IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 15, Issue 2 Ser. III (Mar – Apr 2019), PP 49-54
www.iosrjournals.org

I_{λ_2} −Statistical Convergence of Double Sequence of Order α in Topological Groups

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Abstract: *We shall in analogy to the recently introduced notion of I_λ −statistical convergence of order α; introduce and study, I_{λ_2} −statistical convergence for double sequence of order α in topological groups and also establish some inclusion theorems.*

Keywords and Phrases: *Filter, I_2-Statistical Convergence, Double sequences, Topological groups.*
2010 Mathematics subject classification: Primary 40F05, 40J05, 40G05

Date of Submission: 10-04-2019 Date of acceptance: 25-04-2019

I. Introduction

The concept of statistical convergence was formally introduced by Fast [13] and Schoenberg [36] independently. Although statistical convergence was introduced over fifty years ago, it has become an active area of research in recent years. It has been applied in various areas such as summability theory (Fridy [15] and Salat [33]), topological groups (Cakalli [3], [4]), topological spaces (Di Maio and Kocinac [11]), locally convex spaces (Maddox [26]), measure theory (Cheng et al [8], Connor and Swardson [9] and Miller [27]), Fuzzy Mathematics (Nuray and Savas [31] and Savas [34]). In recent years generalization of statistical convergence has appeared in the study of strong summability and the structure of ideals of bounded functions, (Connor and Swardson [9]). Kostryrko et al., [23]) further extended the idea of statistical convergence to I−convergence using the notion of ideals of $\mathbb{N}$ with many interesting consequences. Das and Savas [10] introduced and studied I−statistical and I−lacunary statistical convergence of order α. Brono *et al.*, [2] introduced and studied the concept I_2- statistical and I_2 −Lacunary statistical convergence for double sequence of order α. Also Mursaleen [29] introduced λ −statistical convergence for real sequence, Hazarika and Savas [22] introduced and studied the notion of λ −statistical convergence in n −normed spaces. Quite recently, Brono and Ali [1] introduced and studied the concept of λ_2 −statistical convergence in 2n-normed spaces. Savas [35] introduced and studied the notion of I_λ −statistical convergence in topological groups. In this paper in analogy to Savas [35], we shall introduce and study I_{λ_2}−statistical convergence for double sequence of order α in topological groups.

Let $K \subseteq \mathbb{N} \times \mathbb{N}$ be a two-dimensional set of positive integers and let $K(n,m)$ be the numbers of (i,j) in K such that $i \leq n$ and $j \leq m$. Then the two-dimensional analogue of natural density can be defined as follows.

The lower asymptotic density of a set $K \subseteq \mathbb{N} \times \mathbb{N}$ is defined as

$$\underline{\delta_2}(K) = \liminf_{n,m} \frac{K(n,m)}{nm}$$

In case the sequence $\left(\frac{K(n,m)}{nm}\right)$ has a limit in Pringsheim's sense then we say that K has a double natural density and is defined as

$\lim_{n,m} \frac{K(n,m)}{nm} = \delta_2(K)$.

For example, let $K = \{(i^2, j^2): i,j \in \mathbb{N}\}$.Then,

$\delta_2 = \lim_{n,m} \frac{K(n,m)}{nm} \leq \lim_{n,m} \frac{\sqrt{n}\sqrt{m}}{nm} = 0$,

i.e., the set K has double natural density zero, while the set $K = \{(i, 2j): i,j \in \mathbb{N}\}$ has double natural density $\frac{1}{2}$.

Note that, if we set $n = m$, we have a two dimensional natural density considered by Christopher [7].

Statistical analogue of double sequences $x = (x_{jk})$ was defined as follows.

Definition 1.1: ([30]): A real double sequence $x = (x_{jk})$ is statistically convergent to a number l if for each $\varepsilon > 0$, the set

$$\{(j,k), j \leq n \text{ and } k \leq m : |x_{jk} - l| \geq \varepsilon\}$$

has double natural density zero. In this case we write $st_2 - \lim_{jk} x_{jk} = l$ and we denote the set of all statistically convergent double sequences by st_2.
Let $\lambda = (\lambda_m)$ be a non decreasing sequence of positive numbers tending to ∞ such that
$\lambda_{m+1} \leq \lambda_m + 1, \lambda_1 = 1$.
The collection of such sequences λ will be denoted by Δ.
The generalized de la Vallèe −Poussin mean is defined by

$$t_m(x) = \frac{1}{\lambda_m} \sum_{k \in I_m} x_k,$$

Where $I_m = [m - \lambda_m + 1, m]$.

