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Quotient Finite Group Automata

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Abstract: *Let (Q, *, Σ, δ, q_0, F) be a Finite Group Automaton. Let (S, *, E, γ, q_0, T) be a Finite Sub-group Automaton of Q. Then (Q/S,* **o** *, Σ, ∧ , q_0*S, F') is a finite group automaton. This finite group automaton is known as the Quotient Finite Group Automaton (Quotient FGA) corresponding to the Finite Subgroup Automaton (S, *, E, γ, q_s, T). If a string w is accepted by (Q, *, Σ, δ, q_0, F), then w is accepted by (Q/S,* **o** *, Σ, ∧ , q_0*S, F'). If L is a language accepted by a finite group automaton (Q, *, Σ, δ, q_0, F) , then L is accepted by (Q/S,* **o** *, Σ, ∧ , q_0*S, F').*

Keywords: *Finite Group Automaton, Finite Sub-group Automaton, Quotient Finite Group Automaton*

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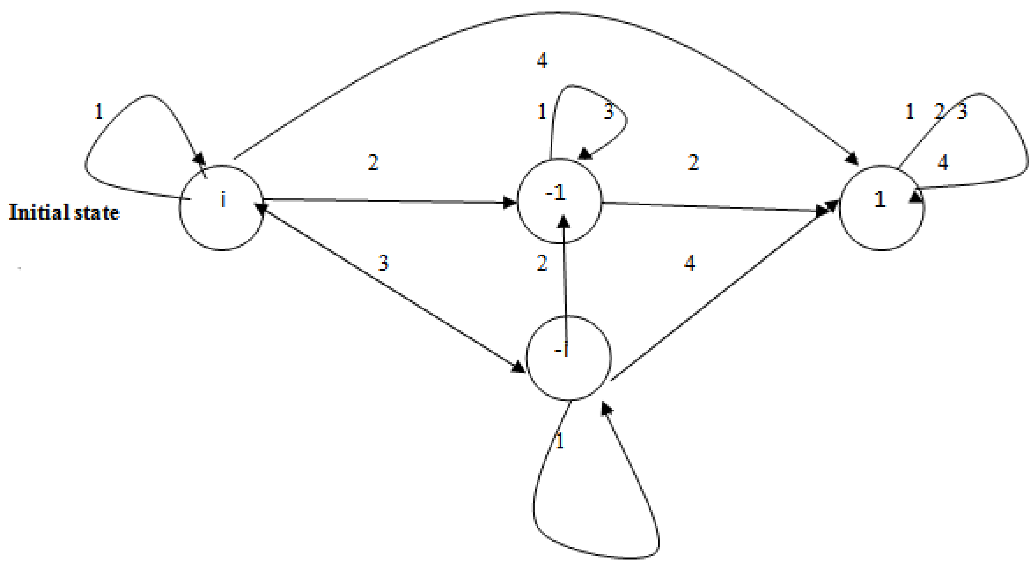
Definition : Finite Automaton: A finite automaton is a 5–tuple (Q, Σ, δ, q_0, F), where Q is a finite set of states, Σ is a finite input alphabet, q_0 in Q is the initial state, F ⊆ Q is the set of final states, and δ is the transition function mapping Q x Σ to Q.
That is δ(q, a) is a state for each state and input symbol a.
Finite Sub-group Automaton: Let B= (Q, *, Σ, δ, q_0, F) be a Finite Group Automaton, where Q is a finite set of states, * is a mapping from Q×Q to Q, Σ is a finite set of integers, q_0 in Q is the initial state and F⊆Q is the set of final states and δ is the transition function mapping from Q×Σ to Q defined by δ(q,n) = q^n. A Finite Sub-group Automaton S of B is a 6-tuple (R, *, E, γ, q_s, T), where R⊆Q for all p,q ϵ R, p * q ϵ R, q_s ϵ R is the initial state where q_s = q_0 or q_s = δ(q_0,n) for some nϵ∑, , E is the set of all n in Σ such that n ≤ m for all m ϵ Σ, ie , E = {n ϵ Σ / n ≤ m, for some m ϵ Σ }, γ is the restriction function of δ restricted to R×E→R, q_0 in R is the initial state and T⊆R and T⊆F.
Definition: Let (Q, (*, Σ, δ, q_0, F)) be a Finite Group Automaton. Let (S,(*, E, γ, q_0, T)) be a Finite Sub-group Automaton of Q, where S⊆Q such that q_0 ϵ S and for all p,q ϵ S, p * q ϵ S , E is the set of all n in Σ such that n ≤ m for all m ϵ Σ, ie , E = {n ϵ Σ / n ≤ m, for some m ϵ Σ }, γ is the restriction function of δ restricted to S×E→S, q_0 in R is the initial state and T⊆S.
For each a in Q, we define a* S = {a*s / s ϵS}.
Let Q/S = {a*S / a ϵ Q}
Define an operation **o** on Q/S by (a*S) **o** (b*S) = (a*b)*S
Since a,b ϵQ and Q is a group under * , a * b ϵ Q
Therefore, (a*b)*S ϵ Q/S
Therefore, **o** is a binary operation on Q/S.
(a*S) **o** ((b*S) **o** (c*S)) = (a*S) **o** ((b*S) **o** (c*S))
= (a*S) **o** (b*c)*S)
= (a * (b*c))*S
= (a * b)*c))*S (Since Q is a group under * ,
a * (b * c) = (a * b) * c for ball a,b,c ϵ Q)
= ((a * b)* S) **o** (c *S)
= ((a *S) **o** (b* S)) **o** (c *S)
Since Q is a group under * , there exists 0 ϵ Q such that a * 0 = a = 0 * a, for all a ϵ Q.
Therefore, 0 * S ϵ Q/S
Also (a * S) **o** (0 * S)= (a * 0) * S
= a * S
(0 * S) **o** (a * S)= (0 * a) * S
= a * S

