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Degree based topological indices of Penta chains

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Abstract
In this paper we obtain degree based topological indices for the graphs formed of concatenated 5-cycles in one rows and in two rows of various lengths.
***Mathematics subject classification**:05C12*
***Keywords**: topological indices, Pentachains.*

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I. Introduction

Topological index is the first effective choice in QSAR research and Weiner index is the most important index used to build correlation model between the chemical structure of various chemicals compounds. Yang and Yen in 1995 computed general expression for Wiener indices of class of polycyclic graphs with different lengths [**1**]. N.Prabhakar Rao and A. Laxmi Prassanna obtained the Wiener index for pentachains in two rows of different lengths [**2**]. Ali A. Ali and Ahmed M.Ali computed Hosoya Polynomial for different types of graphs consisting of coencatenated 5-cycles[**3**]. Motivated with all these works in chemical graph theory, in this paper we compute degree based topological indices for pentachains .
Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The number of edges incident on vertex v is called *degree*, $d_G(v)$ of vertex. The distance between the vertices u and v is the length of shortest path. It is denoted by $d_G(u,v)$. The diameter $diam(G)= p$ is maximum distance between two vertices of a graph G .

II. Degree based topological indices

1. Randić Index: The first degree-based topological index was put forward in 1975 by Milan Randić [4] . It is defined as

$$R(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)d_G(v)}}$$

With summation going over all pairs of adjacent vertices of the molecular graph G. Randić himself named “branching index”, but soon it was renamed to connectivity index. Now a days it is known as Randić index.
2. Reciprocal Randić Index: The reciprocal Randić index is defined as

$$RR(G)=\sum_{uv\in E(G)}\sqrt{d_G(u)d_G(v)}$$

3. Zagreb Indices: In analyzing the structure dependency of π-electron energy [5] the first and second Zagreb indices are defined as

$$M_1(G)=\sum_{uv\in E(G)}(d_G(u)+d_G(v))$$

$$M_2(G)=\sum_{uv\in E(G)}d_G(u)d_G(v)$$

4. A**tom-Bond Connectivity Index:** Ernesto Estrada defined atom-bond connectivity index [6]As

$$ABC(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}$$

5. **Augmented Zagreb Index:** Motivated the success of the *ABC* index, Furtula et. Al. [7] put forward its modified version as Augmented Zagreb index. It is defined as

$$AZI(G)=\sum_{uv\in E(G)}\left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2}\right)^3$$

6. Geometric-Arithmetic Index: This index was introduced by Vukićević [8].It is defined as

$$GA(G)=\sum_{uv\in E(G)}\frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$$

Where $\sqrt{d_G(u)d_G(v)}$ and $\frac{1}{2}\left(d_G(u)+d_G(v)\right)$ are the Geometric and Arithmetic means respectively of the degrees of the end vertices of an edges.

7. Harmonic Index: Siemion Fajtlowicz [9] created computer program for automatic generation of conjectures in graph theory. Then he examined the possible relations between countless graph invariants, among which there was a degree-based quantity Harary index . It is defined as

$$H(G)=\sum_{uv\in E(G)}\frac{2}{d_G(u)+d_G(v)}$$

8. Sum Connectivity Index: The sum connectivity index was put forward by Bo Zhou and NenadTrinajstić [10] and it is given by

$$SCI(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)+d_G(v)}}$$

2.1. Straight chaining of pentagons

A Straight chaining is a graph consisting of *n* pentagonal cycle, every two successive cycles have a common edge ,forming a chain denoted by *G(n,S)*.The order of *G(n,S)* is 3*n*+2 and size 4*n*+1,and the diameter is *n* + 2, *n* ≥ 2.