Definition 1.2 ([24]): A sequence is $x = (x_k)$ is said to be (V, λ)–summable to a number ℓ if

$$t_m(x) \to \ell, \quad as \ m \to \infty.$$

If, $\lambda_m = m$, then (V, λ)–summability reduces to $(C, 1)$ −summability. We write

$$[C,1] = \left\{ x = (x_k) : \exists\, \ell \in \mathbb{R}, \ \lim_{m \to \infty} \frac{1}{m} \sum_{k=1}^{m} |x_k - \ell| = 0 \right\}$$

$$[V,\lambda] = \left\{ x = (x_k) : \exists\, \ell \in \mathbb{R}, \ \lim_{m \to \infty} \frac{1}{\lambda_m} \sum_{k \in I_m} |x_k - \ell| = 0 \right\}$$

For the sets of sequences $x = (x_k)$ which are strongly Cesáro summable (see [14]) and strongly (V, λ) −summable to ℓ, i.e. $(x_k) \xrightarrow{[C,1]} \ell$ and $(x_k) \xrightarrow{[V,\lambda]} \ell$ respectively.

Definition 1.3 ([29])**:** A sequence $x = (x_k)$ is said to be λ −statistcially convergent or S_λ −convergent to ℓ if for every $\varepsilon > 0$

$$\lim_{m \to \infty} \frac{1}{\lambda_m} |\{k \in I_m : |x_k - \ell| \geq \varepsilon\}| = 0.$$

In this case we write

$$S_\lambda - \lim x = \ell \ or \ (x_k) \xrightarrow{S_\lambda} \ell \ and$$
$$S_\lambda = \{x = (x_k) : \exists \ell \in \mathbb{R}, \ S_\lambda - \lim x = \ell\}.$$

It is clear that if $\lambda_m = m$, then S_λ is same as St.
Analogously;
Let $\lambda_2 = (\lambda_{mn})$ be a non decreasing double sequence of positive numbers tending to ∞ such that
$\lambda_{(m+1)(n+1)} \leq \lambda_{mn} + 1, \lambda_{11} = 1$.
The collection of such double sequences will be denoted by Δ_2.
The generalized de la Vallée −Poussin mean for double sequences will defined by

$$t_{mn}(x) = \frac{1}{\lambda_{mn}} \sum_{j,k \in I_{m,n}} \sum x_{jk},$$

Where $I_{m,n} = [mn - \lambda_{mn} + 1, mn]$.

Definition 1.2.1: A double sequence $x = (x_{jk})$ is said to be (V, λ_2) −summable to a number ℓ, if $t_{mn} \to \ell$, as $m, n \to \infty$.
If, $\lambda_{mn} = mn$, then (V, λ_2) −summability reduces to $(C, 1.1)$ −summability. We write

$$(C, 1.1) = \left\{ x = (x_{jk}) : \exists \ell \in \mathbb{R}, \lim_{m,n} \frac{1}{mn} \sum_{j=1}^{m} \sum_{k=1}^{n} |x_{jk} - \ell| = 0 \right\} \ and$$

$$[V, \lambda_2] = \left\{ x = (x_{jk}) : \exists \ell \in \mathbb{R}, \lim_{m,n} \frac{1}{\lambda_{mn}} \sum_{j \in I_m}^{m} \sum_{k \in I_n}^{n} |x_{jk} - \ell| = 0 \right\}$$

For the sets of double sequences $x = (x_{jk})$ which are strongly Cesáro summable (see [29]) and strongly (V, λ_2) −summable to ℓ, that is $x_{jk} \xrightarrow{[C,1.1]} \ell$ and $x_{jk} \xrightarrow{[V,\lambda_2]} \ell$ respectively.

Definition 1.3.1: A double sequence $x = (x_{jk})$ is said to be λ_2 −statistically convergent or S_{λ_2} −convergent to ℓ if for every $\varepsilon > 0$,

$$\lim_{m,n\to\infty} \frac{1}{\lambda_{mn}} \left|\{(j,k) \in I_{m,n} : |x_{jk} - \ell| \geq \varepsilon\}\right| = 0$$

In which case we write $S_{\lambda_2} - \lim x = \ell$ or$(x_{jk}) \xrightarrow{S_{\lambda_2}} \ell$ and

$$S_{\lambda_2} = \{x = (x_{jk}) : \exists\, \ell \in \mathbb{R}, S_{\lambda_2} - \lim x = \ell\}.$$

It is clear that if $\lambda_{mn} = mn$, then, S_{λ_2} is same as St_2

The main purpose of this article is to introduce and study, I_{λ_2}−statistical convergence for double sequence of order α in topological groups and to give some important inclusion theorems.

