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Homomorphism on Finite Group Automata and Homomorphic Images of Finite Group Automata

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Abstract : *Let B = (Q, *, Σ, δ, q_0, F) and B = (Q', Δ , Σ, δ', q_0', F') be two Finite Group Automata. Then a mapping Ψ : B → B' is said to be a Finite Group Automata Homomorphism or simply FGA Homomorphism if 1.Ψ(a*b) = Ψ(a) Δ Ψ(b), 2.Ψ(δ(a,n)) = δ' (Ψ(a),n), 3.Ψ(q_0) = q_0', 4.a ϵ F if and only if Ψ(a) ϵ F'. Let B = (Q, *, Σ, δ, q_0, F) and B' = (Q', Δ , Σ, δ', q_0', F') be two Finite Group Automata. Let f : B → B' be a FGA homomorphism. Then*
(1) if S is a Finite Subgroup Automata of B, then f(S) is a Finite Subgroup Automata of B'.
(2) If B is Finite Abelian Group Automaton, then f(B) is Finite Abelian Subgroup Automaton of B.
(3) If N is a Finite Normal Subgroup Automaton of B, then f(N) is a Finite Normal Subgroup Automaton of B'.
Keywords: *Finite Group Automata, Finite Group Automata Homomorphism (FGA Homomorphism)*

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I. Introduction

Finite Binary Automata, Finite Semigroup Automata, Finite Monoid Automata, Finite Group Automata, and Finite Subgroup Automata were defined and many results were obtained. Commutative Finite Binary Automata, Associative Finite Binary Automata were defined. AC Finite Binary Automata was also defined. Many useful results were obtained.

Now we define a homomorphism on Finite Group Automata. We study the homomorphic images of Finite Group Automata.

II. Preliminaries

Finite Group Automaton: A Finite Group Automaton B is a 6-tuple (Q, *, Σ, δ, q_0, F), where Q is a finite set of elements called states, Σ is a subset of non-negative integers, q_0 ϵ Q, q_0 is a state in Q called the initial state, F⊆Q and the set states (element) of F is said to be the set of final states, $\delta : Q\times\sum \rightarrow Q$ is the transition function defined by δ (q, n) = q^n = q * q *q *.......*q (n times) and * is a mapping from Q×Q to Q satisfying the following conditions.
(i) p * (q * r) = (p * q) * r , for all p,q,r in Q.
(ii) there exists a state denoted by 0 in Q such that p * 0 = p = 0 * p , for all p in Q
(iii) for each state p in Q there exists a state q in Q such that p * q = 0 = q * p.
Note : For n = 0, δ (q, n) = q^n => δ (q, 0) = q^0 , it is taken as 0

Definition : If for a state p in Q there exists a state q in Q such that p * q = 0 = q * p, then the state q is called the inverse state and the state p is called a invertible state in Q.
If a state p is invertible in Q and p * q = 0 = q * p, then the state q is also invertible.
If Σ* is the set of strings of inputs, then the transition function δ is extended as follows :
For m ϵ Σ* and n ϵ Σ, δ': Q×Σ* → Q is defined by δ'(q,mn) = δ(δ'(q,m),n).
If no confusion arises δ' can be replaced by δ.

Finite Sub-group Automaton: Let B= (Q, *, Σ, δ, q_0, F) be a Finite Group Automaton, where Q is a finite set of states, * is a mapping from Q×Q to Q, Σ is a finite set of integers, q_0 in Q is the initial state and F⊆Q is the set of final states and δ is the transition function mapping from Q×Σ to Q defined by δ(q,n) = q^n. A Finite Sub-group Automaton S of B is a 6-tuple (R, *, E, γ, q_s, T), where R⊆Q for all p,q ϵ R, p * q ϵ R, q_s ϵ R is the initial state where q_s = q_0 or q_s = δ(q_0,n) for some n϶∑, , E is the set of all n in Σ such that n ≤ m for all m ϵ Σ, ie , E =

$\{n \in \Sigma \ / \ n \leq m$, for some $m \in \Sigma\}$, γ is the restriction function of δ restricted to $R \times E \rightarrow R$, q_0 in R is the initial state and $T \subseteq R$ and $T \subseteq F$.

Finite Normal Subgroup Automata of B : Let B= (Q, *, Σ, δ, q_0, F) be a finite Group Automaton. Let S = (R, *, E, γ, q_s, T) be a Finite Sub-group Automaton of B. Then S = (R, *, E, γ, q_s, T) is said to be a Finite Normal Subgroup Automata of B if $a * S = S * a$, for all $a \in Q$.

Definition : Let B = (Q, *, Σ, δ, q_0, F) and B = (Q', Δ , Σ, δ', q_0', F') be two Finite Group Automata. Then a mapping Ψ : B → B' is said to be a **Finite Group Automata Homomorphism** or simply **FGA**

Homomorphism if

1. $\Psi(a*b) = \Psi(a) \Delta \Psi(b)$
2. $\Psi(\delta(a,n)) = \delta' (\Psi(a),n)$
3. $\Psi(q_0) = q_0'$
4. $a \in F$ if and only if $\Psi(a) \in F'$.

Theorem : Let B = (Q, *, Σ, δ, q_0, F) and B' = (Q', Δ , Σ, δ', q_0', F') be two Finite Group Automata. Let f : B → B' be a FGA homomorphism. If S is a Finite Subgroup Automata of B, then f(S) is a Finite Subgroup Automata of B'.

