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On Fuzzy Round Digraphs

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Abstract: *In this paper we introduce Fuzzy Round Digraph by means of labelling. Also, some basic results of Fuzzy Round Digraph are discussed.*

Keywords: *Fuzzy digraph, Round Digraph, Locally semicomplete Digraph, Fuzzy Round Digraph.*

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I. Introduction

The concept of fuzzy graph was introduced by Rosenfeld [1] in 1975. Fuzzy graph theory has a vast area of applications. It is used in evaluation of human cardiac function, fuzzy neural networks, etc. Fuzzy graphs can be used to solve traffic light problem, time table scheduling, etc. In fuzzy set theory, there are different types of fuzzy graphs which may be a graph with crisp vertex set and fuzzy edge set or fuzzy vertex set and crisp edge set or fuzzy vertex set and fuzzy edge set or crisp vertices and edges with fuzzy connectivity, etc. A lot of works have been done on fuzzy graphs [3], [4], [5]..

The Fuzzy Directed Graph, Fuzzy competition digraphs are well known topic. In this article Fuzzy Round digraphs are defined and their properties are investigated. Locally semicomplete fuzzy digraph, ordinary arc in a fuzzy round digraph are discussed

II. Preliminaries

Definition 2.1:

Fuzzy digraph$\vec{\xi}$ = (V,σ,$\vec{\mu}$) is a non-empty set V together with a pair of functions σ : V ⟶ [0,1] and $\vec{\mu}$: V ×V ⟶ [0,1] such thatfor all x,y ∈ V , $\vec{\mu}$(x,y) ≤ σ(x)∧σ(y).Since $\vec{\mu}$ is well defined, a fuzzy digraph has at most two directed edges (which must have opposite directions) between any two vertices. Here $\vec{\mu}$(u,v) is denoted by the membership value of the edge $\overrightarrow{(u,v)}$.The loop at a vertex x is represented by $\vec{\mu}$(x,x) ≠ 0. Here $\vec{\mu}$ need not be symmetric as $\vec{\mu}$(x,y) and $\vec{\mu}$(y,x) may have different values. The underlying crisp graph of directed fuzzy graph is the graph similarly obtained except the directed arcs are replaced by undirected edges

Definition 2.2:

Fuzzy out-neighbourhood of a vertex v of a directed fuzzy graph $\vec{\xi}$= (V,σ,$\vec{\mu}$) is the fuzzy set$N^+(v)$ = $({X_v}^+, {m_v}^+)$ where ${X_v}^+$ = {u| $\vec{\mu}$(v,u) > 0} and${m_v}^+$:${X_v}^+$⟶ [0,1] defined by${m_v}^+$ (u) = $\vec{\mu}$(v,u). Similarly, ***fuzzy in-neighbourhood*** of a vertex v of a directed fuzzy graph$\vec{\xi}$ = (V,σ,$\vec{\mu}$) is the fuzzy set$N^-(v)$ = $({X_v}^-, {m_v}^-)$ where${X_v}^-$= {u|$\vec{\mu}$(u,v) > 0} and${m_v}^-$: ${X_v}^-$⟶ [0,1] defined by${m_v}^-$ (u) = $\vec{\mu}$(u,v).

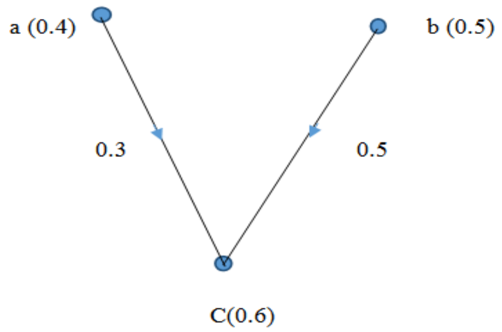
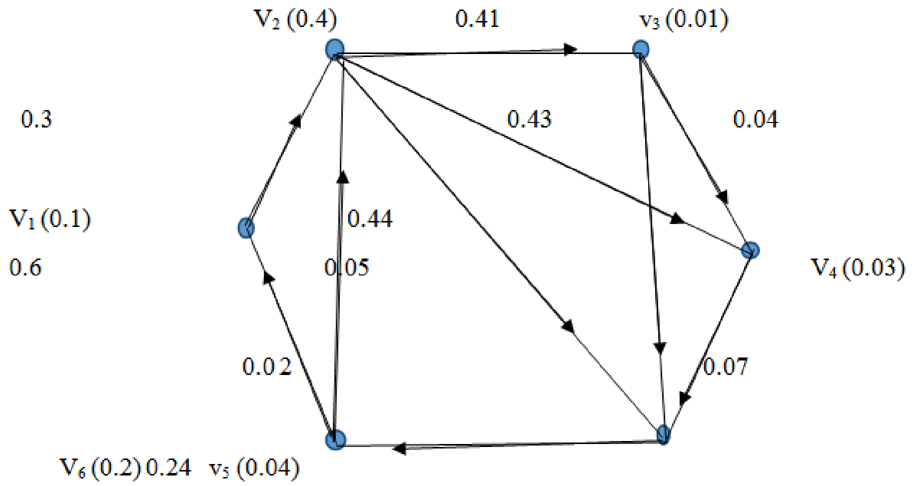