II. Definitions and Preliminaries

The following definitions will be required in the sequel.

Definition 2.1: If X is a non-empty set then a family of set $I \subset P(X)$ is called an ideal in X if and only if (i) $\Phi \in I$; (ii) for each $A, B \in I$ we have $A \cup B \in I$; (iii) for each $A \in I$ and $B \subset A$ we have $B \in I$.

Definition 2.2: Let X is a non-empty set. A non-empty family of sets $F \subset P(X)$ is called a filter on X if and only if (i) $\Phi \notin F$; (ii) for each $A, B \in F$ we have $A \cap B \in F$; (iii) for each $A \in F$ and $B \supset A$ we have $B \in F$.

An ideal I is called non-trivial if $I \neq \Phi$ and $X \notin I$.

Definition 2.3: A non-trivial ideal $I \subset P(X)$ is called an admissible ideal in X if and only if it contains all singletons, i.e., if it contains $\{\{x\} : x \in X\}$.

For further study we shall take $X = \mathbb{N}^2$ and I will denote an ideal of subsets of $\mathbb{N}^2$. The following proposition express a relation between the notions of an ideal and a filter.

Proposition 2.1: Let $I \subset P(\mathbb{N}^2)$ be a non- trivial ideal. Then the class $F = F(I) = \{M \subset \mathbb{N}^2 : M = \mathbb{N}^2 - A, for\ some\ A \in I\}$ is a filter on $\mathbb{N}^2$ (we shall call $F = F(I)$ $the\ filter\ associated\ with\ I$).

Definition 2.4: Let $I \subset P(\mathbb{N}^2)$ be a non-trivial ideal in $\mathbb{N}^2$. A double sequence $x = (x_{jk})$ of real numbers is said to be I −convergent to a number L if for each $\varepsilon > 0$ the set $A(\varepsilon) = \{(i,j) \in \mathbb{N}^2 : |x_{jk} - L| \geq \varepsilon\}$ belongs to I.The number L is called the I −limit of the sequence (x_{ij}) and we write $I - \lim_{jk} x_{jk} = L$.

Remark 2.1: If we take, $I = \{E \subset \mathbb{N}^2 : E\ is\ contained\ (\mathbb{N} \times A) \cup (A \times \mathbb{N})where\ A\ is\ a\ finite\ subset\ of\ N\}$. Then $I -$ convergent is equivalent to the usual Pringsheim's convergence.

A double sequence $x = (x_{jk})$ is said to be $I_2 - [V, \lambda_2]$ −summable to ℓ, if

$I_2 - \lim_{m,n} t_{mn}(x) = \ell$ ($t_{mn}(x)$ is the de la Vall$\acute{e}$e poisson mean of $x = (x_{jk})$)

That is for any $\delta > 0$,

$$\left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \left|\lim_{m,n} t_{mn}(x) - \ell\right| \geq \delta\right\} \in I_2$$

If, $I = I_f = \{K \subseteq N : K\ is\ finite\ subset\}$. $I_2 - [V, \lambda_2]$ −summability becomes $[V, \lambda_2]$ Summability.

Definition 2.5: A double sequence $x = (x_{jk})$ is said to be I_2 −statistically convergent to ℓ, if for each

$$\left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \left\{m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn}\left|\{j \leq m, k \leq n : |x_{jk} - \ell| \geq \varepsilon\}\right| \geq \delta\right\}\right\} \in I_2.$$

In this case we write $x_{jk} \to \ell(St(I_2))$. The class of all I_2 −statistically convergent sequences will be denoted by $St(I_2)$.

Definition 2.6: A double sequence $x = (x_{jk})$ is said to be I_{λ_2} −statistically convergent to ℓ or $S_{\lambda_2}(I_2)$ convergent to ℓ if for any $\varepsilon > 0$ and $\delta >$

$$\left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{\lambda_{mn}}\left|\{j \leq m, k \leq n : |x_{jk} - \ell| \geq \varepsilon\}\right| \geq \delta\right\}\right\} \in I_2$$

In this case we write $x_{jk} \to \ell(S_{\lambda_2}(I_2))$. The class of all I_{λ_2} − statistically convergent sequences will be denoted by $S_{\lambda_2}(I_2)$.