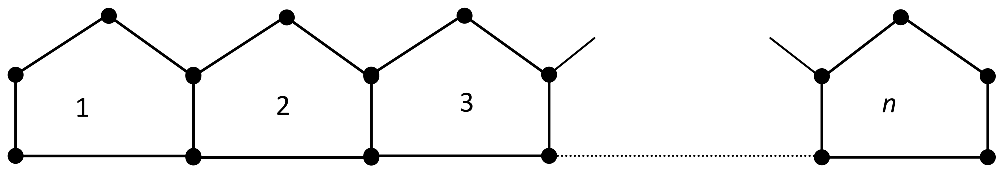


Figure 1. Straight chaining of pentagons

The partition of 4*n*+1 edges of Straight chaining of pentagons is as shown below.

Edge	$(d_G(u),d_G(v))$	(2,2)	(2,3)	(3,3)
Vertices	frequencies	4	2*n*	2*n*-3

Table1.Edge partition of straight chaining of pentagons

Theorem 1.1. The Randić Index of Straight chaining pentagon is given by $R(G(n,S))=\frac{2n+3}{3}+\frac{2n}{\sqrt{6}}$

Proof: Using Table 1 values in the definition of Randić Index we have

$$R(G(n,S)) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$$
$$= 4.\frac{1}{\sqrt{2\cdot 2}} + 2n.\frac{1}{\sqrt{2\cdot 3}} + (2n-3).\frac{1}{\sqrt{3\cdot 3}}$$
$$= \frac{2n+3}{3} + \frac{2n}{\sqrt{6}}$$

Theorem 1.2. The reciprocal Randić Index of Straight chaining pentagon is given by
$RR(G(n,S)) = 6n-1+2n\sqrt{6}$

Proof: Using Table 1 values in the definition of reciprocal Randić Index we have
$$RR(G(n,S)) = \sum_{uv\in E(G)} \sqrt{d_G(u)d_G(v)}$$
$$= 4.\sqrt{2\cdot 2} + 2n.\sqrt{2\cdot 3} + (2n-3).\sqrt{3\cdot 3}$$
$$= 6n-1+2n\sqrt{6}$$

Theorem 1.3. The Zagreb Indices of Straight chaining pentagon is given by
$M_1(G(n,S)) = 22n-2$ and $M_2(G(n,S)) = 30n-11$
Proof: Using Table 1 values in the definition of Zagreb Indices we have
$$M_1(G(n,S)) = \sum_{uv\in E(G)} (d_G(u)+d_G(v))$$
$$= 4(2+2)+2n(2+3)+(2n-3)(3+3)$$
$$= 22n-2$$
$$M_2(G(n,S)) = \sum_{uv\in E(G)} d_G(u)d_G(v)$$
$$= 4(2\times 2)+2n(2\times 3)+(2n-3)(3\times 3)$$
$$= 30n-11$$
Theorem 1.4. The Atom-Bond Connectivity Index of Straight chaining pentagon is given by
$ABC(G(n,S)) = \sqrt{2}\,(n+2) + \frac{4n-6}{3}$
Proof: Using Table 1 values in the definition of Atom-Bond Connectivity Index we have
$$ABC(G(n,S)) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}$$
$$= 4\sqrt{\frac{2+2-2}{2\cdot 2}} + 2n\sqrt{\frac{2+3-2}{2\cdot 3}} + (2n-3)\sqrt{\frac{3+3-2}{3\cdot 3}}$$
$$= \sqrt{2}\,(n+2) + \frac{4n-6}{3}$$
Theorem 1.5. The Augmented Zagreb Index of Straight chaining pentagon is given by
$AZI(G(n,S)) = \frac{1474n-1675}{16}$
Proof: Using Table 1 values in the definition of Augmented Zagreb Index we have

$$AZI(G(n,S)) = \sum_{uv\in E(G)} \left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2} \right)^3$$

$$= 4\left(\frac{2\cdot 2}{2+2-2} \right)^3 + 2n \left(\frac{2\cdot 3}{2+3-2} \right)^3 + (2n-3)\left(\frac{3\cdot 3}{3+3-2} \right)^3$$

$$= 32 + 16n + (2n-3)\frac{729}{16}$$

$$= \frac{1474n - 1675}{16}$$

Theorem 1.6. The Geometric-Arithmetic Index of Straight chaining pentagon is given by

