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Universal Portfolios Generated by the Pseudo f-Divergences

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Abstract: *Universal portfolios generated by the f-divergences have been proposed recently. The f-divergence of Csiszar is generated by a non-negative convex function on the positive axis. In this paper, the pseudo f-divergence is defined for two types of convex functions not satisfying the usual requirements. The first type is a non-negative function convex on a subset of the positive axis and the second is a function convex on the positive axis but non-negative on a subset of the axis. Five portfolios are selected from the local stock exchange for the empirical study of the generated universal portfolio. High investment returns are observed for some of the selected portfolios, indicating the suitability of practical usage.*

Keywords*-Pseudo$f$-divergence, universal portfolio, convex function, investment*

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I. Introduction

Cover and Ordentlich [1] proposed a universal portfolio by weighting the current and past price-relatives in a stock portfolio by the moments of a probability distribution with emphasis on the Dirichlet distribution. In implementation of this portfolio, a vast amount of computer memory and time needed is obvious. To avoid this problem Tan [2] introduced the finite-order universal portfolio where some of the most recent price-relatives need to be weighted. Empirical studies indicated that the distant-past history of price relatives is not significant in improving the performance of the portfolio. The use of the Kullback-Leibler divergence (one of the well-known f-divergences) in generating a universal portfolio is studied by Helmbold et al. [3]. The general use of f-divergences and Bregman divergences in generating universal portfolios is proposed by Tan and Kuang [4].

The f-divergence is generated by a non-negative convex function $f(t)$ on the positive axis. The crucial property of the f-divergence is that the function achieves the value 0 when t is 1. It may occur that this property $f(1) = 0$ is satisfied but other requirements of the generating function are not satisfied. It is the focus of this paper to define pseudo f-divergences retaining the property $f(1) = 0$ for non-negative functions convex on a subset of the positive axis and functions convex on the positive axis but non-negative on a subset of the positive axis. The former is known as phi-disparity difference which is studied by Pardo [5]. For the two types of pseudo f-divergences, the universal portfolios generated will be derived.

II. Some Preliminaries

Definition 2.1: A market with m stocks is assumed and the market behaviour is described by the price-relative vector $\boldsymbol{x}_n = (x_{ni})$on the n^{th} trading day, where x_{ni} is the price relative of the i^{th} stock which is defined as the ratio of the closing price of the i^{th} stock on the n^{th} trading day to the opening price on the same day. A portfolio strategy $\boldsymbol{b}_n = (b_{ni})$on the n^{th} trading day is the vector of the proportions of the current wealth $S_n(\boldsymbol{x}_n)$ invested in the stocks, where b_{ni} is the proportion of the current wealth invested on the i^{th} stock, for $0 \le b_{ni} \le 1, i = 1, 2, \dots, m$ and $\sum_{i=1}^{m} b_{ni} = 1$. The initial investment wealth is assumed to be 1 unit. The wealth $S_n(\boldsymbol{x}_n)$ at the end of the n^{th} trading day is calculated according to:

$$S_n(\boldsymbol{x}_n) = \prod_{j=1}^{n} \boldsymbol{b}_j^t \boldsymbol{x}_j = \prod_{j=1}^{n} \left(\sum_{i=1}^{m} b_{ji} x_{ji} \right). \tag{1}$$

Let $f(t)$ be a convex function on $(0, \infty)$ and is strictly convex at $t = 1$ and satisfies $f(1) = 0$. The f-divergence of two probability distributions $\boldsymbol{p} = (p_i)$ and $\boldsymbol{q} = (q_i)$ is defined as

$$D_f(\boldsymbol{p}||\boldsymbol{q}) = \sum_{i=1}^{m} q_i f\left[\frac{p_i}{q_i}\right]. \tag{2}$$

The f-divergence (2) is known as the f-*disparity difference* between two probability distributions $\boldsymbol{p} = (p_i)$ and

$\boldsymbol{q} = (q_i)$ if $f(t)$ is an f-disparity function which is defined as follows. The continuous function $f(t)$ on $(0, \infty)$ is an *f-disparity function* if

(i) $f(t)$ is decreasing for $0 < t < 1$.
(ii) $f(t)$ is increasing for $1 < t < \infty$.
(iii) $f(1) = 0$.
(iv) $f(0)$ is determined by the continuous extension of $f(t)$ (see [5], pg. 29)

The f-disparity function is also known as the phi-disparity function in the statistical inference literature. The convex function $f(t)$ used in the f-divergence is an f-disparity function. The converse is not true. An f-disparity function may not be convex on $(0, \infty)$. Thus, the f-disparity difference (2) which may not be a divergence is a weaker form of (2). The f-disparity difference (2) is known as a *pseudo f-divergence*.