Throughout in this article X will denote a topological Hausdorff group, written additively, which satisfies the first axiom of countability. A double sequence $x = (x_{jk})$ in X is said statistically convergent to an element ℓ of X if for each neighbourhood U of 0,

$$\lim_{mn} \frac{1}{mn} \left|\{j \le m, k \le n : x_{jk} - \ell \notin U\}\right| = 0,$$

Where, the vertical bars denote the cardinality of the enclosed set. The set of statistically convergent sequences in X is denoted by $St_2(X)$. Furthermore we define λ_2 −statistically convergence in topological groups as follows:

A double sequence $x = (x_{jk})$ is said to be S_{λ_2} −convergent to ℓ (or λ_2 −statistically convergent to ℓ) if for each neighbourhood U of 0,

$$\lim_{mn} \frac{1}{\lambda_{mn}} \left|\{(j,k) \in I_{m,n} : x_{jk} - \ell \notin U\}\right| = 0.$$

In this case, we write

$S_{\lambda_2} - \lim_{jk} x_{jk} = \ell$ or $x_{jk} \to \ell(S_{\lambda_2})$ and define

$$S_{\lambda_2}(X) = \left\{x = (x_{jk}) : for\ some\ \ell, \quad S_{\lambda_2} - \lim_{jk} x_{jk}\right\}.$$

III. I_{λ_2} − Convergence for double sequence of order α

In this section, we shall introduce and study I_{λ_2} −statistical convergence for double sequences of order α in topological groups and we shall also present some inclusion theorems.

We now have

Definition 3.1: A double sequence $x = (x_{jk})$ in X is said to be statistically convergent of order α to ℓ of X if for each neighbourhood U of 0,

$$\lim_{mn} \frac{1}{(mn)^{\alpha}} \left|\{j \le m, k \le n : x_{jk} - \ell \notin U\}\right| = 0.$$

The set of all statistically convergent of order α sequences in X is denoted by ${St_2}^{\alpha}(X)$.

Also we define λ_2 −statistical convergence of order α in topological groups as follows:

Definition 3.2: A double sequence $x = (x_{jk})$ is said to be S_{λ_2} −convergent of order α to ℓ (or λ_2 −statistically convergent of order α to ℓ) if for each neighbourhood U of 0,

$$\lim_{mn} \frac{1}{\lambda_{mn}^{\alpha}} \left|\{(j,k) \in I_{m,n} : x_{jk} - \ell \notin U\}\right| = 0$$

In this case, we define

$$S_{\lambda_2}^{\alpha}(X) = \left\{x = (x_{jk}) : for\ some\ \ell, S_{\lambda_2}^{\alpha} - \lim_{jk} x_{jk} = \ell\right\}$$

If we take $\lambda_{mn} = mn$, $S_{\lambda_2}^{\alpha}(X)$ reduce to ${St_2}^{\alpha}(X)$:

Now we shall give the definitions of I_2 −statistical convergence and I_{λ_2} −Statistical convergence of order α in topological groups as follows:

Definition 3.3: A double sequence $x = (x_{jk})$ is said to be I_2 −statistically convergent of order α to ℓ or $S^{\alpha}(I_2)$ −convergent to ℓ if for each $\delta > 0$ and each neighbourhood U of 0,

$\left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{(mn)^{\alpha}} \left|\{j \le m, k \le n : x_{jk} - \ell \notin U\}\right| \ge \delta\right\} \in I_2$.

In this case we write $x_{jk} \to \ell(S^{\alpha}(I_2))$. The class of all I_2 −statistically convergent sequences will be denoted by simply $S^{\alpha}(I_2)(X)$.

For, $I = I_f = \{K \subseteq \mathbb{N} : K\ is\ a\ finite\ subset\}$, I_2 −statistical convergence of order α becomes statistical convergence of double sequences of order α in topological groups which has not been study till now. Finally for $I = I_{f,} \lambda_{mn} = mn$ and $\alpha = 1$ it becomes statistical convergence of double in topological groups.

