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A novel scrutiny on preemptive priority in fuzzy queueing theory

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Abstract: *In this paper we deal with the fuzzy optimization of the mean number of customers and the mean waiting time of a customer in the queue in a preemptive priority discipline with two priority classes where the preemptive units do not return to service but are lost. Poisson arrival, Exponential service time, single server and infinite waiting line are assumed. Fuzzyfying the parameters in the mean number of customers and the mean waiting time of a customer in the queue, optimization is obtained using statistical technique. A Numerical example for fuzzy optimization is illustrated.*

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I. Introduction

Nowadays priority queueing problems play an important role in queueing theory. If r<s then the first came unit in class(r) is selected for service in preference to a unit in class (s). A unit of class (r) cannot go for service until the class (r-1) is empty. Within each class, first-come first-served policy is adopted. When an unit of higher priority arrives, the unit of a lower priority in service may be allowed to complete its service or its service be interrupted immediately and it be displaced. The displaced unit may return to service again or be lost. When it resumes its service again, the time which it already spent in service may be deducted or be allowed to start the service from the beginning. So we consider that the arrival and service rates are independent. [2, 4] Bailey and Barry (1956) introduced the preemptive queueing system and the work was extended by white, Christie and Stephan in (1958). White proposed a difference between preemptive resume and repeat. [3] have given an interesting applications of combinatorics of lattice paths to solve some non-trivial problems in queueing theory. Classical problems like Ma/Mb/I systems with and without global blocking, queueing models related to random walks in a quarter planes like Flatto-Hahn model with preemptive priorities have been discussed.

Later many researchers have stated many results on preemptive priority queueing models [l, 5, 9]. [6, 8,12] stated many results on the elementary model and time based priority queueing system. However most of the time the parameters used is not deterministic because of the nature of the problem. So, the fuzzy concept was introduced in queueing theory to develop the uncertain optimality analysis. [13] Introduced the fuzzy concept in the year 1978. [9] Later, R.J.Lie, E.S.Lee (1989) introduced fuzzy queueing model and investigated fuzzy Erlang queueing model. [7] have proposed a mathematical programming and developed a membership function of the system performance after giving fuzzy triangular numbers for the arrival and service rates of the two priority classes, for the convenience of testing the validity of the proposal. Using α-cuts, by Zadeh's extension principle they have reduced the fuzzy queue to a family of crisp queues. [11] Considered the mathematical model of Preemptive priority as a multidimensional random walk of a limited environment.

II. Description of the preemptive priority model [11]:

The preemptive priority queue discipline considered is a single server model with infinite waiting line and two priority classes (1) and (2) only, where the preempted units do not return to service but are lost. The arrivals in the priority classes (1) and (2) form Poisson processes with arrival rates λ_1 *and* λ_2 respectively. The service times in priority classes (1) and (2) form Exponential distribution with service rates μ_1 and μ_2 respectively. The arrival and service times in each class are independent. The characteristics of each class are described by the corresponding M/M/1 model.

If there are s priority types, the types (1),(2),…(r), r<s are independent of (r+1),(r+2),…(s) on the input process. It behaves similar to a system where only first r priorities are taken. If t_r' is the time spent by an item of class (r), it has the distribution

$$P(t_r' < t) = 1 - P(t_r > t,\ T_1 > t, T_2 > t, \ldots T_{r-1} > t),$$
$$= 1 - P(t_r > t).P(T_1 > t).P(T_2 > t)\ldots P(T_{r-1} > t)$$
$$= 1 - \exp\left[-\left(\mu_r + \sum_{i=1}^{r-1}\lambda_i\right)t\right],$$

Where t_r is the service time of a class (r) item with no interruption in service T_i; i=1, 2, 3, …r-1 is the length of the arriving interval to class(i).

The average time spent in service by a class (r) item = $\dfrac{1}{\mu_r + \sum_{i=1}^{r-1}\lambda_i}$.

The necessary and condition for the existence of steady-state solution is $\sum_{i=1}^{s}\dfrac{\lambda_r}{\mu_r + \sum_{i=1}^{r-1}\lambda_i} < 1$.