Example. Let $f(t) = \frac{(1-t)^2}{(1+t^2)}$ for $t \geq 0$. Then

$$f'(t) = -2\left[\frac{1-t^2}{(1+t^2)^2}\right] \tag{3}$$

is negative for $0 < t < 1$ and positive for $t > 1$. Noting that

$$f''(t) = \frac{-4t}{(1+t^2)^3}[t^2 - 3],$$

it is clear that $f''(t) > 0$ for $0 < t < \sqrt{3}$ and$f''(t) < 0$ for $t > \sqrt{3}$. Thus, $f(t)$ is not convex on $(0, \infty)$. However, $f(t)$ is an f-disparity function.

III. Main Results

The fact that the f-disparity difference has the same form as the f-divergence implies that the mathematical form of the universal portfolio generated by the f-disparity difference is the same as that generated by the f-divergence. Let $f(t)$ be a given f-disparity difference. Then from [4], then the Type-1 universal portfolio $\boldsymbol{b}_{n+1}$ generated by $f(t)$ corresponding to the objective function

$$\hat{F}(\boldsymbol{b}_{n+1}; \lambda) = \eta\left[\log(\boldsymbol{b}_n^t \boldsymbol{x}_n) + \left(\frac{\boldsymbol{b}_{n+1}^t \boldsymbol{x}_n}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right) - \mathbf{1}\right] - D_f(\boldsymbol{b}_{n+1} || \boldsymbol{b}_n) + \lambda\left(\sum_{j=1}^{m} b_{n+1,j} - 1\right) \tag{4}$$

is given by

$$f'\left(\frac{b_{n+1,i}}{b_{ni}}\right) = \eta\left(\frac{x_{ni}}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right) + \xi, \quad i = 1, 2, \cdots, m, \tag{5}$$

where λ is the Lagrange multiplier and without loss of generality, the parameters η and ξ are assumed to be constants.

Proposition 3.1: For the f-disparity function $f(t) = \frac{(1-t)^2}{(1+t^2)}, t \geq 0$, a valid version of the universal portfolio $\boldsymbol{b}_{n+1}$generated by $f(t)$ is given by

$$b_{n+1,i} = \frac{b_{ni}\left[v_i^{-1} - 1 + v_i^{-1}(1 - 4v_i)^{\frac{1}{2}}\right]^{\frac{1}{2}}}{\sum_{j=1}^{m} b_{nj}\left[v_j^{-1} - 1 + v_j^{-1}(1 - 4v_j)^{\frac{1}{2}}\right]^{\frac{1}{2}}}, \quad i = 1, 2, \cdots, m \tag{6}$$

for $0 < v_i < \frac{1}{4}$, where

$$v_i = \frac{\eta x_{ni}}{\boldsymbol{b}_n^t \boldsymbol{x}_n} + \xi; \tag{7}$$

η and ξ are parameters.

Proof: From (3), (5) and (7),

$$f'\left(\frac{b_{n+1,i}}{b_{ni}}\right) = -2\left[\frac{\left(1 - \frac{b_{n+1,i}^2}{b_{ni}^2}\right)}{\left(1 + \frac{b_{n+1,i}^2}{b_{ni}^2}\right)^2}\right] = v_i, \quad \text{for} \quad i = 1, 2, \cdots, m. \tag{8}$$

Simplifying (8),

$$v_i\left[b_{n+1,i}^4 + 2b_{ni}^2 b_{n+1,i}^2 + b_{ni}^4\right] + 2b_{ni}^4 - 2b_{ni}^2 b_{n+1,i}^2 = 0$$

or

$$v_i b_{n+1,i}^4 + 2(v_i - 1)b_{ni}^2 b_{n+1,i}^2 + v_i b_{ni}^4 + 2b_{ni}^4 = 0.$$

Solving the quadratic in $b_{n+1,i}^2$, it follows that

$$b_{n+1,i}^2 = \frac{1}{2v_i}\left\{2(1 - v_i)b_{ni}^2 \pm \sqrt{4b_{ni}^4(1 - 4v_i)}\right\},\ \ i = 1, 2, \cdots, m.$$

Choosing the positive root,

$$b_{n+1,i} = b_{ni}\left[v_i^{-1} - 1 + v_i^{-1}(1 - 4v_i)^{\frac{1}{2}}\right]^{\frac{1}{2}},\ \ i = 1, 2, \cdots, m. \tag{9}$$

For a valid portfolio, $0 \leq b_{n+1,i} \leq 1$. Thus normalizing (9) leads to (6).

