IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 15, Issue 1 Ser. II (Jan – Feb 2019), PP 12-18
www.iosrjournals.org

A Proposed Method for Solving Quasi-Concave Quadratic Programming Problems by Multi-Objective Technique with Computer Algebra

Zahidul Islam Sohag[1], M. Asadujjaman[2]
[1](*Mathematics Department, University of Dhaka, Bangladesh)*
[2](*Mathematics Department, University of Dhaka, Bangladesh)*
Corresponding Author: Zahidul Islam Sohag

Abstract:*In this paper a new method is proposed to solve Quasi-Concave Quadratic Programming problems in which the objective function is in the form of product of two linear functions and constraints functions are in the linear inequalities form. In this method we convert the problem into Multi-Objective Linear Programming problem by splitting those two linear functions and considering them as different maximize/ minimize (depending on main objective function type) type linear objective functions under same constraints and then solve the problem by Chandra Sen's method. For developing this method, we use programming language MATLAB 2017. To demonstrate our propose method, numerical examples are also illustrated.*

Date of Submission: 22-02-2019 Date of acceptance: 08-03-2019

I. Introduction

Non-linear programming is an essential part of operations research. Non-linear programming (NLP) is the process of solving an optimization problem defined by a system of equalities and inequalities, collectively termed constraints, over a set of unknown real variables, along with an objective function to be maximized or minimized, where some of the constraints or the objective function is non-linear. It is the sub-field of mathematical optimization that deals with problems that are not linear. Non-Linear Programming problems are concerned with the efficient use or allocation of limited resources to meet desired objectives. In different sectors like design, construction, maintenance, producing planning, financial and corporate planning and engineering, decision makers have to take decisions and their ultimate goal is to minimize effort or maximize profit. A quadratic programming (QP) is a special type of mathematical optimization problem. It is the problem of optimizing (minimizing or maximizing) a quadratic function of several variables subject to linear constraints on these variables. A large number of algorithms for solving QP problems have been developed. Some of them are extensions of the simplex method and others are based on different principles. In the conversance, a great number of methods (Wolfe[1], Beale[2], Frank and Wolfe[3], Shetty[4], Lemke[5], Best and Ritter[6], Theil and van de Panne[7], Boot[8], Fletcher[9], Swarup[10], Gupta and Sharma[11], Moraru[12, 13], Jensen and King[14], Bazaraa, Sherali and Shetty[15]) are designed to solve QP problems in a finite number of steps. Among them, Wolfe's method[1], Swarup's simplex method[10] and Gupta and Sharma's method[11] are more popular than the other methods. Jayalakshmi and Pandian[16] suggested a method to solve Quadratic Programming problems having linearly factorized objective function.

In order to extend this work, in this paper we propose and algorithm to solve Quasi-Concave Quadratic Programming (QCQP) problems. In this method we convert our problem into Multi-Objective Linear Programming (MOLP) problem[17] and solve this MOLP problem using Chandra Sen's Method[18]. We develop a computer technique for this method by using programming language MATLAB 2017. We also illustrate numerical examples to demonstrate our method.

II. Quadratic Programming Problems

The general QP problem can be written as

$$\text{Maximize } Z = cX + \frac{1}{2}X^TQX$$
$$\text{Subject to:} AX \leq b \text{ and } X \geq 0$$

Where c is an n-dimensional row vector describing the coefficients of the linear terms in the objective function, and Q is a$(n \times n)$ symmetric real matrix describing the coefficients of the quadratic terms. If a constant term exists it is dropped from the model. As in LP, the decision variables are denoted by the n-dimensional column vector X, and the constraints are defined by an $(m \times n)A$ matrix and an m-dimensional column vector b of right-hand side coefficients. We assume that a feasible solution exists and that the constraints region is bounded. When the objective function Z is strictly convex for all feasible points the problem has a unique local maximum which is also the global maximum. A sufficient condition to guarantee strictly convexity is for Q to be positive definite.