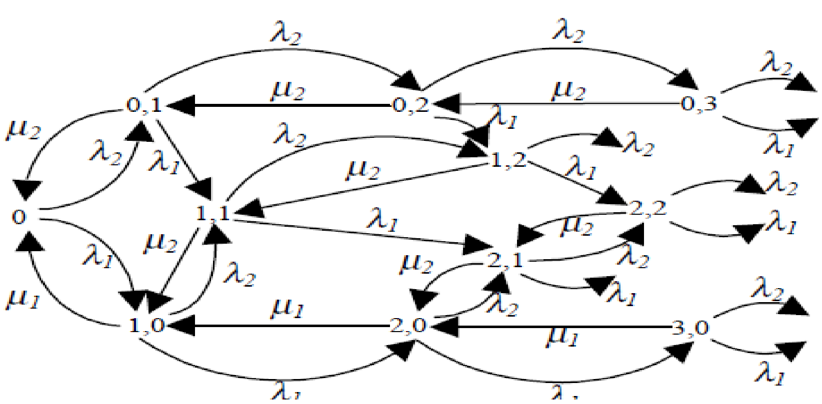
If $\sum_{r=1}^{L}\dfrac{\lambda_r}{\mu_r + \sum_{i=1}^{r-1}\lambda_i} < 1$ and $\sum_{r=1}^{L+1}\dfrac{\lambda_r}{\mu_r + \sum_{i=1}^{r-1}\lambda_i} > 1$ for some L, $1 \le L < s$, then the queue gets saturated with (L+1) class items. The first L Priority classes will be in statistical equilibrium.

If $\rho_1 = \dfrac{\lambda_r}{\mu_r + \sum_{i=1}^{r-1}\lambda_i}$, then $\rho_1 + \rho_2 + \rho_3 + \ldots + \rho_s < 1$, the probability $p_0 = 1 - \rho_1 - \rho_2 - \ldots - \rho_s$ is the stationary probability for no item in any class. Also and $P_r = \rho_r$ is the stationary probability that a class (r) item is in service.

The probability for a class (r) item not to finish its service

$$p\left(\min_{1\le i\le r-1} T_i = t_r\right) = 1 - P(T_1 > t_r, T_2 > t_r, \ldots T_{r-1} > t_r)$$
$$= 1 - \prod_{i=1}^{r-1} P(T_i > b_r)$$
$$= 1 - \prod_{i=1}^{r-1}\int_0^\infty \left(1 - e^{-\mu_r t}\right)\lambda_i e^{-\lambda_i t}\,dt$$
$$= 1 - \prod_{i=1}^{r-1}\frac{\mu_r}{(\mu_r + \lambda_i)}$$

For the Particular case, we have consider s=2 , we get $\rho_1 + \rho_2 = \dfrac{\lambda_1}{\mu_1} + \dfrac{\lambda_2}{\mu_2 + \lambda_1} < 1.$



State Transition diagram for 2-priority M/M/1 Queue with preemptive priority

If p_{nk}, n, k = 0, 1, 2, 3,… are the steady state probabilities that there are n class (1) items and k class (2) item, then the steady state equations are

$$\mu_1 p_{10} + \mu_2 p_{01} - (\lambda_1+\lambda_2)p_{00} = 0$$
$$\lambda_1 p_{00} + \lambda_1 p_{01} + \mu_1 p_{20} - (\lambda_1+\lambda_2+\mu_1)p_{10} = 0$$
$$\lambda_2 p_{0k-1} + \mu_1 p_{1k} + \mu_2 p_{0k+1} - (\lambda_1+\lambda_2+\mu_2)p_{0k} = 0\ ,k>0$$
$$\lambda_1 p_{n-10} + \mu_1 p_{n+10} - (\lambda_1+\lambda_2+\mu_1)p_{n0} = 0\ ,n>1$$
$$\lambda_1 p_{0k+1} + \lambda_2 p_{1k-1} + \mu_1 p_{2k} - (\lambda_1+\lambda_2+\mu_1)p_{1k} = 0, k>0$$
$$\lambda_1 p_{n-1k} + \lambda_2 p_{nk-1} + \mu_1 p_{n+1k} - (\lambda_1+\lambda_2+\mu_1)p_{nk} = 0, n>1, k>0$$

They determine the probabilities p_{nk} with the condition $\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} p_{nk} = 1$

From the balance equations we get the number of customers in the system is,
$L_s = \frac{\rho_1}{1-\rho_1} + \frac{\rho_1\left(\frac{\lambda_2}{\mu_1-\lambda_1}\right)+\rho_2(1-\rho_1)}{(1-\rho_1-\rho_2)} = L^1 + L^2$, and the mean numbers of type(1) items and of type (2) items are $L^1 = \frac{\rho_1}{1-\rho_1}$ and $L^2 = \frac{\rho_1\left(\frac{\lambda_2}{\mu_1-\lambda_1}\right)+\rho_2(1-\rho_1)}{(1-\rho_1-\rho_2)}$ respectively.