Remarks. (i) For an empirical study, the parameters η and ξ in (7) are chosen so that $0 < v_i < \frac{1}{4}$.

(ii) For small v_i, $(1 - 4v_i)^{\frac{1}{2}}$ can be approximated as $(1 - 2v_i)$. The portfolio (6) can be replaced by

$$b_{n+1,i} = \frac{b_{ni}\left[2v_i^{-1} - 3\right]^{\frac{1}{2}}}{\sum_{j=1}^{m} b_{nj}\left[2v_j^{-1} - 3\right]^{\frac{1}{2}}},\ \ i = 1, 2, \cdots, m. \tag{10}$$

A non-negative, continuously differentiable convex function $g(t)$ on $(0, \infty)$ may not satisfy the condition $g(1) = 0$. The function $g(t)$ translated by $g(1)$ can satisfy $f(1) = 0$, where $f(t) = g(t) - g(1)$. Consider function $f(t) = g(t) - g(1)$ on $(0, \infty)$ where $g(t)$ is a given non-negative, continuously differentiable convex function on $(0, \infty)$. For a translated convex function $g(t)$, a *pseudo f-divergence* can be defined for

$$f(t) = g(t) - g(1) \tag{11}$$

by replacing $f(t)$ in (2) by (11). For the new function $f(t)$ defined by (11), if $f(t) \geq 0$ is not satisfied for all $0 \leq t < \infty$, the f-divergence defined is a pseudo f-divergence.

For $t \geq 0$, consider the convex function

$$g(t) = \frac{d_1}{(\beta + 1)}(t + c_1)^{\beta+1} + \frac{d_2}{(1 - \beta)}\frac{1}{(t + c_2)^{\beta-1}}, \tag{12}$$

for constants $c_1 > 0$, $c_2 > 0$, $d_1 > 0$, $d_2 < 0$ and the parameter $\beta > 1$. For $\beta = 1$, $g(t)$ is defined as

$$g(t) = \frac{d_1}{2}(t + c_1)^2 + d_2\log(t + c_2). \tag{13}$$

The first two derivatives of $g(t)$ are:

$$g^{'(t)} = d_1(t + c_1)^{\beta} + d_2(t + c_2)^{-\beta}, \tag{14}$$

$$g^{''(t)} = d_1\beta(t + c_1)^{\beta-1} - d_2\beta(t + c_2)^{-\beta-1}. \tag{15}$$

It is clear that $g^{''}(t) > 0$ for

$$d_1(t + c_1)^{\beta-1}(t + c_2)^{\beta+1} > d_2 \tag{16}$$

and hence $g(t)$ is convex on $(0, \infty)$ for $d_1 > 0$ and $d_2 < 0$. A pseudo f-divergence is defined by (2) for $f(t)$ and $g(t)$ given by (11) and (12) or (13) respectively.

Proposition 3.2. Consider the convex function $g(t)$ given by (12) or (13), $f(t)$ given by (11) and the objective function (4).

(i) For $c_1 = c_2 = c > 0$ and $\beta > 1$, a valid version of the universal portfolio $\boldsymbol{b}_{n+1}$ generated by $f(t)$ is given by

$$b_{n+1,j} = \frac{b_{ni}\left\{\left[\frac{1}{2d_1}\left(v_i + \sqrt{v_i^2 - 4d_1d_2}\right)\right]^{\frac{1}{\beta}} - c\right\}}{\sum_{j=1}^{m} b_{nj}\left\{\left[\frac{1}{2d_1}\left(v_j + \sqrt{v_j^2 - 4d_1d_2}\right)\right]^{\frac{1}{\beta}} - c\right\}}, \tag{17}$$

for $i = 1, 2, \cdots, m$ where v_i is given by (7) and $\left[\frac{1}{2d_1}\left(v_i + \sqrt{v_i^2 - 4d_1 d_2}\right)\right]^{\frac{1}{\beta}} > c$ for selected values of v_i and c.