III. Quasi-Concave Quadratic Programming Problems

The quasi-concave quadratic programming (QCQP) problem subject to linear constraints with a quadratic objective function which is multiplication of factorize form of two linear functions. QCQP problem can be written as:

$$\text{Maximize } Z = (cX + \alpha)(dX + \beta)$$
$$\text{Subject to: } \quad AX \le b \text{ and } X \ge 0$$

where, A is an $(m \times n)$ matrix, $b \in \mathbb{R}^m$, and $X, c, d \in \mathbb{R}^n$ and $\alpha, \beta \in \mathbb{R}$. Here we assume that

(i)$(cX + \alpha)$and$(dX + \beta)$are positive for all feasible solution.

(ii) The constraints set $S = \{ X : AX = b, X \ge 0 \}$is non-empty and bounded.

Let us assume that $(cX + \alpha), (dX + \beta) > 0$ for all $x = (x_1, x_2, \cdots\cdots, x_m, x_{m+1}, \cdots\cdots, x_n)^T \in S$, where S denotes a feasible set defined by the constraints. Also assume that S is non-empty.

IV. Mathematical Formulation of MOLP Problems

Mathematical general form of MOLP problem is given as:

$$\begin{aligned} &Max\ Z_1 = C_1^t X + \alpha_1 \\ &Max\ Z_2 = C_2^t X + \alpha_2 \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &Max\ Z_r = C_r^t X + \alpha_r \\ &Min\ Z_{r+1} = C_{r+1}^t X + \alpha_{r+1} \\ &Min\ Z_{r+2} = C_{r+2}^t X + \alpha_{r+2} \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &Min\ Z_s = C_s^t X + \alpha_s \end{aligned} \qquad \cdots\cdots\cdots\cdots (1)$$

Subject to:

$$AX \le b$$
$$X \ge 0$$

Where, X is n-dimensional and b is m-dimensional vectors. A is $m \times n$matrix. $\alpha_1, \alpha_2, \dots\dots.., \alpha_s$are scalars. Here, Z_i is need to be maximized for $i = 1,2, \dots\dots\dots, r$ and need to be minimized for $i = r + 1, \dots\dots., s$.

V. Chandra Sen's Method

In this method, firstly all objective functions need to be maximized or minimized individually by Simplex method. By solving each objective function of equation (1) following equations are obtained:

$$\begin{aligned} &Max\ Z_1 = \varphi_1 \\ &Max\ Z_2 = \varphi_2 \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &Max\ Z_r = \varphi_r \\ &Min\ Z_{r+1} = \varphi_{r+1} \\ &Min\ Z_{r+2} = \varphi_{r+2} \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &Min\ Z_s = \varphi_s \end{aligned}$$

Where, $\varphi_1, \varphi_2, \dots\dots\dots.., \varphi_s$ are the optimal values of objective functions.

These values are used to form a single objective function by adding (for maximum) and subtracting (for minimum) of each result of dividing each Z_i by φ_i.Mathematically,

$$Max\ Z = \sum_{i=1}^{r} \frac{Z_i}{|\varphi_i|} - \sum_{i=r+1}^{s} \frac{Z_i}{|\varphi_i|}$$

Where, $|\varphi_i| \ne 0$.

Subject to the constraints are remain same as equation (1). Then this single objective linear programming problem is optimized.

VI. Mathematical Formulation of Proposed Method

Our proposed method is all out splitting the factor form of objective function of QCQP problem then solving it as MOLP problem. The general QCQP problem

$$\text{Max} \quad Z = Z_1(x) \cdot Z_2(x) = \left(\sum_{j=1}^{n} c_j x_j + \alpha\right) \cdot \left(\sum_{j=1}^{n} d_j x_j + \beta\right)$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,\cdots\cdots,m$$

Let us assume that $Z_1(x)$, $Z_2(x) > 0$ for all $x = (x_1, x_2, \cdots\cdots, x_m, x_{m+1}, \cdots\cdots, x_n)^T \in S$, where S denotes a feasible set defined by the constraints. Also assume that S is non-empty.