Also, we shall denote the waiting time distribution for a class (1) item and the waiting time distribution for a class (2) item respectively by $w^1(t)$ *and* $w^2(t)$.

Then the mean value $\overline{W}$ of the waiting time is $\overline{w} = \frac{1}{\lambda_1+\lambda_2}\left(\frac{\rho_1^2}{1-\rho_1} + \frac{\rho_1\frac{\lambda_2}{\mu_1-\lambda_1}+\rho_2^2}{1-\rho_1-\rho_2}\right)$ where

$w^1(t) = \frac{1}{\lambda_1+\lambda_2}\left(\frac{\rho_1^2}{1-\rho_1}\right)$ and $w^2(t) = \frac{1}{\lambda_1+\lambda_2}\left(\frac{\rho_1\frac{\lambda_2}{\mu_1-\lambda_1}+\rho_2^2}{1-\rho_1-\rho_2}\right)$ which corresponds to the relation

$\overline{W} = \frac{L_q}{(\lambda_1+\lambda_2)}$, where $L_q = L - \rho_1 - \rho_2$ is the expected number of items in the queue.

I. Fuzzy Optimization Procedure

The arrival and service rates of fuzzy queueing parameters denoted by $\overline{\lambda}$ and $\overline{\mu}$ are defined as $\overline{\lambda} = \{(x, \mu_{\overline{\lambda}}(x))/x \in X\}$, $\overline{\mu} = \{(y, \mu_{\overline{\lambda}}(y))/y \in Y\}$ where X & Y are the Universal sets. Using α-cuts, the fuzzy membership functions of $\overline{\lambda}$ and $\overline{\mu}$ are

$$\hat{\lambda}_\alpha = [x_\alpha^l, x_\alpha^u] = \left[\min_{x\in X}\{x/\mu_\lambda(x)\ge\alpha\},\ \max_{x\in X}\{x/\mu_\lambda(x)\ge\alpha\}\right]$$
$$\hat{\mu}_\alpha = [y_\alpha^l, y_\alpha^u] = \left[\min_{y\in Y}\{x/\mu_\mu(y)\ge\alpha\},\ \max_{y\in Y}\{y/\mu_\mu(y)\ge\alpha\}\right]$$

where, $x_\alpha^l, x_\alpha^u, y_\alpha^l$ *and* y_α^u are the lower and upper bounds of the arrival and service rates respectively. The inverse membership functions through α-cuts are given by $\mu_{\hat{\lambda}}(x) = \alpha$ and $\mu_{\hat{\mu}}(y) \ge \alpha$ or $\mu_{\hat{\lambda}}(x) \ge \alpha$ and $\mu_{\overline{\mu}}(y) = \alpha$.Then the average number of customers in the system for class (1) and class (2) are derived from reference [10]. By Little's formula, we find the average number of customers in the queue as

$\mu_{\overline{L_q^U}}(z) = \sup \{\mu_{\bar{\lambda}}(x), \mu_{\bar{\mu}}(y) / z = L_q^1 + L_q^2\}$ and $\mu_{\overline{L_q^l}}(z) = \inf \{\mu_{\bar{\lambda}}(x), \mu_{\bar{\mu}}(y) / z = L_q^1 + L_q^2\}$ where $L_q^1 = \left(\frac{\lambda_1}{\mu_1 - \lambda_1}\right) - \frac{\lambda_1}{\mu_1}$ and $L_q^2 = \frac{\rho_1 \frac{\lambda_2}{\mu_1 - \lambda_1} + \rho_2(1-\rho_1)}{(1-\rho_1-\rho_2)} - \frac{\lambda_2}{\mu_2 + \lambda_1}$. Also the average number of customers waiting in the queue is $\mu_{\overline{W_q^U}}(z) = \sup \{\mu_{\bar{\lambda}}(x), \mu_{\bar{\mu}}(y) / z = W_q^1 + W_q^2\}$ and $\mu_{\overline{W_q^l}}(z) = \inf \{\mu_{\bar{\lambda}}(x), \mu_{\bar{\mu}}(y) / z = W_q^1 + W_q^2\}$ where $W_q^1 = \left(\frac{1}{\lambda_1 + \lambda_2}\right)\left(\frac{\rho_1^2}{1-\rho_1}\right)$ and $W_q^2 = \left(\frac{1}{\lambda_1 + \lambda_2}\right)\left(\frac{\rho_1 \frac{\lambda_2}{\mu_1 - \lambda_1} + \rho_2^2}{(1-\rho_1-\rho_2)}\right)$.Then we find the upper bounds ${}^U L_q^1$ and ${}^U L_q^2$ of L_q^1 and L_q^2 respectively and the lower bounds of ${}^l L_q^1$ and ${}^l L_q^2$ of L_q^1 and L_q^2 respectively with the help of α-cuts and Mixed integer non-linear programming technique. Using Chi-Square test for goodness of fit we optimize L_q^1 , L_q^2 and L_q. Similarly we optimize W_q. Finally we trace the regression line using the α -cut values of L_q and w_q to find the fuzzy optimum values.