(ii) For $\beta = 1$, a valid version of the universal portfolio $\boldsymbol{b}_{n+1}$ generated by $f(t)$ is given by

$$b_{n+1,i} = \frac{b_{ni}\left\{\begin{array}{c}\frac{1}{2d_1}[v_i - d_1(c_1 + c_2)] \\ +\frac{1}{2d_1}\sqrt{\{v_i - d_1(c_1 + c_2)\}^2 - 4d_1[d_1 c_1 c_2 + d_2 - v_i c_2]}\end{array}\right\}}{\sum_{j=1}^{m} b_{nj}\left\{\begin{array}{c}\frac{1}{2d_1}[v_j - d_1(c_1 + c_2)] \\ +\frac{1}{2d_1}\sqrt{\{v_j - d_1(c_1 + c_2)\}^2 - 4d_1[d_1 c_1 c_2 + d_2 - v_j c_2]}\end{array}\right\}} \quad (18)$$

for $i = 1, 2, \cdots, m$, where the numerator of (18) is positive for selected values of v_i, c_1 and c_2.

***Proof*:** (i) The universal portfolio $\boldsymbol{b}_{n+1}$ generated by the pseudo f-divergence (2) is given by

$$f'(t) = g'(t) = d_1(t + c)^{\beta} + d_2(t + c)^{-\beta} = v_i \quad (19)$$

for $i = 1, 2, \cdots, m$ from (11), (12), (5) and (7) where $t = \frac{b_{n+1,i}}{bni}$.

Simplifying the equation (19),

$$d_1(t + c)^{2\beta} - v_i(t + c)^{\beta} + d_2 = 0.$$

Solving the quadratic in $(t + c)$ and taking the positive root,

$$t = \left[\frac{1}{2d_1}\left(v_i + \sqrt{v_i^2 - 4d_1 d_2}\right)\right]^{\frac{1}{\beta}} - c. \quad (20)$$

Normalizing (20) leads to (17).

(ii) The universal portfolio $\boldsymbol{b}_{n+1}$ generated by (2) is given by

$$f'(t) = g'(t) = d_1(t + c) + d_2(t + c_2)^{-1} = v_i \quad (21)$$

for $i = 1, 2, \cdots, m$ from (11), (13), (5) and (7) where $t = \frac{b_{n+1,i}}{b_{ni}}$.

Simplifying (21),

$$d_1(t + c_1)(t + c_2) + d_2 - v_i(t + c_2) = 0$$

or

$$d_1 t^2 + [d_1(c_1 + c_2) - v_i]t + [d_1 c_1 c_2 + d_2 - v_i c_2] = 0,$$

for $i = 1, 2, \cdots, m$.

Solving the quadratic in t and normalizing $b_{n+1,i}$, the universal portfolio (18) is obtained.

IV. Empirical Results

Table 1 gives the list of Malaysian Companies selected from Kuala Lumpur Stock Exchange for the empirical study. The data is collected for the period 3rd January 2005 until 4th September 2015, which consists of 2500 trading days. The stock-price data consists of five sets J, K, L, M and N.

The universal portfolio (6) generated by the f-disparity differences in Proposition 3.1 is run over data sets J, K, L, M and N. The wealth achieved after 2500 trading days are listed in Table 3. Table 3 gives the accumulated wealth S_{2500} after 2500 trading days for selected values of the parameters η and ξ together with the final portfolio $\boldsymbol{b}_{2501}$. The best wealth is obtained for data set M while the lowest wealth is obtained for data set N. The empirical result from Table 3 shows that data sets J and M are good performing portfolios achieving maximum wealth of 16.2445 and 19.9981 respectively. Table 3 also shows that data sets K, L and N are poor portfolios, achieving maximum wealth of 7.9554, 7.1999 and 5.4044 respectively. The fourth stock of data set J and third stock of data set M are performing well in the market. Hence the portfolios assign more weights on these stocks and lead to higher wealth return. Equivalently, the first, second and fifth stock for both data sets J and L are not performing well. Thus, the portfolios assign lower weights are assigned to them.

Table 2 shows the wealth achieved by the Type 1 Helmbold universal portfolio (see [4]). A comparison of the performance between Type 1 Helmbold universal portfolio and universal portfolio (6) is done. The results from Table 2 and Table 3 show that the universal portfolio (6) performs slightly better for the data set L and N while the Type 1 Helmbold universal portfolio performs better for the data set K. There is no significant differences between the performance of these two universal portfolios for the data sets J and M.