Here, both $Z_1(x)\&Z_2(x)$ are linear. $Z$is the product of $Z_1(x)\&Z_2(x)$. So, Z will be maximize when we get maximized value of $Z_1(x)\&Z_2(x)$. So, after splitting the objective function we will get two maximum type linear objective functions. Then the form of the problem will become

$$\text{Max} \; Z_1(x) = \left(\sum_{j=1}^{n} c_j x_j + \alpha\right)$$

$$\text{Max} \; Z_2(x) = \left(\sum_{j=1}^{n} d_j x_j + \beta\right)$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,\cdots\cdots,m$$

Which is the form of MOLP problem. Now we will apply Chandra Sen's Method. So, firstly we need to maximize every objective function individually under same constraints. Let, the maximum value of $Z_1(x) = \varphi_1$ and the maximum value of$Z_2(x) = \varphi_2$. Now fromChandra sen's method, the combined objective function will be

$$\text{Max} Z_c = \frac{Z_1(x)}{|\varphi_1|} + \frac{Z_2(x)}{|\varphi_2|}$$

So the problem becomes,

$$\text{Max} \; Z_c = \frac{Z_1(x)}{|\varphi_1|} + \frac{Z_2(x)}{|\varphi_2|}$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,2,\cdots\cdots,m$$

Now we need to solve this problem to obtain the solution of the main problem.

VII. Algorithm of Proposed Method

Step 1: Split the objective function into two linear maximum type objective function.
Step 2: Optimize those objective functions for performing Chandra Sen's method.
Step 3: Construct a single objective function by adding objective functions dividing by their modulus optimized value respectively.
Step 4: Perform simplex method to optimize the converted single objective function.
Step 5: Calculate the optimal value of the main problem using the result obtaining from the converted MOLP problem.

VIII. Numerical Example

Consider the following QCQP problem:

$$\text{Max} \; Z = (2x_1 + 4x_2 + x_3 + 1) \cdot (6x_1 + x_2 + 2x_3 + 2)$$

Subject to: $x_1 + 3x_2 \le 15$

$$2x_1 + x_2 \le 20$$
$$x_2 + 4x_3 \le 28$$

$$x_1, x_2, x_3 \geq 0$$

Now we will split the objective function into two maximum type objective functions. Then the problem becomes

Max $Z_1 = 2x_1 + 4x_2 + x_3 + 1$

Max $Z_2 = 6x_1 + x_2 + 2x_3 + 2$

Subject to: $x_1 + 3x_2 \leq 15$

$$2x_1 + x_2 \leq 20$$
$$x_2 + 4x_3 \leq 28$$
$$x_1, x_2, x_3 \geq 0$$

Now taking,

$$\text{Max } Z_1 = 2x_1 + 4x_2 + x_3 + 1$$

Subject to: $x_1 + 3x_2 \leq 15$

$$2x_1 + x_2 \leq 20$$
$$x_2 + 4x_3 \leq 28$$
$$x_1, x_2, x_3 \geq 0$$

Now the standard form will be,

$$\text{Max } Z_1 = 2x_1 + 4x_2 + x_3 + 1$$

Subject to: $x_1 + 3x_2 + s_1 = 15$

$$2x_1 + x_2 + s_2 = 20$$
$$x_2 + 4x_3 + s_3 = 28$$
$$x_1, x_2, x_3 \geq 0$$

Table 1: Initial table of 1st objective function

C_B	C_j	2	4	1	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
0	s_1	1	3	0	1	0	0	**15**	5→
0	s_2	2	1	0	0	1	0	**20**	20
0	s_3	0	1	4	0	0	1	**28**	28
$\bar{C}_j = C_j - Z_j$		2	4↑	1	0	0	0	$Z_1 = 1$	

Table 2

C_B	C_j	2	4	1	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
4	x_2	1/3	1	0	1/3	0	0	**5**	-
0	s_2	5/3	0	0	-1/3	1	0	**15**	-
0	s_3	-1/3	0	4	-1/3	0	1	**23**	23/4→
$\bar{C}_j = C_j - Z_j$		2/3	0	1↑	-4/3	0	0	$Z_1 = 21$	

Table 3

C_B	C_j	2	4	1	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
4	x_2	1/3	1	0	1/3	0	0	**5**	15
0	s_2	5/3	0	0	-1/3	1	0	**15**	9→
1	x_3	-1/12	0	1	-1/12	0	1/4	**23/4**	-
$\bar{C}_j = C_j - Z_j$		3/4↑	0	0	-5/4	0	-1/4	$Z_1 = 107/4$	