Figure: 1

Example 2.2.1: Let $\vec{\xi}$ be a directed fuzzy graph. Let the vertex set be {a,b,c} with membership values σ(a) = 0.4, σ(b) = 0.5, σ(c) = 0.6. The membership values of arcs are $\vec{\mu}$ (a,b) = 0.3, $\vec{\mu}$(b,c) = 0.5. So $N^{+}(a)$ = {(b,0.3)}. $N^{-}(c)$ = {(a,0.3),(b,0.5)}. (Note that (a,σ(a)) represents the vertex a with membership value σ(a)). It is shown in Figure 1.

Definition 2.3:

A ***Fuzzy Digraph*** on 'n' vertices is ***round*** if its vertices are labelled as $v_1, v_2, \dots \dots \dots v_n$ so that or each ' I'$N^{+}(v_i) = \{(X_{v_{i+1}}{}^{+}, m_{v_{i+1}}{}^{+})$ ………$(X_{v_{i+d^{+}(v_i)}}{}^{+}, m_{v_{i+d^{+}(v_i)}}{}^{+})\}$ and $N^{-}(v_i) = \{(X_{v_{i-d^{-}(v_i)}}{}^{-}, m_{v_{i-d^{-}(v_i)}}{}^{-}) \dots \dots (X_{v_{i-1}}{}^{-}, m_{v_{i-1}}{}^{-})\}$



Definition 2.4: An ***arc***$(x\sigma(x), y\sigma(y))$ of a fuzzy digraph $\mathcal{D}$ is ordinary if$(y\sigma(y), x\sigma(x))$ is not in $\mathcal{D}$. A cycle or path Q of a fuzzy digraph $\mathcal{D}$ is ***ordinary*** if all arcs of Q are ordinary.

III. Fuzzy Round Digraph

Proposition 3.1 Every Fuzzy round digraph is locally semicomplete.

Proof: Let $\mathcal{D}_{\mathcal{R}}$ be a Fuzzy round digraph and let $v_1, v_2, \dots \dots \dots v_n$ be a round labelling of $\mathcal{D}_{\mathcal{R}}$. Consider an arbitrary vertex, say $v_i\sigma(v_i)$. Let $(x\sigma(x), y\sigma(y))$be a pair of out-neighbours of $v_i\sigma(v_i)$. We show that and $y\sigma(y)$are adjacent. Assume without loss of generality that$(v_i\sigma(v_i),, y\sigma(y))$ appear in that circular order in the round labelling. Since $v_i\sigma(v_i)$. →$y\sigma(y)$and the in-neighbours of $y\sigma(y)$appear consecutively preceding $y\sigma(y)$, i.e., $x\sigma(x)$→ $y\sigma(y)$. Thus the out-neighbours of $v_i\sigma(v_i)$ are pairwise adjacent. Similarly, we can show that the in-neighbours of $v_i\sigma(v_i)$are also pairwise adjacent. Therefore, $\mathcal{D}_{\mathcal{R}}$ is locally semicomplete.
(a$\sigma(a)$ represents the vertex a with member ship value $\sigma(a)$)

Result 3.2:
Let$\mathcal{D}_{\mathcal{R}}$ be a Fuzzy round digraph then the following is true:
(a) Every induced fuzzysubdigraph of$\mathcal{D}_{\mathcal{R}}$ is round.
(b) For each $x\sigma(x) \in V(\mathcal{D}_{\mathcal{R}})$, the subdigraphs induced by $N^{+}(x\sigma(x)) - N^{-}(x\sigma(x))$ and $N^{-}(x\sigma(x)) - N^{+}(x\sigma(x))$ are transitive tournament.

Lemma 3.3:

Let$\mathcal{D}_{\mathcal{R}}$ be a fuzzy round digraph. Then, for each vertex$x\sigma(x)$ of$\mathcal{D}_{\mathcal{R}}$, the fuzzysubdigraph induced by $N^{+}(x\sigma(x)) - N^{-}(x\sigma(x))$ contains no ordinary cycle.