Definition 3.4: A double sequence $x = (x_{jk})$ is said to be I_{λ_2} −statistically convergent of order α to ℓ or $S_{\lambda_2}^{\alpha}$ −convergent of order α to ℓ, if for each $\delta > 0$ and each neighbourhood U of 0

$$\left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{\lambda_2^{\alpha}} \left|\{jk \in I_{mn} : x_{jk} - \ell \notin U\}\right| \ge \delta\right\} \in I_2.$$

In this case, we write

$$S_{\lambda_2}^{\alpha}(I_2)(X) = \left\{x = (x_{jk}) : for\ some\ \ell, S_{\lambda_2}^{\alpha}(I_2) - \lim_{jk} x_{jk} = \ell\right\}$$

If we take $\alpha = 1$, we have

$$S_{\lambda_2}(I_2)(X) = \left\{x = (x_{jk}) : for\ some\ \ell, S_{\lambda_2}(I_2) - \lim_{jk} x_{jk} = \ell\right\}$$

For, $I = I_f,\ I_{\lambda_2}$ −statistical convergence of order α becomes λ_2 −statistical convergence of order α in topological groups which has not been study till now. If $\lambda_{mn} = mn$, I_{λ_2} −statistical convergence of order α becomes double statistical convergence of order α in topological groups

IV. Inclusion Theorems

We shall prove the following theorems

Theorem 4.1: Let $0 < \alpha \le \beta \le 1$. Then $S^{\alpha}_{\lambda_2}(I)(X) \subset S^{\beta}_{\lambda_2}(I)(X)$.

Proof: $0 < \alpha \le \beta \le 1$. Then

$$\frac{\left|\{(j,k) \in I_{m,n} : x_{jk} - \ell \notin U\}\right|}{\lambda^{\beta}_{mn}} \le \frac{\left|\{(j,k) \in I_{m,n} : x_{jk} - \ell \notin U\}\right|}{\lambda^{\alpha}_{mn}}$$

And so for any $\delta > 0$ and any neighbourhood U of 0

$$\left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{\left|\{(j,k) \in I_{m,n} : x_{jk} - \ell \notin U\}\right|}{\lambda^{\beta}_{mn}} \ge \delta\right\} \subset \left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{\left|\{(j,k) \in I_{m,n} : x_{jk} - \ell \notin U\}\right|}{\lambda^{\alpha}_{mn}} \ge \delta\right\}.$$

Hence if the set on the right hand side belongs to the ideal I_2 then obviously the set on the left hand side also belongs to I_2. This shows that $S^{\alpha}_{\lambda_2}(I)(X) \subset S^{\beta}_{\lambda_2}(I)(X)$.

Corollary 4.1: If a double sequence is I_{λ_2} −statistically convergent of order α to ℓ for some $0 < \alpha \le 1$ then, it is, I_{λ_2} −statistically convergent to ℓ i.e. $S^{\alpha}_{\lambda_2}(I) \subset S_{\lambda_2}(I)$.

Similarly we can show that

Theorem 4.2: Let $0 < \alpha \le \beta \le 1$. Then

(i) $St^{\alpha}_2(I) \subset St^{\beta}_2(I)$.

(ii) In particular $St^{\alpha}_2(I) \subset St_2(I)$.

Theorem 4.3: $St^{\alpha}_2(I)(X) \subset S^{\alpha}_{\lambda_2}(I)(X)$ if $\lim_{mn} inf \frac{\lambda^{\alpha}_{mn}}{(mn)^{\alpha}} > 0$

Proof: Let us take any neighbourhood U of 0. Then,

$$\frac{1}{(mn)^{\alpha}}\left|\{j \le m, k \le n : x_{jk} - \ell \notin U\}\right| \ge \frac{1}{mn}\left|\{(j,k) \in I_{m,n} : x_{jk} - \ell \notin U\}\right|$$
$$\ge \frac{\lambda^{\alpha}_{mn}}{(mn)^{\alpha}} \frac{1}{\lambda^{\alpha}_{mn}}\left|\{(j,k) \in I_{m,n} : x_{jk} - \ell \notin U\}\right|.$$

If $\lim_{mn} inf \frac{\lambda^{\alpha}_{mn}}{(mn)^{\alpha}} = a$

Then from definition $\left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{\lambda^{\alpha}_{mn}}{(mn)^{\alpha}} < \frac{a}{2}\right\}$ is finite. For $\delta > 0$ and for each neighbourhood U of 0,

$$\left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{\lambda_{mn}}\left|\{(j,k) \in I_{m,n} : x_{jk} - \ell \notin U\}\right| \ge \delta\right\} \subset$$
$$\left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{1}{mn}\left|\{(j,k) \in I_{m,n} : x_{jk} - \ell \notin U\}\right| \ge \frac{a}{2}\delta\right\} \cup \left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \frac{\lambda^{\alpha}_{mn}}{(mn)^{\alpha}} < \frac{a}{2}\right\}.$$