Table 4: Optimal table of 2nd objective function

C_B	C_j	2	4	1	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
4	x_2	0	1	0	2/5	-1/5	0	**2**	-
2	x_1	1	0	0	-1/5	3/5	0	**9**	-
1	x_3	0	0	1	-1/10	1/20	1/4	**13/2**	-
$\bar{C}_j = C_j - Z_j$		0	0	0	-11/10	-9/20	-1/4	$Z_1 = 67/2$	

Since all $(C_j - Z_j) \leq 0$ in **Table 4**, this table gives the optimal solution. So, $|Z_1| = \frac{67}{2}$.

Again, Max $Z_2 = x_1 + x_2 + 2x_3 + 2$

Subject to: $x_1 + 3x_2 \leq 15$

$$2x_1 + x_2 \leq 20$$
$$x_2 + 4x_3 \leq 28$$
$$x_1, x_2, x_3 \geq 0$$

Now the standard form will be,

$$\text{Max } Z_2 = 6x_1 + x_2 + 2x_3 + 2$$

Subject to: $x_1 + 3x_2 + s_1 = 15$

$$2x_1 + x_2 + s_2 = 20$$
$$x_2 + 4x_3 + s_3 = 28$$
$$x_1 , x_2 , x_3 \geq 0$$

Table 5: Initial table of 2nd objective function

C_B	C_j	6	1	2	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
0	s_1	1	3	0	1	0	0	**15**	15
0	s_2	2	1	0	0	1	0	**20**	10→
0	s_3	0	1	4	0	0	1	**28**	-
$\bar{C}_j = C_j - Z_j$		6↑	1	2	0	0	0	$Z_2 = 2$	

Table 6

C_B	C_j	6	1	2	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
0	s_1	0	5/2	0	1	-1/2	0	**5**	-
6	x_1	1	1/2	0	0	1/2	0	**10**	-
0	s_3	0	1	4	0	0	1	**28**	7→
$\bar{C}_j = C_j - Z_j$		0	-2	2↑	0	-3	0	$Z_2 = 62$	

Table 7: Optimal table of 2nd objective function

C_B	C_j	6	1	2	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
0	s_1	0	5/2	0	1	-1/2	0	**5**	-
6	x_1	1	1/2	0	0	1/2	0	**10**	-
2	x_3	0	1/4	1	0	0	1/4	**7**	-
$\bar{C}_j = C_j - Z_j$		0	-5/2	0	0	-3	-2	$Z_2 = 76$	

Since all $(C_j - Z_j) \leq 0$ in **Table 7**, this table gives the optimal solution. So, $|Z_2| = 76$.
Now the single objective function will be,

$$\text{Max } Z_c = \frac{Z_1(X)}{|Z_1|} + \frac{Z_2(X)}{|Z_2|}$$
$$= \frac{2x_1 + 4x_2 + x_3 + 1}{67/2} + \frac{6x_1 + x_2 + 2x_3 + 2}{76}$$
$$= \frac{4x_1 + 8x_2 + 2x_3 + 2}{67} + \frac{6x_1 + x_2 + 2x_3 + 2}{76}$$
$$= \frac{353}{2546}x_1 + \frac{675}{5092}x_2 + \frac{143}{2546}x_3 + \frac{143}{2546}$$
$$\therefore \text{Max } Z_c = \frac{353}{2546}x_1 + \frac{675}{5092}x_2 + \frac{143}{2546}x_3 + \frac{143}{2546}$$

Subject to: $x_1 + 3x_2 \leq 15$

$$2x_1 + x_2 \leq 20$$
$$x_2 + 4x_3 \leq 28$$
$$x_1 , x_2 , x_3 \geq 0$$

Now the standard form will be,

$$\text{Max } Z_c = \frac{353}{2546}x_1 + \frac{675}{5092}x_2 + \frac{143}{2546}x_3 + \frac{143}{2546}$$