$$GA(G(n,S)) = 2n + 1 + \frac{4\sqrt{6}n}{5}$$

Proof: Using Table 1 values in the definition of Geometric-Arithmetic Index we have

$$GA(G(n,S)) = \sum_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$$

$$= 4\cdot\frac{2\sqrt{2\cdot 2}}{2+2} + 2n\cdot\frac{2\sqrt{2\cdot 3}}{2+3} + (2n-3)\cdot\frac{2\sqrt{3\cdot 3}}{3+3}$$

$$= 2n + 1 + \frac{4\sqrt{6}n}{5}$$

Theorem 1.7. The Harmonic index of Straight chaining pentagon is given by

$$H(G(n,S)) = \frac{6}{5}(n+2)$$

Proof: Using Table 1 values in the definition of Harmonic index we have

$$H(G) = \sum_{uv\in E(G)} \frac{2}{d_G(u)+d_G(v)}$$

$$= 4\cdot \frac{2}{2+2} + 2n\cdot\frac{2}{2+3} + (2n-3)\cdot\frac{2}{3+3}$$

$$= \frac{6}{5}(n+2)$$

Theorem 1.8. The sum connectivity index of Straight chaining pentagon is given by

$$H(G(n,S)) = 2 + \frac{2n}{\sqrt{5}} + \frac{2n-3}{\sqrt{6}}$$

Proof: Using Table 1 values in the definition of sum connectivity index we have

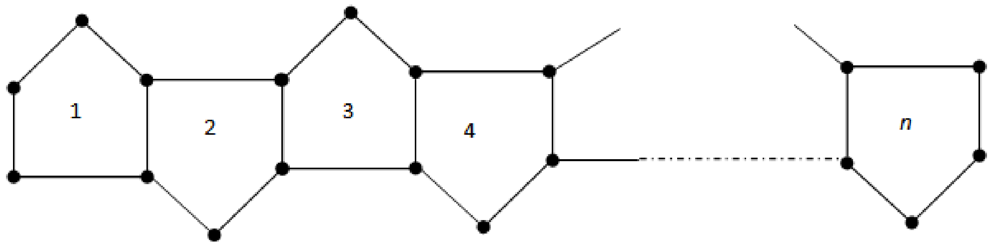
$$SCI(G(n,S)) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}}$$

$$= 4\cdot \frac{1}{\sqrt{2+2}} + 2n\cdot\frac{1}{\sqrt{2+3}} + (2n-3)\cdot\frac{1}{\sqrt{3+3}}$$

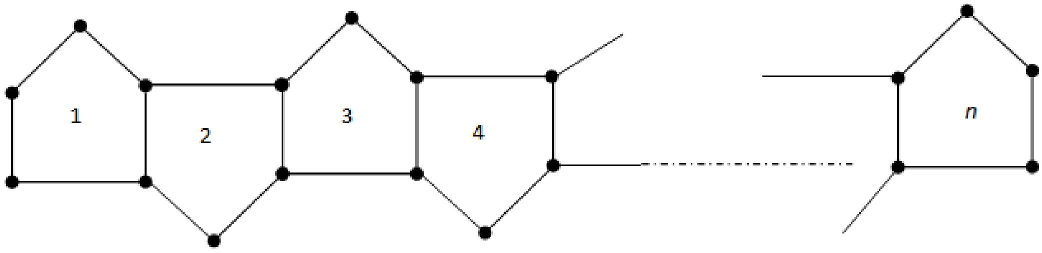
$$= 2 + \frac{2n}{\sqrt{5}} + \frac{2n-3}{\sqrt{6}}$$

2.2.Alternate chaining of pentagons

An alternate chaining is a graph consisting of *n* pentagonal cycle, every two successive cycles have a common edge ,forming a chain denoted by *G(n,A)*.The order of *G(n,A)* is 3*n*+2 and size 4*n*+1,and the diameter is $\left\lfloor \frac{3n+2}{2} \right\rfloor, n \geq 2$



***n* is even**



***n* is odd**

Figure 2.Alternate chaining of pentagons

The partition of 4*n*+1 edges of alternate chaining of pentagons in the case of *n* is even or odd is as shown below.