Therefore, (a * S) **o** (0 * S)= (0 * S) **o** (a * S)
Therefore, 0 * S is the identity element of Q/S.
Now for each a * S ϵ Q/S there exists a^{-1} * S ϵ Q/S such that
(a * S) **o** (a^{-1} * S)= (a^{-1} * S) **o** (a * S) = 0 * S
Hence (Q/S, **o**) is a group.
Consider q_0*S, where q_0 is the initial state of B = (Q, *, Σ, δ, q_0, F)
Let F' = {f * S / f ϵ F}
Define ∧ : Q/S× Σ → Q/S by ∧(a*S , n) = δ(a , n) * S
Clearly it is a well defined mapping.
We shall prove that (Q/S, **o** , Σ, ∧ , q_0*S, F') is a finite group automaton.
The elements of Q/S are considered as states.
The set ∑ of the same input symbols are taken
The function ∧ : Q/S× Σ → Q/S by ∧(a*S , n) = δ(a , n) * S is our transition function.
q_0*S is taken as the initial state.
F' = { a * S / a ϵ F }
F' is taken as the set of final states.
Then (Q/S, **o** , Σ, ∧ , q_0*S, F') is a finite group automaton.

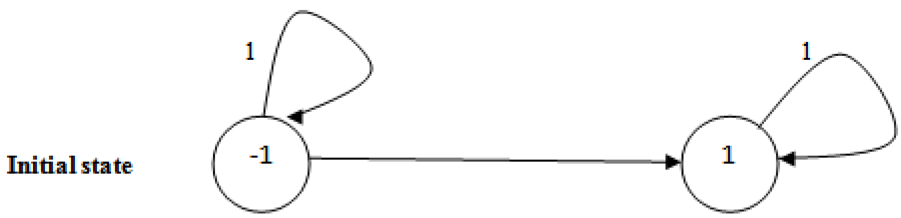
Definition : Quotient Finite Group Automaton : Let (Q, *, Σ, δ, q_0, F) be a Finite Group Automaton. Let (S, *, E, γ, q_0, T) be a Finite Sub-group Automaton of Q. Then (Q/S, **o** , Σ, ∧ , q_0*S, F') is a finite group automaton. This finite group automaton is known as the Quotient Finite Group Automaton (Quotient FGA) corresponding to the Finite Subgroup Automaton(S, *, E, γ, q_s, T).