Subject to: $x_1 + 3x_2 + s_1 = 15$

$$2x_1 + x_2 + s_2 = 20$$
$$x_2 + 4x_3 + s_3 = 28$$
$$x_1 , x_2 , x_3 \geq 0$$

Table 8: Initial table of converted single objective function

C_B	C_j	353/2546	675/5092	143/2546	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
0	s_1	1	3	0	1	0	0	**15**	15
0	s_2	2	1	0	0	1	0	**20**	10→
0	s_3	0	1	4	0	0	1	**28**	-
$\bar{C}_j = C_j - Z_j$		353/2546↑	675/5092	143/2546	0	0	0	Z_c = **143/2546**	

Table 9

C_B	C_j	353/2546	675/5092	143/2546	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
0	s_1	0	5/2	0	1	-1/2	0	**5**	2→
353/2546	x_1	1	1/2	0	0	1/2	0	**10**	20
0	s_3	0	1	4	0	0	1	**28**	28
$\bar{C}_j = C_j - Z_j$		0	161/2546↑	143/2546	0	-353/5092	0	Z_c = **3673/2546**	

Table 10

C_B	C_j	353/2546	675/5092	143/2546	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
675/5092	x_2	0	1	0	2/5	-1/5	0	**2**	-
353/2546	x_1	1	0	0	-1/5	3/5	0	**9**	-
0	s_3	0	0	4	-2/5	1/5	1	**26**	13/2
$\bar{C}_j = C_j - Z_j$		0	0	143/2546	-161/6365	-1443/25460	0	Z_c = **1.56912**	

Table 11: Optimal table of converted single objective function

C_B	C_j	353/2546	675/5092	143/2546	0	0	0	X_B	Ratio
	Basis	x_1	x_2	x_3	s_1	s_2	s_3		
675/5092	x_2	0	1	0	2/5	-1/5	0	**2**	-
353/2546	x_1	1	0	0	-1/5	3/5	0	**9**	-
143/2546	x_3	0	0	1	-1/10	1/20	1/4	**13/2**	-
$\bar{C}_j = C_j - Z_j$		0	0	0	-501/25460	-3029/50920	-143/10184	Z_c = **147/76**	

Since all $(C_j - Z_j) \leq 0$ in **Table 11**, this table gives the optimal solution.

Now, $x_1 = 9$, $x_2 = 2$ and $x_3 = \frac{13}{2}$.

So, the optimal solution of our main problem is Max Z= 2378.5 and $(x_1, x_2, x_3) = \left(9, 2, \frac{13}{2}\right)$

IX. Computer Code for solving QCQP Problems

In this section, we use MATLAB programming language to solve our QCQP problems. There is a built in command "**quadprog**" to solve quadratic programming problems. But here we present a code according to our proposed algorithm and compare result and elapsed time with the existing command.

```
%QCQP Problem Solving Using Our ProposedAlgorithm
tic;
f1=[-2;-4;-1];          %first factors coefficients
f2=[-6;-1;-2];          %second factors coefficients
C=[1;2];                %constant of factors
A=[1 3 0;2 1 0;0 1 4];  % matrix for linear inequality constraints
b=[15;20;28];           % vector for linear inequality constraints
lb=[0;0;0];             % vector of lower bounds
[p,xval1] = linprog(f1,A,b,[],[],lb,[])
F1=abs(-xval1+C(1));
[q,xval2] = linprog(f2,A,b,[],[],lb,[])
F2=abs(-xval2+C(2));
fnew=f1/F1+f2/F2;       %New objective function
x = linprog(fnew,A,b,[],[],lb,[]);
fprintf('\n Optimal solutions are: \n')
fprintf('\n x1=%15.7f\n',x(1))
fprintf('\n x2=%15.7f\n',x(2))
fprintf('\n x3=%15.7f\n',x(3))
max= (-f1'*x+C(1))*(-f2'*x+C(2));
fprintf('\n Maximum value is %15.7f\n',max)
toc;
```

Output

```
Optimal solutions are:
x1=      9.0000000
x2=      2.0000000
x3=      6.5000000
 Maximum value is    2378.4999982
Elapsed time is 0.022649 seconds.
```