Edge	$(d_G(u),d_G(v))$	(2,2)	(2,3)	(3,3)
Vertices	frequencies	4	2*n*	2*n*-3

Table 2.Edge partition of alternate chaining of pentagons

Theorem 2.1. The degree based topological indices for straight chaining and alternate chaining equal .i.e

$$R(G(n,S)) = R(G(n,A))$$

$$RR(G(n,S)) = RR(G(n,A))$$

$$M_1(G(n,S)) = M_1(G(n,A))$$

$$M_2(G(n,S)) = M_2(G(n,A))$$

$$ABC(G(n,S)) = ABC(G(n,A))$$

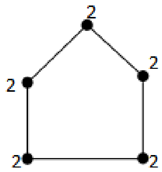
$$H(G(n,S)) = H(G(n,A))$$

$$AZI(G(n,S)) = AZI(G(n,A))$$

$$SCI(G(n,S)) = SCI(G(n,A))$$

Proof: By the observation inTable 1and Table 2. We have the proof.

Corollary 2.2 . For *n*=1



$$R(G(1,S)) = SCI(G(1,S)) = H(G(1,S)) = \frac{5}{2}$$

$$M_1(G(1,S)) = M_2(G(1,S)) = 20$$

$$RR(G(1,S)) = 10$$

$$AZI(G(1,S)) = 40$$

$$ABC(G(1,S)) = \frac{5}{\sqrt{2}}$$

$$GA(G(1,S)) = 5$$

2.3.Double Row Pentachains

In this section we obtain degree based topological indices of graphs consisting of two rows of straight chains with *n* pentagons in the two rows combined as shown in Figure.3 and Figure.4.We denote them as $G(n,S_1)$ and $G(n,S_2)$.

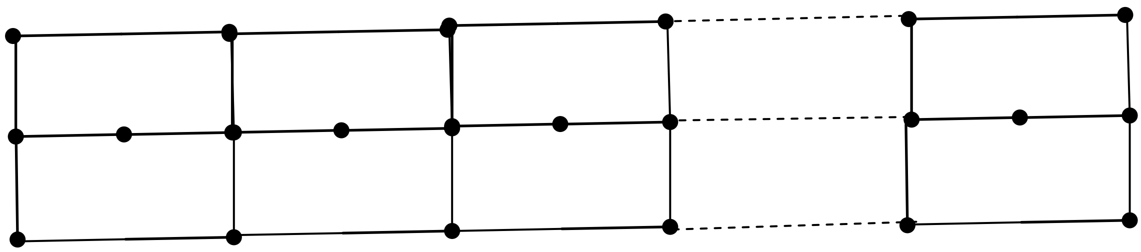


Figure3. Double row pentachains $G(n,S_1)$

The order and size of the graphs $G(n,S_1)$is $4n+3$ and $6n+2$ respectively, and the diameter is $n+2$, $n \geq 2$.
The partition of $6n+2$ edges of Double row pentachains for both even and odd is as shown below.

Edge	$(d_G(u),d_G(v))$	(2,3)	(2,4)	(3,3)	(3,4)
Vertices	frequencies	10	$2n$-2	$2(2n$-2)	$2(2n$-1)

Table 3.Edge partition of Double row pentachains $G(n,S_1)$

Theorem 3.1. The Randić Index of Double row pentachains $G(n,S_1)$is given by

$$R(G(n,S_1)) = \frac{10}{\sqrt{6}} + \frac{2n-4}{3} + \frac{n-1}{\sqrt{6}}(\sqrt{2}+\sqrt{3})$$

Proof: Using Table 3 values in the definition of Randić Index we have

$$R(G(n,S_1)) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$$
$$= 10.\frac{1}{\sqrt{2\cdot 3}} + (2n-2).\frac{1}{\sqrt{2\cdot 4}} + (2n-4).\frac{1}{\sqrt{3\cdot 3}} + (2n-2).\frac{1}{\sqrt{3\cdot 4}}$$
$$= \frac{10}{\sqrt{6}} + \frac{2n-4}{3} + \frac{n-1}{\sqrt{6}}(\sqrt{2}+\sqrt{3})$$