III. Numerical Example

Let us consider the system follows the preemptive priority discipline with different possibility level of arrival and service rates. These queue parameters are considered as λ_1=[1 2 3], λ_2=[2 5 6] and μ_1= [10,11,12] and μ_2= [12,14,15]. We Find the expected number of customers in the queue and analyze the performance measures of the system.

Using α-cuts we find the upper and lower bounds of the arrival and service rates as

$[x_\alpha^l, x_\alpha^u]$ = [α+1, 3-α] and [3 α+2, 6- α] $[y_\alpha^l, y_\alpha^u]$ =[α+10, 12-α] and [2α+12, 15-α].

The total number of customers in the queue is $L_q = L_q^1 + L_q^2$,

where $L_q^1 = \frac{\rho_1}{1-\rho_1} = \left(\frac{\lambda_1}{\mu_1 - \lambda_1}\right) - \frac{\lambda_1}{\mu_1}$, and

$$L_q^2 = \frac{\rho_1 \frac{\lambda_2}{\mu_1 - \lambda_1} + \rho_2(1-\rho_1)}{(1-\rho_1-\rho_2)} - \frac{\lambda_2}{\mu_2 + \lambda_1} = \frac{\lambda_1\lambda_2\mu_2 + 2\lambda_1^2\lambda_2 + \mu_1^2\lambda_2 - 2\lambda_1\lambda_2\mu_1}{\mu_1^2(\mu_2 + \lambda_1 - \lambda_2) - 2\lambda_1\mu_1\mu_2 - 2\mu_1\lambda_1^2 + \lambda_1^2\mu_2 + \lambda_1^3 + \lambda_1\lambda_2\mu_1},$$

Applying MILP technique we get the upper and lower bound value of

$${}^l L_q^1 = \left(\frac{\lambda_1}{\mu_1 - \lambda_1}\right) - \frac{\lambda_1}{\mu_1} == \frac{\lambda_1(\mu_1 - \mu_1 + \lambda_1)}{\mu_1(\mu_1 - \lambda_1)} = \frac{\lambda_1^2}{\mu_1(\mu_1 - \lambda_1)} = \frac{(\alpha+1)^2}{(11-2\alpha)(12-\alpha)} = \frac{\alpha^2 + 2\alpha + 1}{2\alpha^2 - 35\alpha + 121}$$

Similarly upper bound of ${}^U L_q^1 = \left(\frac{\lambda_1}{\mu_1 - \lambda_1}\right) - \frac{\lambda_1}{\mu_1} = \frac{(3-\alpha)^2}{(7+2\alpha)(\alpha+10)} = \frac{\alpha^2 - 6\alpha + 9}{2\alpha^2 + 27\alpha + 70}$

$${}^l L_q^2 = \frac{\lambda_1\lambda_2\mu_2 + 2\lambda_1^2\lambda_2 + \mu_1^2\lambda_2 - 2\lambda_1\lambda_2\mu_1}{\mu_1^2(\mu_2 + \lambda_1 - \lambda_2) - 2\lambda_1\mu_1\mu_2 - 2\mu_1\lambda_1^2 + \lambda_1^2\mu_2 + \lambda_1^3 + \lambda_1\lambda_2\mu_1} = \frac{2\alpha^3 - 24\alpha^2 + 275\alpha + 274}{-4\alpha^3 + 39\alpha^2 - 904\alpha + 1672}$$