Table 12: Comparison

Methods	Execution Time (Seconds)	Result (Value of Z)	Comment
Existing Command	0.041086	2378.4999982	Our proposed method take less time than existing method
Code according to Our Proposed Method	0.022649	2378.4999982	

X. Conclusion

The goal of the research is to develop a simple technique to solve QCQP problems. So, we proposed a new method involving multi-objective technique. We illustrate numerical example to demonstrate our method. We also use MATLAB code according to our algorithm and compare with existing command. Though the result is identical to the existing one but elapsed time is lesser. We therefore hope that our proposed method for solving QCQP problems can be used as an effective tool for solving QCQP problems and hence our time and labor can be saved.

References

[1]. Wolfe, P., 1959. The simplex method for quadratic programming, Econometrica, 27, 3. 382-398.
[2]. Beale, E.M.L., 1959. On quadratic programming, Naval Research Logistics Quarterly, 6, 227-243.
[3]. Frank, M., P. Wolfe, 1956. An algorithm for quadratic programming, Naval Research Logistics Quarterly, 3, 95-110.
[4]. Shetty C.M., 1963. A simplified procedure for quadratic programming, Operations Research, 11, 248-260.
[5]. Lemke C.E., 1965. Bi-matrix equilibrium points and mathematical programming, Management Science, 11, 681-689.
[6]. Best M.J., K. Ritter K, 1988. A quadratic programming algorithm, Zeitschrift for Operational Research,32, 271-297.
[7]. Theil H., C. Van de Panne, 1961. Quadratic programming as an extension of conventional quadratic maximization, Management Science, 7, 1-20.
[8]. J.C.G., Boot, 1961. Notes on quadratic programming: The kuhn-Tucker and Theil-van de Panne conditions, degeneracy and equality constraints, Management Science, 8, 85-98.
[9]. Fletcher R., 1971. A general quadratic programming algorithm, J. Inst., Maths. Applics, 7, 76-91.
[10]. Swarup K., 1966. Quadratic programming, CCERO (Belgium), 8, 132-136.
[11]. Gupta A.K., J.K. Sharma, 1983. A generalized simplex technique for solving quadratic programming problem, Indian Journal of Technology, 21, 198-201.
[12]. Moraru V., 1997. An algorithm for solving quadratic programming problems, Computer science Journal of Moldova, 5, 223-235.
[13]. Moraru V., 2000. Primal-dual method for solving convex quadratic programming problems, Computer science Journal of Moldova, 8, 209-220.
[14]. Jensen D.L., A.J. King, 1992. A decomposition method for quadratic programming, IBM Systems Journal, 31, 39-48.
[15]. Bazaraa M., H. Sherali, C.M. Shetty, 2006. Nonlinear programming: Theory and algorithm (John Wiley, New York).
[16]. Jayalakshmi M., Pandian P., 2014, A method for solving quadratic programming problems having linearly factorized objective function, International Journal Of Modern Engineering Research, 4, 20-24.
[17]. Zahidul Islam Sohag, Md. Asadujjaman. A proposed new average method for solving multi-objective linear programming problem using various kinds of mean techniques. Mathematics Letters. Vol. 4, No. 2, 2018, pp. 25-33. doi: 10.11648/j.ml.20180402.11
[18]. Sen. Chandra, A new approach for multi-objective rural development planning, The Indian Economic Journal, Vol. 30, No.4, PP.91-96, 1983.
[19]. M. Asadujjaman, M.B. Hasan, A proposed technique for solving quasi-concave quadratic programming problems with bounded variables, The Dhaka University Journal of Science, 63(2):111-117, 2015 (July).
[20]. M. Asadujjaman, M.B. Hasan, A proposed technique for solving quasi-concave quadratic programming problems with bounded variables by objective separable Method, The Dhaka University Journal of Science, 64(1): 51-58, 2016 (January).

Zahidul Islam Sohag."A Proposed Method for Solving Quasi-Concave Quadratic Programming Problems by Multi-Objective Technique with Computer Algebra." IOSR Journal of Mathematics (IOSR-JM) 15.1 (2019): 12-18.