The set on the right hand side belongs to I_2 and this completed the proof. Finally, we prove the reverse inclusion state in theorem 4.3

Theorem 4.4: If $\lambda_2 \in \Delta_2$ be such that for a particular α, $0 < \alpha < 1$, $\lim_{mn} \frac{mn - \lambda_{mn}}{(mn)^{\alpha}} = 0$ then $S^{\alpha}_{\lambda_2}(I)(X) \subset St_2(I)^{\alpha}(X)$.

Proof: Let $\delta > 0$ be given. Since $\lim_{mn} \frac{\lambda_{mn}}{mn} = 1$, we can choose $m,n \in \mathbb{N} \times \mathbb{N}$ such that $\left|\frac{\lambda^{\alpha}_{mn}}{(mn)^{\alpha}} - 1\right| < \frac{\delta}{2}$, for all$m,n \ge p$. Let us take any neighbourhood U of 0. Now

$$\frac{1}{(mn)^\alpha}\left|\{j \le m, k \le n: x_{jk} - \ell \notin U\}\right| \frac{1}{mn}\left|\{j \le m, k \le n: x_{jk} - \ell \notin U\}\right| + \frac{1}{(mn)^\alpha}\left|\{(j,k) \in I_{m,n}: x_{jk} - \ell \notin U\}\right|$$
$$\le \frac{mn - \lambda_{mn}}{(mn)^\alpha} + \frac{1}{mn}\left|(j,k) \in I_{m,n}: x_{jk} - \ell \notin U\right|$$
$$\le 1 - \left(1 - \frac{\delta}{2}\right) + \frac{1}{(mn)^\alpha}\left|\{(j,k) \in I_{m,n}: x_{jk} - \ell \notin U\}\right|$$
$$= \frac{\delta}{2} + \frac{1}{(mn)^\alpha}\left|\{(j,k) \in I_{m,n}: x_{jk} - \ell \notin U\}\right|,$$

For all $m, n \ge p$. Hence for $\delta > 0$ and for each neighbourhood U of 0,

$$\left\{(m,n) \in \mathbb{N} \times \mathbb{N}: \frac{1}{(mn)^\alpha}\left|\{j \le m, k \le n: x_{jk} - \ell \notin U\}\right| \ge \delta\right\}$$
$$\subset \left\{(m,n) \in \mathbb{N} \times \mathbb{N}: \frac{1}{(mn)^\alpha}\left|\{j \le m, k \le n: x_{jk} - \ell \notin U\}\right| \ge \frac{\delta}{2}\right\} \cup \{1,2,3,4,5, \dots p\}.$$

If $S_{\lambda_2}^I - lim x_{jk} = \ell$ then the set on the right hand side belongs to I_2 and also the set on the left hand side also belongs to I_2. This shows that $x = (x_{jk})$ is I_2 −statistically convergent to ℓ.