$${}^U L_q^2 = \frac{-3\alpha^3 + 2\alpha^2 + 2\alpha + 564}{6\alpha^3 + 13\alpha^2 + 437\alpha + 297},$$

such that α+1< λ_1 < 3-α, 3α+2 < λ_2 < 6-α, α+10< μ_1<12-α and 2α+12< μ_2 < 15-α

Thus the interval values of Lq are

$$\left[{}^l L_q^1, {}^l L_q^2\right] = \left[\frac{\alpha^2 + 2\alpha + 1}{2\alpha^2 - 35\alpha + 121}, \quad \frac{2\alpha^3 - 24\alpha^2 + 275\alpha + 274}{-4\alpha^3 + 39\alpha^2 - 904\alpha + 1672}\right]$$ and

$$\left[{}^{U}L_q^1, {}^{U}L_q^2\right] = \left[\frac{\alpha^2-6\alpha+9}{2\alpha^2+27\alpha+70}, \quad \frac{-3\alpha^3+2\alpha^2+2\alpha+564}{6\alpha^3+13\alpha^2+437\alpha+297}\right]$$

Table: 1 11 values for the α cuts of the performance measures are as follows:

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
${}^{l}L_q$	0.08607	0.10037	0.11620	0.13383	0.15358	0.17586	0.201195	0.23027	0.264	0.30362	0.35087
${}^{U}L_q$	1.01378	0.88552	0.785055	0.70406	0.63725	0.58111	0.53322	0.491825	0.455655	0.42375	0.39536

Table: 2 Chi-Square test table

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\frac{\left({}^{l}L_q - \overline{{}^{l}L_q}\right)^2}{\overline{{}^{l}L_q}}$	0.05872	0.04398	0.03015	0.01780	0.00781	0.001414	0.000407	0.00748	0.02669	0.06437	0.13064
$\frac{\left({}^{U}L_q - \overline{{}^{U}L_q}\right)^2}{\overline{{}^{U}L_q}}$	0.23719	0.10573	0.03935	0.0092454	0.00014	0.00348	0.014268	0.02948	0.047236	0.066359	0.08610

To find the value of number of customers waiting in the Queue

The Expected number of customers waiting in the queue is $W_q = W_q^1 + W_q^2$,

where $W_q^1 = \left(\frac{1}{\lambda_1+\lambda_2}\right)\left(\frac{\rho_1^2}{1-\rho_1}\right) = \frac{\lambda_1^2}{\mu_1(\mu_1-\lambda_1)(\lambda_1+\lambda_2)}$ and

$$W_q^2 = \left(\frac{1}{\lambda_1+\lambda_2}\right)\left(\frac{\rho_1\frac{\lambda_2}{\mu_1-\lambda_1}+\rho_2^2}{(1-\rho_1-\rho_2)}\right) = \left(\frac{1}{\lambda_1+\lambda_2}\right)\frac{\lambda_1\lambda_2(\mu_2+\lambda_1)^2+\lambda_2^2(\mu_1^2-\lambda_1\mu_1)}{(\mu_2+\lambda_1)(\mu_1-\lambda_1)(\mu_1\mu_2+\mu_1\lambda_1-\lambda_1\mu_2-\lambda_1^2-\lambda_2\mu_1}\ ,$$

Applying MILP technique we get the upper and lower bound value of

$${}^{l}W_q^1 = \frac{\lambda_1^2}{\mu_1(\mu_1-\lambda_1)(\lambda_1+\lambda_2)} = = \frac{\alpha^2+2\alpha+1}{8\alpha^3-134\alpha^2+423\alpha+396}$$ and upper bound of

$${}^{U}W_q^1 = \frac{\lambda_1^2}{\mu_1(\mu_1-\lambda_1)(\lambda_1+\lambda_2)} = \frac{\alpha^2-6\alpha+9}{-4\alpha^3-36\alpha^2+103\alpha+630}$$

$${}^{l}W_q^2 = \left(\frac{1}{\lambda_1+\lambda_2}\right)\frac{\lambda_1\lambda_2(\mu_2+\lambda_1)^2+\lambda_2^2(\mu_1^2-\lambda_1\mu_1)}{(\mu_2+\lambda_1)(\mu_1-\lambda_1)(\mu_1\mu_2+\mu_1\lambda_1-\lambda_1\mu_2-\lambda_1^2-\lambda_2\mu_1} = \frac{18\alpha^4-291\alpha^3+1544\alpha^2+2724\alpha+1040}{-384\alpha^4-6624\alpha^3-70676\alpha^2+57568\alpha+80256}$$