Table 1: List of Malaysian companies in data sets J, K, L, M and N

Data Set	Portfolio of Five Malaysian Companies
J	Public Bank, Nestle Malaysia, Telekom Malaysia, Eco World Development Group, Gamuda
K	AMMB Holding, Air Asia, Encorp, IJM Corp, Genting Plantations
L	Alliance Financial Group, DiGi.com, KSL Holdings, IJM Corp, Kulim Malaysia
M	Hong Leong Bank, DiGi.com, Eco World Development Group, Zecon, United Malacca
N	RHB Capital, Carlsberg Brewery Malaysia, KSL Holdings, Crest Building Holdings, Kulim Malaysia

Table 2

The wealth S_{2500} achieved after 2500 trading days by running the Type 1 Helmbold universal portfolio with parameter γ (see [4]) over data sets J, K, L, M, N for selected values of γ together with the final portfolio $\boldsymbol{b}_{2501}$

Set	γ	S_{2500}	b_1	b_2	b_3	b_4	b_5
	0.3	15.4957	0.126763	0.127786	0.114715	0.513119	0.117617
	0.4	16.1562	0.103203	0.104335	0.090455	0.608522	0.093485
J	0.5	16.30937	0.082917	0.084071	0.070421	0.689242	0.073349
	0.6	15.99526	0.066127	0.067247	0.054443	0.755045	0.057138
	0.7	15.30857	0.052549	0.053598	0.041955	0.807542	0.044356
	-4.2	18.57109	0.068924	0.226932	0.646187	0.057511	0.000447
	-4.1	18.6742	0.080392	0.263809	0.587355	0.067859	0.000585
K	-4	18.71009	0.092274	0.301252	0.526939	0.078782	0.000754
	-3.9	18.67303	0.10426	0.338003	0.466762	0.09002	0.000956
	-3.8	18.5588	0.11605	0.372854	0.40859	0.10131	0.001196
	-2	4.42198	0.261983	0.009207	0.504003	0.198672	0.026135
	-1.9	4.436701	0.268802	0.011158	0.481837	0.208073	0.03013
L	-1.8	4.444895	0.274631	0.013474	0.460337	0.216948	0.03461
	-1.7	4.44681	0.279396	0.016213	0.439609	0.225169	0.039612
	-1.6	4.442872	0.283043	0.01944	0.419736	0.232612	0.045169
	0.3	19.11034	0.114547	0.168995	0.489913	0.108615	0.117929
	0.4	19.74197	0.089628	0.150149	0.581495	0.085567	0.093161
M	0.5	19.97019	0.068846	0.130784	0.661432	0.066695	0.072243
	0.6	19.80705	0.0522	0.112291	0.728585	0.051626	0.055297
	0.7	19.30544	0.03923	0.095428	0.783594	0.039797	0.041952
	-2.3	4.992359	0.045748	0.199536	0.601143	0.13437	0.019203
	-2.2	5.00734	0.050722	0.207542	0.575474	0.144158	0.022104
N	-2.1	5.012886	0.056009	0.214906	0.549948	0.153793	0.025344
	-2	5.008729	0.061599	0.22155	0.524765	0.16314	0.028947
	-1.9	4.994809	0.067479	0.227411	0.500107	0.172066	0.032937

Table 3: The wealth S_{2500} obtained by running universal portfolio generated by f-disparity differences on Stock J, K, L, M, N for 2500 days for selected value of η and ξ with the final portfolio $\boldsymbol{b}_{2501}$