Theorem 3.2. The reciprocal Randić Index of Double row pentachains $G(n,S_1)$is given by
$RR(G(n,S_1) = 6n+8+2(n-1)(\sqrt{6}+2\sqrt{3})$

Proof: Using Table 3 values in the definition of reciprocal Randić Index we hav
$$RR(G(n,S_1)) = \sum_{uv\in E(G)} \sqrt{d_G(u)d_G(v)}$$
$$=10.\sqrt{2\cdot 2} + (2n-2).\sqrt{2\cdot 3} + (2n-4).\sqrt{3\cdot 3} + (2n-2).\sqrt{3\cdot 4}$$
$$= 6n+8+2(n-1)(\sqrt{6}+2\sqrt{3})$$

Theorem 3.3. The Zagreb Indices of Double row pentachains $G(n,S_1)$is given by
$M_1(G(n,S_1)) = 38n-10$ and $M_2(G(n,S_1)) = 58n-36$

Proof: Using Table 3 values in the definition of Zagreb Indices we have
$$M_1(G(n,S_1)) = \sum_{uv\in E(G)} (d_G(u)+d_G(v))$$
$$=10(2+2)+(2n-2)(2+4)+(2n-4)(3+3)+(2n-2)(3+4)$$
$$=38n-10$$
$$M_2(G(n,S_1)) = \sum_{uv\in E(G)} d_G(u)d_G(v)$$
$$=10(2\times 2)+(2n-2)(2\times 4)+(2n-4)(3\times 3)+(2n-2)(3\times 4)$$
$$=58n-36$$

Theorem 3.4. The Atom-Bond Connectivity Index of Double row pentachains $G(n,S_1)$is given by
$$ABC(G(n,S_1)) = \sqrt{2}\,(n-4) + \frac{4}{3}(n-2) + \sqrt{\frac{5}{3}}\,(n-1)$$

Proof: Using Table 3 values in the definition of Atom-Bond Connectivity Index we have
$$ABC(G(n,S_1)) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}$$
$$=10\sqrt{\frac{2+2-2}{2\cdot 2}} + (2n-2)\sqrt{\frac{2+4-2}{2\cdot 4}} + (2n-4)\sqrt{\frac{3+3-2}{3\cdot 3}} + (2n-2)\sqrt{\frac{3+4-2}{3\cdot 4}}$$
$$= \sqrt{2}\,(n-4) + \frac{4}{3}(n-2) + \sqrt{\frac{5}{3}}\,(n-1)$$

Theorem 3.5. The Augmented Zagreb Index of Double row pentachains $G(n,S_1)$is given by
$AZI(G(n,S_1)) = 52.63n - 43.4$

Proof: Using Table 3 values in the definition of Augmented Zagreb Index we have

$$AZI(G(n,S_1)) = \sum_{uv \in E(G)} \left(\frac{d_G(u) d_G(v)}{d_G(u) + d_G(v) - 2} \right)^3$$

$$= 10\left(\frac{2\cdot 2}{2+2-2}\right)^3 + (2n-2)\left(\frac{2\cdot 4}{2+4-2}\right)^3 + (2n-4)\left(\frac{3\cdot 3}{3+3-2}\right)^3 + (2n-2)\left(\frac{3\cdot 4}{3+4-2}\right)^3$$

$$= 52.63n - 4.6$$

Theorem 3.6. The Geometric-Arithmetic Index of Double row pentachains $G(n,S_1)$is given by

$$GA(G(n,S_1)) = \frac{140\sqrt{6} + (20n-20)\sqrt{2} + 70n - 280 + (8n-8)\sqrt{3}}{35}$$