Example : Consider the Finite Binary Automaton (Q, *, Σ, δ, q_0, F), where Q = {1,-1,i,-i}, Σ = {1,2,3,4} q_0 = i is the initial state and F, the set of final states is Q, δ is the transition function mapping from Q×Σ to Q defined by δ(q,n) = q^n, and
* is the mapping from Q×Q to Q defined as in the example 3.2.1



Transition Diagram of (Q, *, Σ, δ, q_0, F)

Then the Finite Binary Automaton (Q, *, Σ, δ, q_0, F) is a Finite Group Automaton.
1) Let (S, *, E, γ, q_s, T), where S = { 1,-1}, E = {1,2}, q_s = -1, T = {1,-1}
$q_s = (q_0)^2 = (i)^2 = -1$.
Here * is the usual multiplication.



Transition Diagram of (S, *, E, γ, q_0, T)

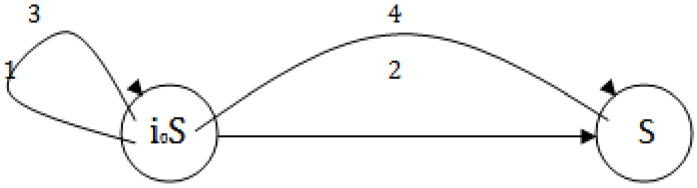
Then (S, *, E, γ, q_s, T) is a Finite Subgroup Automaton of the Finite group Automaton (Q, *, Σ, δ, q_0, F).
Now S = { 1,-1}
1.S = {1.1, 1(-1) }
= {1, -1}
= S
-1.S = {(-1).1, (-1).(-1)}
= {-1, 1 }
= {1, -1 }
= S
Therefore, 1.S = (-1).S
i.S = {(i).1, (i).(-1)}
= {i, -i}
(-i).S = {(-i).1, (-i).(-1)}
= {-i, i}
= {i, -i}
= S
Therefore, i.S = (-i).S
Now Q/S = { 1.S, -1.S, i.S, -i.S }
Q/S = { 1.S, i.S, }
= {S, i.S
q_0.S = i.{1, -1}
= {i, -i}
Let F' = {f * S / f ϵ F}
Define ∧ : Q/S× Σ → Q/S by ∧(a*S , n) = δ(a , n) * S
= δ(a , n) 0 S

∧(i 0 S , 1) = δ(i , 1) 0 S
= i^1 0 S
= i 0 S
∧(i 0 S , 2) = δ(i , 2) 0 S
= i^2 0 S
= (-1) 0 S
= -S
= S
∧(i 0 S , 3) = δ(i , 3) 0 S
= i^3 0 S
= (-i) 0 S
= iS
∧(i 0 S , 4) = δ(i , 4) 0 S
= i^4 0 S
= 1 0 S
= S

∧(-i 0 S , 1) = δ(-i , 1) 0 S
= $-i^1$ 0 S
= -i 0 S
= i.S
∧(-i 0 S , 2) = δ(-i , 2) 0 S
= $(-i)^2$0 S
= (-1) 0 S

