associated with the small triangle (*ΔABC)* is calculated by putting equation (4) into equation (2).Therefore, total circumference of the circle will be

$$S_T = r\,\theta_r * number\ of\ samples = r\theta_r \times \left(\frac{360^0}{\theta_d}\right) \tag{5}$$

## III. The Value of Pi (π) in Degree

The Value of *Pi (π)* is defined by the ratio between the circumference ($S_T$) and diameter (*2r*) of a circle.

$$\pi = \frac{S_T}{2r} = \frac{\theta_r * 360^0}{2\theta_d}$$
$$= \lim_{\theta_d \to 0} \frac{360^0 \times \sqrt{2(1-\cos\theta_d)}}{2\theta_d}$$
$$= \lim_{\theta_d \to 0} \left(\frac{360^0}{\sqrt{2}}\right) \times \left(\frac{\sin\theta_d}{2\sqrt{(1-\cos\theta_d)}}\right)$$

Since $\left[\frac{0}{0}\right]$, using L. Hospital Rule

$$= \lim_{\theta_d \to 0} \left(\frac{360^0}{\sqrt{2}}\right) \times \left(\frac{\sqrt{(1-(\cos\theta_d)^2)}}{2\sqrt{(1-\cos\theta_d)}}\right)$$
$$= \lim_{\theta_d \to 0} \left(\frac{360^0}{\sqrt{2}}\right) \times \left(\frac{\sqrt{(1-\cos\theta_d)\times(1+\cos\theta_d)}}{2\sqrt{(1-\cos\theta_d)}}\right)$$

$$= \lim_{\theta_d \to 0} \left(\frac{360^0}{\sqrt{2}}\right) \times \left(\frac{\sqrt{(1+cos\theta_d)}}{2}\right)$$

Putting the limit,

$$\pi = \left(\frac{360^0}{\sqrt{2}}\right) \times \left(\frac{\sqrt{(1+1)}}{2}\right) = 180^0 \tag{6}$$

Therefore, the value of the *Pi (π)* in degree is equal to the *$180^0$*. So, we can conclude that the equation (4) gives a perfect relationship between the degree and radian.

## IV. The Value of Pi (π) in Radian

Consider a very small angle in degree is $\theta_d$ and its corresponding value in radian $\theta_r$ is calculated by the equation (4). In previous section, it has been shown that value of *Pi (π)* in degree is equal to the $180^0$.

Therefore, the value of the *Pi (π)* will be

$$\pi = \theta_r * number\ of\ samples$$
$$= \theta_r * \left(\frac{180}{\theta_d}\right) \qquad \text{Where} \quad \theta_d \cong 0$$
$$= 180 * \left(\frac{\theta_r}{\theta_d}\right) \tag{7}$$

Where $\left(\frac{\theta_r}{\theta_d}\right)$ is linear ratio between radian and degree, which is called the one radian as shown in Table I. Equation (8) shows that this linear ratio does not valid in classical level other than the quantum level.

$$\theta_r = \sqrt{2(1 - cos\theta_d)}$$

Using Taylor Series expansion

$$= \sqrt{\left(2(1 - \left[1 - \frac{\theta_d^{\ 2}}{2} + \frac{\theta_d^{\ 4}}{4!} - \frac{\theta_d^{\ 6}}{6!} + \cdots\right]\right)} \tag{8.a}$$

When value of $\theta_d$ in quantum level($\theta_d \ll 1$), we can neglect the higher order terms of this equation.
Therefore,

$$\theta_r \cong \theta_d \tag{8.b}$$

Equation (8.b) shows a linear relationship between radian and degree in quantum level. So, the $\frac{\theta_r}{\theta_d}$ can be directly used to measure value of 1 radian.

At classical level, ($\theta_d \geq 1$) we cannot neglect the higher order terms of the equation (8.a). Therefore equation (8.a) becomes

$$\theta_r \cong \sqrt{\theta_d^{\ 2} - \frac{\theta_d^{\ 4}}{12}} \tag{8.c}$$

Equation (8.c) shows a nonlinear relationship between radian and degree in classical level. So we cannot calculate the value of 1 radian using($\frac{\theta_r}{\theta_d}$).

**Table. I**

| Level | Degree($\theta_d$) | Radian($\theta_r$) | 1 Radian $(\frac{\theta_r}{\theta_d})$ | Value of Pi(π)( $\frac{\theta_r * 180}{\theta_d}$) |
|---|---|---|---|---|
| Quantum Level | 0.00001 | 1.744136200952474*10^-7 | 0.01744136200952474 | 3.1394451617144536 |
| | 0.0001 | 0.0000017453452194240843 | 0.017453452194240843 | 3.1416213949633516 |
| | 0.001 | 0.00001745329316723716 | 0.01745329316723716 | 3.141592770102689 |
| | 0.01 | 0.00017453292467515142 | 0.017453292467515142 | 3.1415926441527255 |
| Classical Level | 0.1 | 0.0017453290304737818 | 0.017453290304737818 | 3.1415922548528075 |
| | 1 | 0.017453070996746128 | 0.017453070996746128 | 3.1415527794143028 |

In Table I, a set of value of 1 radian and ***Pi (π)*** is calculated. The calculated value of ***Pi (π)*** in quantum level is more accurate than classical level because at the quantum level, $1\,Rad = (\frac{\theta_r}{\theta_d})$ is valid but at classical level that 1 rad relation does not valid due to nonlinearity. Therefore, among the values of ***Pi (π)*** in Table I, 3.1394451617144536... is the accurate value of ***Pi (π) .***

## V. Conclusion

We have established a quantum relationship between degree and radian. It has been shown that calculated value of ***Pi (π)*** in quantum level is more accurate than classical. The calculated the value of *Pi (π)* in degree and radian are equal to the $180^0$ and 3.1394451617144536… respectively.

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