Proof: Using Table 3 values in the definition of Geometric-Arithmetic Index we have

$$GA(G(n,S_1)) = \sum_{uv \in E(G)} \frac{2\sqrt{d_G(u) d_G(v)}}{d_G(u) + d_G(v)}$$

$$= 10 \cdot \frac{2\sqrt{2\cdot 3}}{2+3} + (2n-2)\cdot\frac{2\sqrt{2\cdot 4}}{2+4} + (2n-4)\cdot\frac{2\sqrt{3\cdot 3}}{3+3} + (2n-2)\cdot\frac{2\sqrt{3\cdot 4}}{3+4}$$

$$= \frac{140\sqrt{6} + (20n-20)\sqrt{2} + 70n - 280 + (8n-8)\sqrt{3}}{35}$$

Theorem 3.7. The Harmonic index of Double row pentachains $G(n,S_1)$is given by

$$H(G(n.S_1)) = \frac{40n+30}{21}$$

Proof: Using Table 3 values in the definition of Harmonic index we have

$$H(G(n.S_1)) = \sum_{uv \in E(G)} \frac{2}{d_G(u) + d_G(v)}$$

$$= 10 \cdot \frac{2}{2+3} + (2n-2)\cdot\frac{2}{2+4} + (2n-4)\cdot\frac{2}{3+3} + (2n-2)\cdot\frac{2}{3+4}$$

$$= \frac{40n+30}{21}$$

Theorem 3.8. The sum connectivity index of Double row pentachains $G(n,S_1)$is given by

$$SCI(G(n,S_1)) = 5 + \frac{4n-6}{\sqrt{6}} + \frac{2n-2}{\sqrt{7}}$$

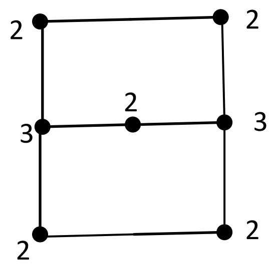
Proof: Using Table 3 values in the definition of sum connectivity index we have

$$SCI(G(n,S_1)) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}$$

$$= 10 \cdot \frac{1}{\sqrt{2+2}} + (2n-2)\cdot\frac{1}{\sqrt{2+4}} + (2n-4)\cdot\frac{1}{\sqrt{3+3}} + (2n-2)\cdot\frac{1}{\sqrt{3+4}}$$

$$= 5 + \frac{4n-6}{\sqrt{6}} + \frac{2n-2}{\sqrt{7}}$$

Corollary 3.9. For n = 1



$$R(G(1,S_1)) = 1+\sqrt{6}$$
$$RR(G(1,S_1)) = 4+6\sqrt{6}$$
$$M_1(G(1,S_1)) = 38$$
$$M_2(G(1,S_1)) = 44$$
$$ABC(G(1,S_1)) = 4\sqrt{2}$$
$$AZI(G(1,S_1)) = 43.77$$
$$GA(G(1,S_1)) = 4+\sqrt{\frac{6}{5}}$$
$$H(G(1,S_1)) = 3.4$$
$$SCI(G(1,S_1)) = 1+\frac{6}{\sqrt{5}}$$

The order and size of the graph $G(n,S_2)$ is $5n+3$ and $7n+2$ respectively, and the diameter is $n+2$, $n \geq 2$. The partition of $7n+2$ edges of Double row pentachains for both even and odd is as shown below.

Edge	$(d_G(u),d_G(v))$	(2,2)	(2,3)	(3,4)	(4,4)
Vertices	Frequencies	4	$4n$	$2n$	n-2