$${}^{U}W_q^2 = \frac{3\alpha^4-30\alpha^3+439\alpha^2-3297\alpha+6570}{-12\alpha^5-332\alpha^4-2107\alpha^3-5358\alpha^2+14985\alpha+71820},$$

such that $\alpha+1 < \lambda_1 < 3-\alpha$, $3\alpha+2 < \lambda_2 < 6-\alpha$, $\alpha+10 < \mu_1 < 12-\alpha$ and $2\alpha+12 < \mu_2 < 15-\alpha$

Thus the interval values of W_q are

$$\left[{}^{l}W_q^1, {}^{l}W_q^2\right] = \left[\frac{\alpha^2+2\alpha+1}{8\alpha^3-134\alpha^2+423\alpha+396}, \quad \frac{18\alpha^4-291\alpha^3+1544\alpha^2+2724\alpha+1040}{-384\alpha^4-6624\alpha^3-70676\alpha^2+57568\alpha+80256}\right]$$ and

$$\left[{}^{U}W_q^1, {}^{U}W_q^2\right] = \left[\frac{\alpha^2-6\alpha+9}{-4\alpha^3-36\alpha^2+103\alpha+630}, \quad \frac{3\alpha^4-30\alpha^3+439\alpha^2-3297\alpha+6570}{-12\alpha^5-332\alpha^4-2107\alpha^3+5358\alpha^2+14985\alpha+71820}\right]$$

Table: 3 11 values for th e α cuts of the performance measures are as follows:

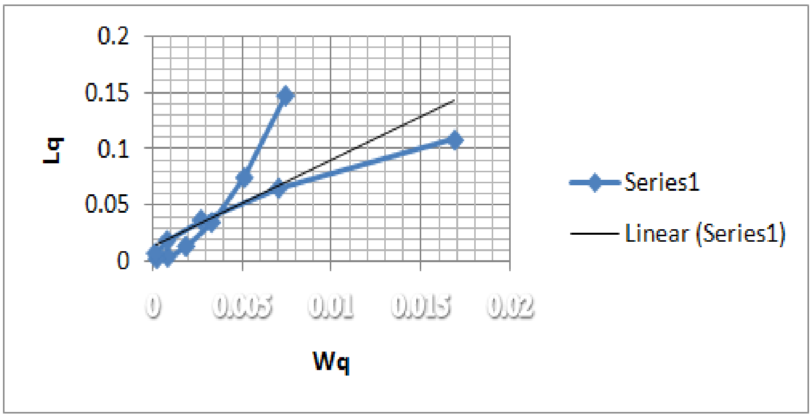
α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
${}^{l}W_q$	0.007742	0.009166	0.010764	0.012584	0.014683	0.017162	0.020158	0.023896	0.028703	0.035242	0.044747
${}^{U}W_q$	0.052883	0.049126	0.045549	0.042159	0.038952	0.03593	0.033088	0.030426	0.027928	0.025596	0.023419

Table : 4 Chi-Square test table

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\frac{({}^{l}W_q - \overline{W_q})^2}{{}^{l}W_q}$	0.007889	0.006219	0.004581	0.003020	0.001622	0.000526	0.000004	0.000584	0.003339	0.010717	0.028902
$\frac{({}^{U}W_q - \overline{W_q})^2}{{}^{U}W_q}$	0.007004	0.004111	0.002068	0.000773	0.000123	0.000022	0.000379	0.001111	0.002149	0.003423	0.004880

Table : 5 Relation between Lq and Wq

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Lq	0.147955	0.074855	0.03475	0.013523	0.003975	0.002448	0.007337	0.01848	0.036963	0.065362	0.108371
W_q	0.007447	0.005164	0.003325	0.001897	0.0008725	0.0002738	0.000191433	0.000848	0.002744	0.007070	0.016891



IV. Result and discussion

In this paper, we found the value of L_q=0.36443 and W_q= 0.05607from crisp values obtained using Robust ranking method. The regression line is traced using the α -cut values of L_q and w_q. From the figure we obtain the fuzzy optimum values of L_q and w_q at α =0.6 as 0.007337 and 0.000191433. This concludes that fuzzy optimization is a better optimization.

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