References

[1]. A. M. Brono and A. G. K. Ali, $\lambda 2$−Statistical convergence in 2n-normed spaces. Far East Journal of Mathematical Sciences, 99(2016), 1551-1569.
[2]. A. M. Brono, A. G. K. Ali and M. T. Bukar, I_2 −statistical and I_2 −lacunary statistical convergence for double sequence of order α. IOSR Journal of Mathematics, 13(2017), 107-112.
[3]. H. Cakalli. On statistical convergence in topological groups, Pure and Applied Mathematical Sciences, 43(1996), 27-31.
[4]. H. Cakalli, A study on statistical convergence. *Funct. Anal. Approx. Comput.*, *1*(2009), 19-24.
[5]. A. Caserta, G. Di Maio and Lj. D. R. Koćinac, Statistical convergence in function spaces, Abstr. Appl. Anal. (2011).
[6]. A. Caserta, and Lj. D. R. Koćinac, On statistical exhaustiveness, Appl. Math. Letters, in press. 25(2012), 1447–1451.
[7]. J. Christopher, "The asymptotic density of some *k*-dimensional sets," *The American Mathematical Monthly*, 63(1956), 399–401.
[8]. L. X. Cheng, G. C. Ling, Y. Y. Lan and H. Liu, Measure theory of statistical convergence, Science in China, Ser. A: Math. 51(2008), 2285-2303.
[9]. J. Connor and M. A. Swardson, Measures and ideals of $C*(X)$, Ann. N.Y. Acad. Sci. 704(1993), 80-91.
[10]. P. Das and E. Savas, On I −Statistical and I −Lacunary Statistical Convergence of order α. Bulleting of Iranian Mathematical Society, 40(2014), 459-472.
[11]. Di. Maio, G. et al., Statistical convergence in topology, Topology Appl. 156(2008), 28-45.
[12]. H. Dutta, On sequence spaces with elements in a sequence of real linear *n*-normed spaces, Appl. Math. Letters, 23(2010), 1109-1113.
[13]. H. Fast, Sur la convergence statistique, Colloq. Math. 2(1951), 241-244.
[14]. A. R. Freedman, J. J. Semba and M. Raphael, Some Cesáro-type summability spaces, Proc. London Math. Soc., 37(1978), 508-520.
[15]. J. A. Fridy, On statistical convergence, Analysis, 5(1985), 301-313.
[16]. S. Gähler, 2 −metrische R¨aume and ihretopologischeStruktur, Math.Nachr. 26(1963), 115-148.
[17]. S. Gähler, Linear 2 −normietre Raume, Math. Nachr. 28, (1965), 1-43.
[18]. H. Gunawan, The spaces of *p*-summable sequences and its natural *n*-norm, Bull. Austral. Math. Soc. 64, (2001), 137-147.
[19]. H. Gunawan and M. Mashadi, On *n*-normed spaces, Int. J. Math. Math. Sci. 27(2001), 631-639.
[20]. H. Gunawan and M. Mashadi, On finite dimensional 2-normed spaces, Soochow J. Math. 27(2001), 147-169.
[21]. M. Gürdal and S. Pehlivan, Statistical convergence in 2-normed spaces, South. Asian Bull. Math. 33(2009), 257-264.
[22]. B. Hazarika and E. Savas. λ −statistical convergence in n-normed spaces.An. St. Univ. Ovidius Constanta, 21(2013), 141-153.
[23]. P. Kostryrko, T. Šalát and W. Wilczynki, I −convergence, Real Anal. Exchange, 26(2000/2001), 669-685.
[24]. L. Leindler, Uber die de la Vall´ee-Pousinsche Summierbarkeit allgenmeiner Othogonalreihen, Acta Math. Acad. Sci. Hungar, 16(1965), 375-
387.
[25]. Z. Lewandowska, On 2 −normed sets, Glas. Math., 38(2003), 99- 110.
[26]. I. J. Maddox, Statistical convergence in a locally convex spaces, Math. Proc. Cambridge Philos. Soc., 104(1988), 141-145.
[27]. H. I. Miller, A measure theoretical subsequence characterization of statistical convergence, Trans. Amer. Math. Soc., 347(1995), 1811-1819.
[28]. A. Misiak, n −inner product spaces, Math. Nachr. 140 (1989), 299-329.
[29]. M. Mursaleen, λ-statistical convergence, Math. Slovaca, 50(2000),111-115.
[30]. M. Mursaleen and O. H. Edely. On statistical convergence of double sequence.Journal of Mathematical Analysis and Application. 288(2003), 223-231.
[31]. F. Nuary and E. Savas. Statistical convergence of sequences of fuzzy numbers, Math. Slovaca, 45(1995), 269-273.
[32]. B. S. Reddy, Statistical convergence in n −normed spaces, Int. Math. Forum, 5(2010), 1185-1193.
[33]. T. Salat, On statistical convergence of real numbers, Math. Slovaca, 30(1980), 139-150.
[34]. E. Savas, Statistical convergence of fuzzy numbers, Inform. Sci., 137(2001), 277-282.
[35]. E. Savas, I_λ −statistical convergence of order α in topological groups. GeneralLetters in Mathematics (GLM), 1(2)(2016), 64-73.
[36]. I. J. Schoenberg, The integrability of certain functions and related summability methods, Amer. Math. Monthly 66 (1959), 361-375.

A. M. BRONO. "I_(λ_2)-Statistical Convergence of Double Sequence of Order α in Topological Groups." IOSR Journal of Mathematics (IOSR-JM) 15.2 (2019): 49-54.