$= -S$
$= S$
$\wedge(-i\ 0\ S, 3) = \delta(-i, 3)\ 0\ S$
$= (i)^3\ 0\ S$
$= (-i)\ 0\ S$
$= i0S$
$\wedge(-i\ 0\ S, 4) = \delta(-i, 4)\ 0\ S$
$= (-i)^4\ 0\ S$
$= 1\ 0\ S$
$= S$
$\wedge(1\ 0\ S, 1) = \delta(1, 1)\ 0\ S$
$= 1^1\ 0\ S$
$= 1\ 0\ S$
$= S$
$\wedge(1\ 0\ S, 2) = \delta(1, 2)\ 0\ S$
$= 1^2\ 0\ S$
$= 10\ S$
$= S$
$\wedge(1\ 0\ S, 3) = \delta(1, 3)\ 0\ S$
$= 1^3\ 0\ S$
$= 1\ 0\ S$
$= S$
$\wedge(1\ 0\ S, 4) = \delta(1, 4)\ 0\ S$
$= 1^4\ 0\ S$
$= 1\ 0\ S$
$= S$
$\wedge(-1\ 0\ S, 1) = \delta(-1, n)\ 0\ S$
$= (-1)^1\ 0\ S$
$= (-1)\ 0\ S$
$= -S$
$= S$
$\wedge(-1\ 0\ S, 2) = \delta(-1, 2)\ 0\ S$
$= (-1)^2\ 0\ S$
$= 1\ 0\ S$
$= S$
$\wedge(-1\ 0\ S, 3) = \delta(-1, 3)\ 0\ S$
$= (-1)^3\ 0\ S$
$= (-1)\ 0\ S$
$= -S$
$= S$
$\wedge(-1\ 0\ S, 4) = \delta(-1, 4)\ 0\ S$
$= (-1)^4\ 0\ S$
$= 1\ 0\ S = S$

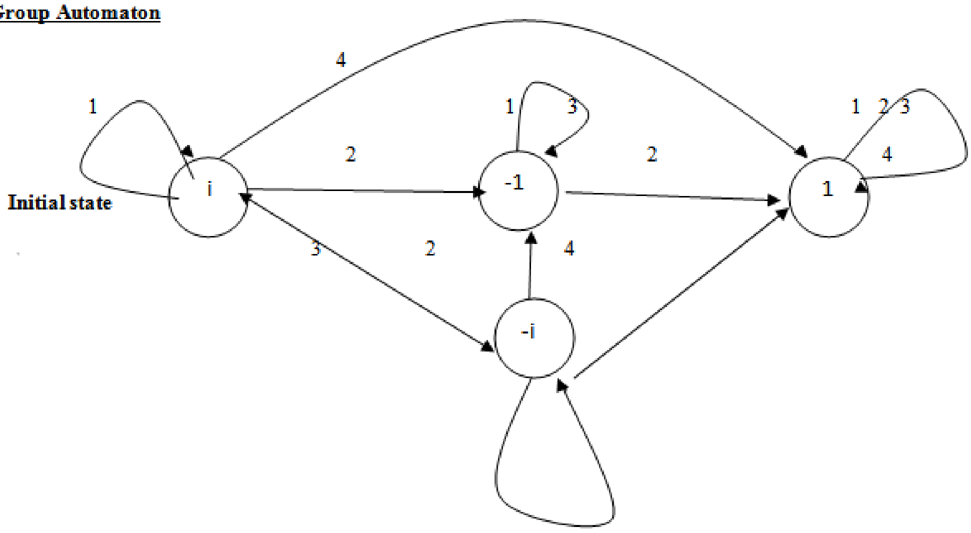
Therefore (Q/S, **o** , Σ, ∧ , q_0*S, F') is a finite group automaton.
Now the Diagram of the Quotient Finite Group Automaton (Q/S, **0** , ∑, ∧ , q_0*S, F') corresponding to the Finite Subgroup Automaton (S, *, E, γ, q_0, T) is given below.