Set	η	ξ	S_{2500}	b_1	b_2	b_3	b_4	b_5
	-0.15	0.33	16.0048	0.02865	0.08488	0.02322	0.81565	0.04760
	-0.15	0.29	15.9767	0.02290	0.06843	0.01953	0.84713	0.04202
	-0.13	0.32	16.1824	0.03737	0.11062	0.03061	0.75952	0.06188
	-0.12	0.22	15.9622	0.02256	0.06779 0.06	0.02005	0.84527	0.04432
J	-0.11	0.2	16.0118	0.02269	0.06826	0.02034	0.84356	0.04515
	-0.10	0.18	15.9716	0.02264	0.06818	0.02045	0.84310	0.04562
	-0.09	0.16	15.8157	0.02238	0.06744	0.02037	0.84413	0.04568
	-0.08	0.29	15.7066	0.06334	0.18797	0.05473	0.58490	0.10906
	-0.04	0.07	16.2445	0.02347	0.07089	0.02184	0.83444	0.04939
	-0.02	0.26	15.7233	0.09227	0.27546	0.08636	0.37454	0.17138
	0.38	-0.27	7.8305	0.06429	0.47703	0.15911	0.27890	0.02066
	0.39	-0.28	7.8439	0.06286	0.48230	0.15671	0.27849	0.01964
	0.4	-0.28	7.8766	0.06788	0.48139	0.14453	0.28575	0.02045
	0.4	-0.29	7.8421	0.06142	0.48794	0.15379	0.27817	0.01867
	0.41	-0.29	7.9185	0.06660	0.48522	0.14350	0.28524	0.01944
K	0.42	-0.3	7.9446	0.06534	0.48932	0.14200	0.28484	0.01849
	0.43	-0.31	7.9554	0.06409	0.49371	0.14004	0.28455	0.01760
	0.44	-0.32	7.9511	0.06286	0.49837	0.13764	0.28436	0.01676
	0.45	-0.33	7.9318	0.06167	0.50331	0.13478	0.28426	0.01598
	0.46	-0.34	7.8964	0.06054	0.50854	0.13135	0.28432	0.01526
L	-0.57	0.69	5.6261	0.00672	0.23427	0.66861	0.02889	0.06152
	-0.49	0.57	5.5615	0.00312	0.29812	0.63980	0.01759	0.04136

	-0.48	0.58	5.6194	0.00838	0.22587	0.66632	0.03506	0.06437
	-0.34	0.41	5.4893	0.01130	0.23297	0.63815	0.04460	0.07298
	-0.05	0.06	7.1999	0.00424	0.07740	0.87411	0.01709	0.02715
	0.45	-0.28	5.2723	0.16843	0.01741	0.23279	0.53960	0.04176
	0.47	-0.31	5.2772	0.17115	0.01624	0.22957	0.53703	0.04600
	0.48	-0.32	5.2868	0.17126	0.01499	0.23087	0.53805	0.04483
	0.49	-0.33	5.2950	0.17134	0.01378	0.23198	0.53913	0.04377
	0.5	-0.34	5.3016	0.17139	0.01254	0.23290	0.54033	0.04284
	-0.18	0.35	19.8899	0.01059	0.08481	0.83751	0.04457	0.02252
	-0.18	0.34	19.6881	0.00967	0.07827	0.84074	0.05077	0.02056
	-0.17	0.32	19.9981	0.01019	0.07982	0.82945	0.05892	0.02163
	-0.17	0.35	19.4965	0.01387	0.10514	0.80774	0.04388	0.02938
M	-0.16	0.34	19.5251	0.01628	0.11697	0.78280	0.04958	0.03436
	-0.12	0.29	19.8063	0.02725	0.15833	0.67550	0.08218	0.05673
	-0.09	0.24	19.4608	0.03716	0.18689	0.58611	0.11324	0.07661
	-0.07	0.19	19.7307	0.04101	0.19383	0.55213	0.12884	0.08419
	-0.04	0.11	19.5588	0.04570	0.20329	0.51215	0.14542	0.09344
	0.09	-0.02	19.9003	0.07633	0.09295	0.03334	0.65396	0.14342
	0.43	-0.29	5.3227	0.03614	0.29255	0.18894	0.42942	0.05294
	0.44	-0.31	5.3005	0.03573	0.27274	0.19327	0.43815	0.06012
	0.45	-0.32	5.3329	0.03518	0.27126	0.19496	0.43748	0.06112
	0.45	-0.33	5.2841	0.03592	0.25827	0.19806	0.42891	0.07883
	0.46	-0.33	5.3663	0.03469	0.27003	0.19664	0.43606	0.06259
N	0.46	-0.34 ..34- 0.34	5.3036	0.03539	0.25607	0.19851	0.42569	0.08434
	0.47	-0.34	5.4044	0.03432	0.26940	0.19855	0.43294	0.06480
	0.47	-0.35	5.3193	0.03480	0.25335	0.19838	0.42115	0.09231
	0.48	-0.36	5.3283	0.03409	0.24952	0.19725	0.41438	0.10476
	0.49	-0.37	5.3221	0.03302	0.24288	0.19389	0.40276	0.12745

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