Table 4.Edge partition of Double row pentachains $G(n,S_2)$

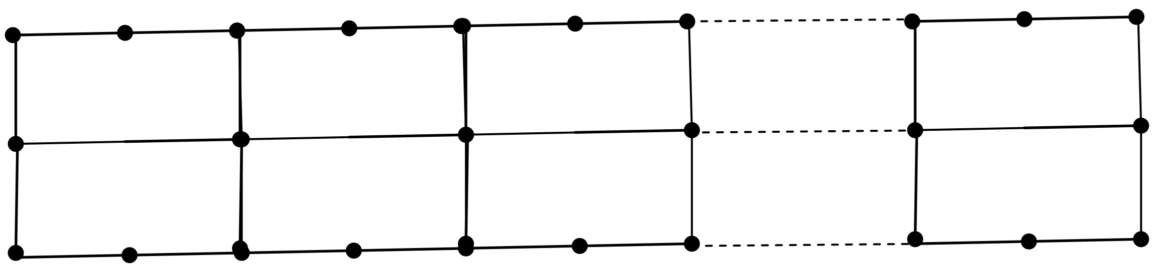


Figure4. Double row pentachains $G(n,S_2)$

Theorem 4.1. The Randić Index of Double row pentachains $G(n,S_2)$is given by

$$R(G(n,S_2)) = \frac{11n-18}{12}+\frac{4n}{\sqrt{6}}$$

Proof: Using Table 4 values in the definition of Randić Index we have

$$R(G(n,S_2)) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$$

$$= 4.\frac{1}{\sqrt{2\cdot 2}} + 4n.\frac{1}{\sqrt{2\cdot 3}} + 2n.\frac{1}{\sqrt{3\cdot 3}} + (n-2).\frac{1}{\sqrt{4\cdot 4}}$$

$$= \frac{11n-18}{12} + \frac{4n}{\sqrt{6}}$$

Theorem 4.2. The reciprocal Randić Index of Double row pentachains $G(n,S_2)$is given by
$RR(G(n,S_2)) = 4n(\sqrt{3}+\sqrt{6}+1)$

Proof: Using Table 4 values in the definition of reciprocal Randić Index we have

$$RR(G(n,S_2)) = \sum_{uv\in E(G)} \sqrt{d_G(u)d_G(v)}$$

$$= 4.\sqrt{2\cdot 2} + 4n.\sqrt{2\cdot 3} + 2n.\sqrt{3\cdot 4} + (n-2).\sqrt{4\cdot 4}$$

$$= 4n(\sqrt{3}+\sqrt{6}+1)$$

Theorem 4.3. The Zagreb Indices of Double row pentachains $G(n,S_2)$is given by
$M_1(G(n,S_2)) = 42n$ and $M_2(G(n,S_2)) = 64n$

Proof: Using Table 4 values in the definition of Zagreb Indices we have

$$M_1(G(n,S_2)) = \sum_{uv\in E(G)} (d_G(u)+d_G(v))$$

$$= 4(2+2)+4n(2+3)+2n(3+4)+(n-2)(4+4)$$

$$=$$

$$M_2(G(n,S_2)) = \sum_{uv\in E(G)} d_G(u)d_G(v)$$

$$= 4(2\times 2)+4n(2\times 3)+2n(3\times 4)+(n-2)(4\times 4)$$

$$= 64n$$

Theorem 4.4. The Atom-Bond Connectivity Index of Double row pentachains $G(n,S_2)$is given by

$$ABC(G(n,S_2)) = \frac{8+8n+(n-2)\sqrt{3}}{2\sqrt{2}} + n\sqrt{\frac{5}{3}}$$

Proof: Using Table 4 values in the definition of Atom-Bond Connectivity Index we have

$$ABC(G(n,S_2)) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}$$

$$= 4\sqrt{\frac{2+2-2}{2\cdot 2}} + 4n\sqrt{\frac{2+3-2}{2\cdot 3}} + 2n\sqrt{\frac{3+4-2}{3\cdot 4}} + (n-2)\sqrt{\frac{4+4-2}{4\cdot 4}}$$

$$= \frac{8+8n+(n-2)\sqrt{3}}{2\sqrt{2}} + n\sqrt{\frac{5}{3}}$$

Theorem 4.5. The Augmented Zagreb Index of Double row pentachains $G(n,S_2)$is given by
$AZI(G(n,S_2)) = 78.61n - 5.92$