Transition Diagram of Quotient Finite Group Automaton (Q/S, **0** , ∑, ∧ , q_0*S, F')

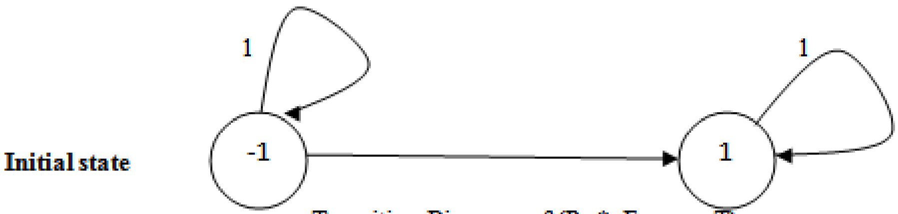
<u>Now the Diagrams for Finite Group Automaton, the Finite Subgroup Automaton and the Quotient Finite Group Automaton follow one by one</u>

<u>Finite Group Automaton</u>



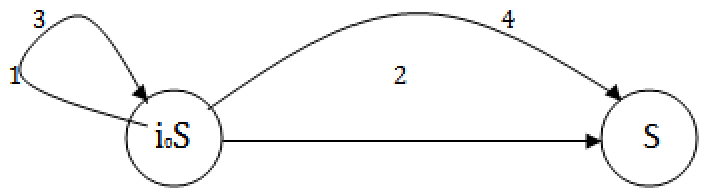
Transition Diagram of $(Q, *, \Sigma, \delta, q_0, F)$ 1

<u>Finite Subgroup Automaton</u>



Transition Diagram of $(R, *, E, \gamma, q_0, T)$

<u>Quotient Finite Group Automaton</u>



Transition Diagram of Quotient Finite Group Automaton $(Q/S, \mathbf{0}, \Sigma, \wedge, q_0*S, F')$

Theorem : If a string w is accepted by $(Q, *, \Sigma, \delta, q_0, F)$, then w is accepted by $(Q/S, \mathbf{o}, \Sigma, \wedge, q_0*S, F')$.
Proof : Let $(Q, *, \Sigma, \delta, q_0, F)$ be a Finite Group Automaton.
Let $(S, *, E, \gamma, q_0, T)$ be a Finite Sub-group Automaton of S.
Then $(Q/S, \mathbf{o}, \Sigma, \wedge, q_0*S, F')$ is a finite group automaton.
where q_0*S = is the initial state of Q/S in which q_0 is the initial state of $(Q, *, \Sigma, \delta, q_0, F)$
$T = \{f * S \,/\, f \in F\}$
$\wedge$ is defined by $\wedge : Q/S \times \Sigma \rightarrow Q/S$ by $\wedge(a*S, n) = \delta(a, n) * S$.
Let w be accepted by $(Q, *, \Sigma, \delta, q_0, F)$
Then $\delta(q_0, w) \in F$
Let $\delta(q_0, w) = f$
Then $f \in F$
Now $\wedge(q_0*S, w) = \delta(q_o, w) * S$
$= f * S \in F'$
Therefore, w is accepted by Q/S.

Theorem : If L is a language accepted by a finite group automaton (Q, *, Σ, δ, q_0, F) , then L is accepted by (Q/S, **o** Σ, ∧ , q_0*S, F').

Proof : Let B = (Q, *, Σ, δ, q_0, F) be a Finite Group Automaton.
Let L is a language accepted by (Q, *, Σ, δ, q_0, F).
Let (S, *, E, γ, q_0, T) be a Finite Sub-group Automaton of Q.
Then (Q/S, **o** , Σ, ∧ , q_0*S, F') is a finite group automaton which is the Quotient finite group automaton.
Let w ϵ L
Then δ(q_0, w) ϵ F
By the above theorem ∧(q_0*S, w) = δ(q_o , w) * S = f * S ϵ F'
Therefore, L is accepted by (Q/S, **o** , Σ, ∧ , q_0*S, F') .

Conclusion

Further research can be done in Quotient Finite Group automata. Many useful results may be obtained in this Quotient Finite Group automata.

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