Proof : Let B = (Q, *, Σ, δ, q_0, F) and B' = (Q', Δ , Σ, δ', q_0', F') be two Finite Group Automata.
Let f : B → B' be a FGA homomorphism.
Let S = (S, *, E, γ, q_s, T) be a Finite Sub-group Automaton of B , where $S \subseteq Q$, for all $p,q \in S$, $p * q \in S$,
$q_s \in S$ is the initial state where $q_s = q_0$ or $q_s = \delta(q_0,n)$ for some $n \in \sum$,
E is the set of all n in Σ such that $n \leq m$ for some $m \in \Sigma$, ie ,$E = \{n \in \Sigma \ / \ n \leq m$, for some $m \in \Sigma\}$,
γ is the restriction function of δ restricted to $S \times E \rightarrow S$, and $T \subseteq S$ and $T \subseteq F$.
Consider f(S).
Clearly $f(S) \subseteq Q'$
Let $p', q' \in f(S)$.
Then there exist $p, q \in S$ such that f(p) = p' and f(q) = q'.
Now p' Δ q' = f(p) Δ f(q)
= f(p * q)
Since S is a Finite Sub-group Automaton and $p, q \in S$, $p * q \in S$
Therefore, $f(p * q) \in f(S)$.
$q_s \in S$ is the initial state where $q_s = q_0$ or $q_s = \delta(q_0,n)$ for some $n \in \sum$
Therefore, $f(q_s) \in f(S)$ and $f(q_s)$ is the initial state of f(S) where $f(q_s)= f(q_0) = q_0'$ or $f(q_s)= \delta'(q_0',n)$ for some $n \in \sum$
E is the set of all n in Σ such that $n \leq m$ for some $m \in \Sigma$, ie , $E = \{n \in \Sigma \ / \ n \leq m$, for some $m \in \Sigma\}$
Let γ' be the restriction function of δ' restricted to $f(S) \times E \rightarrow f(S)$,
Now $f(T) \subseteq Q'$ and $f(T) \subseteq F'$.
Hence, f(S) = (f(S), Δ, E, γ', $f(q_s)$, f(T)) is a Finite Subgroup Automata of B'.

Theorem : Let B = (Q, *, Σ, δ, q_0, F) and B' = (Q', Δ , Σ, δ', q_0', F') be two Finite Group Automata. Let f : B → B' be a FGA homomorphism. If B is Finite Abelian Group Automaton, then f(B) is Finite Abelian Subgroup Automaton of B,

Proof : Let B = (Q, *, Σ, δ, q_0, F) and B' = (Q', Δ , Σ, δ', q_0', F') be two Finite Group Automata.
Let f : B → B' be a FGA homomorphism.
Suppose B is Finite Abelian Group Automaton.
By the previous theorem f(B) is a Finite Subgroup Automaton of B'.
Let $a', b' \in f(B)$;
Then there exist states (elements) $a, b \in B$ such that f(a) = a' and f(b) = b'
Now a' Δ b' = f(a) Δ f(b)
= f(a * b)
= f(b * a) (since B is an Abelian Finite Group Automata)
= f(b) Δ f(a)
= b' Δ a'
Hence f(B) is Finite Abelian Subgroup Automata of B'

Theorem : Let B = (Q, *, Σ, δ, q_0, F) and B' = (Q', Δ , Σ, δ', q_0', F') be two Finite Group Automata. Let f : B → B' be a FGA homomorphism of B onto B'. If N is a Finite Normal Subgroup Automaton of B, then f(N) is a Finite Normal Subgroup Automaton of B'.

Proof : Let B = (Q, *, Σ, δ, q_0, F) and B' = (Q', Δ , Σ, δ', q_0', F') be two Finite Group Automata.
Let f : B → B' be a FGA homomorphism.
Let N be a Finite Normal Subgroup Automaton of B.
Then for each a ϵ Q, a * N = N * a
Since N is a Finite Subgroup Automaton of B, f(N) is a Finite Subgroup Automaton of B'.
Let a' ϵ B'
Now consider a' Δ f(N).
Let x ϵ a' Δ f(N)
Then x = a' Δ f(m), for some m ϵ N
Since a' ϵ B' and f : B → B' is onto, there exists a ϵ B such that f(a) = a'.
Now x = a' Δ f(m)
= f(a) Δ f(m)
= f(a * m)
= f(m * a) (since N is a Normal Finite Normal Subgroup Automaton of B)
= f(m) Δ f(a)
= f(m) Δ a' ϵ f(N) Δ a'
ie x ϵ f(N) Δ a'
Therefore, a' Δ f(N) ⊆ f(N) Δ a'…………(1)
Conversely, let x ϵ f(N) Δ a'
Then x = Then x = f(m) Δ a', for some m ϵ N
Since a' ϵ B' , there exists a ϵ B such that f(a) = a'.
Now x = f(m) Δ a'
= f(m) Δ f(a)
= f(m * a)
= f(a * m) (since N is a Normal Finite Normal Subgroup Automaton of B)
= f(a) Δ f(m)
= a'Δ f(m) ϵ a' Δ f(N)
ie x ϵ a' Δ f(N)
Therefore, f(N) Δ a' ⊆ a' Δ f(N) …………(2)
Therefore from (1) and (2) a' Δ f(N) = f(N) Δ a'
Hence f(N) is a Finite Normal Subgroup Automaton of B'

III. Conclusion

The theory of Finite Group Automata, Homomorphism on Finite Group Automata and the Homomorphic images of Finite Group Automata will be useful in many areas. Researchers can do wonders in this area of research.

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