Proof: Using Table 4 values in the definition of Augmented Zagreb Index we have

$$AZI(G(n,S_2)) = \sum_{uv\in E(G)} \left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2} \right)^3$$

$$= 4\left(\frac{2\cdot 2}{2+2-2} \right)^3 + 4n\left(\frac{2\cdot 3}{2+3-2} \right)^3 + 2n\left(\frac{3\cdot 4}{3+4-2} \right)^3 + (n-2)\left(\frac{4\cdot 4}{4+4-2} \right)^3$$

$$= \frac{256312n-20000}{3375} = 78.61n-5.92$$

Theorem 4.6. The Geometric-Arithmetic Index of Double row pentachains $G(n,S_2)$is given by

$$GA(G(n,S_2)) = n(\frac{\sqrt{6}}{5} + \frac{\sqrt{3}}{7}) + n + 2$$

Proof: Using Table 4 values in the definition of Geometric-Arithmetic Index we have

$$GA(G(n,S_2)) = \sum_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$$

$$= 4\cdot\frac{2\sqrt{2\cdot 2}}{2+2} + 4n\cdot\frac{2\sqrt{2\cdot 3}}{2+3} + 2n\cdot\frac{2\sqrt{3\cdot 4}}{3+4} + (n-2)\cdot\frac{2\sqrt{4\cdot 4}}{4+4}$$

$$= 8n(\frac{\sqrt{6}}{5} + \frac{\sqrt{3}}{7}) + n + 2$$

Theorem 4.7. The Harmonic index of Double row pentachains $G(n,S_2)$is given by

$$H(G(n.S_2)) = 2.42n - 1.5$$

Proof: Using Table 4 values in the definition of Harmonic index we have

$$H(G(n.S_2)) = \sum_{uv\in E(G)} \frac{2}{d_G(u)+d_G(v)}$$

$$= 4\cdot \frac{2}{2+2} + 4n\cdot\frac{2}{2+3} + 2n\cdot\frac{2}{3+4} + (n-2)\cdot\frac{2}{4+4}$$

$$= \frac{339n-210}{140} = 2.42n - 1.5$$

Theorem 4.8. The sum connectivity index of Double row pentachains $G(n,S_2)$)is given by

$$SCI(G(n,S_2)) = 2 + \frac{4n}{\sqrt{5}} + \frac{2n}{\sqrt{7}} + \frac{n-2}{2\sqrt{2}}$$

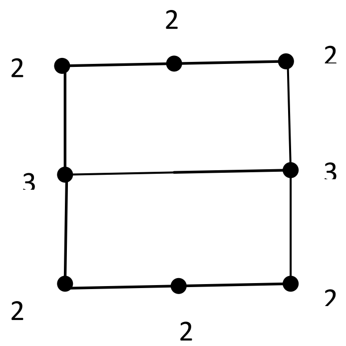
Proof: Using Table 4 values in the definition of sum connectivity index we have

$$SCI(G(n,S_2)) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}}$$

$$= 4\cdot \frac{1}{\sqrt{2+2}} + 4n\cdot\frac{1}{\sqrt{2+3}} + 2n\cdot\frac{1}{\sqrt{3+4}} + (n-2)\cdot\frac{1}{\sqrt{4+4}}$$

$$= 2 + \frac{4n}{\sqrt{5}} + \frac{2n}{\sqrt{7}} + \frac{n-2}{2\sqrt{2}}$$

Corollary.4.9. For n=1



$$R(G(1,S_2)) = \frac{7}{3} + \frac{4}{\sqrt{6}}$$

$$RR(G(1,S_2)) = 11 + 4\sqrt{6}$$

$$M_1(G(1,S_2)) = 45$$

$$M_2(G(1,S_2)) = 49$$

$$ABC(G(1,S_2)) = 4\sqrt{2} + \frac{2}{3}$$

$$AZI(G(1,S_2)) = 75.39$$

$$GA(G(1,S_2)) = 2 + \frac{8\sqrt{6}}{5}$$

$$H(G(1,S_2)) = 3.94$$

$$SCI(G(1,S_2)) = \frac{7}{3} + \frac{4}{\sqrt{6}}$$

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