Proof: Suppose the fuzzy subdigraph induced by some $N^{+}(x\sigma(x)) - N^{-}(x\sigma(x))$ contains an ordinary cycle C. Let $v_1\,\sigma(v_1), v_2\,\sigma(v_2), \dots \dots \dots v_n\ \sigma(v_n)$be a round labelling of$\mathcal{D}_{\mathcal{R}}$. Without loss of generality, assume that $x\sigma(x) = v_1\,\sigma(v_1)$. Then C must contain an arc$(v_i\,\sigma(v_i) v_j\,\sigma(j))$, such that $v_i\,\sigma(v_i), v_j\,\sigma(v_j), \in A(\mathcal{D}_{\mathcal{R}})$ and i > j. We have$v_1\,\sigma(v_1) \in N^{-}(v_i\,\sigma(v_i))$ but $v_j\,\sigma(v_j) \notin N^{-}(v_i\,\sigma(v_i))$ contradicting the assumption that$v_1\,\sigma(v_1), v_2\,\sigma(v_2), \dots \dots \dots v_n\ \sigma(v_n)$ is a round labelling of$\mathcal{D}_{\mathcal{R}}$. ut.

Theorem 3.4: A connected locally semicomplete fuzzy digraph $\mathcal{D}$ is round if and only if the following holds for each vertex x$\sigma(x)$ of$\mathcal{D}$:
(a) N^+(x$\sigma(x)$) − N^-(x$\sigma(x)$) and N^-(x$\sigma(x)$)−N^+(x$\sigma(x)$) induce transitive tournaments and
(b) N^+(x$\sigma(x)$) ∩ N^-(x$\sigma(x)$) induces a (semicomplete) fuzzy subdigraph containing no ordinary cycle.

Proof: The necessity follows from Lemmas 3.3and result 3.2. To prove the sufficiency, we consider two cases.

Case 1:$\mathcal{D}$ has an ordinary cycle. We start by proving that $\mathcal{D}$ contains an ordinary Hamilton cycle. Let C = $x_1\,\sigma(x_1), x_2\,\sigma(x_2), \ldots\ldots\ldots x_k\,\sigma(x_k)$be a longest ordinary cycle in $\mathcal{D}$. Assume that k ≠ n, the number of vertices in $\mathcal{D}$. Since$\mathcal{D}$ is connected there is a vertex v $\sigma(v)$∈V($\mathcal{D}$)−V(C) such that v $\sigma(v)$ is adjacent to some vertex of C. Suppose that there is an ordinary arc between v $\sigma(v)$ and some vertex, say $x_1\,\sigma(x_1)$, of C. We may without loss of generality assume that the ordinary arc
Is$x_1\,\sigma(x_1)$v $\sigma(v)$. The vertices v $\sigma(v)$ and$x_2\,\sigma(x_2)$ are adjacent since they are out-neighbours of $x_1\,\sigma(x_1)$. The arc betweenv $\sigma(v)$ and$x_2\,\sigma(x_2)$ must be ordinary since$\mathcal{D}$ does not contain as an induced subdigraph . Since C is a longest ordinary cycle,v$\sigma(v)$ cannot dominate $x_2\,\sigma(x_2)$. Thus, $x_2\,\sigma(x_2)$→ v $\sigma(v)$Similarly, we can prove that$x_i\,\sigma(x_i)$→ v $\sigma(v)$ for every i = 3,4,...,k. Hence,N^-(v$\sigma(v)$)−N^+(v$\sigma(v)$) contains all vertices of C, which contradicts the assumption thatN^-(v$\sigma(v)$)−N^+(v$\sigma(v)$) induces a transitive tournament. Since there is no ordinary arc between v and C, we may assume that v $\sigma(v)x_1\,\sigma(x_1)$v $\sigma(v)$ is a 2-cycle of $\mathcal{D}$. Using the fact that $\mathcal{D}$ is locally semicomplete, it is easy to derive that V(C) ⊆N^+(v$\sigma(v)$) ∩ N^-(v$\sigma(v)$). This contradicts the assumption that N^+(v$\sigma(v)$) ∩ N^-(v$\sigma(v)$) contains no ordinary cycle. Thus, we have shown that $\mathcal{D}$ contains an ordinary Hamilton cycle. This implies that N^+(x$\sigma(x)$) − N^-(x$\sigma(x)$)≠∅ for every x $\sigma(x)$∈V($\mathcal{D}$). Tofindaroundlabellingof$\mathcal{D}$ consider an arbitrary vertex, say $y_1\sigma(y_1)$, and, for each i = 1,2,..., let $y_{i+1}\sigma(y_{i+1})$ be the vertex of in-degree zero in the (transitive) tournament induced by $N^+y_i\sigma(y_i)$−$N^-y_i\sigma(y_i)$. Let $y_1\sigma(y_1)\; y_2\sigma(y_2)\ldots\ldots\ldots y_r\sigma(y_r)$ be distinct vertices such that w $\sigma(w)$ of in-degree zero in the tournament induced by $N^+y_r\sigma(y_r)$−$N^-y_r\sigma(y_r)$ is in $\{y_1\sigma(y_1)\; y_2\sigma(y_2)\ldots\ldots\ldots y_{r-2}\sigma(y_{r-2})\}$. We show that$w\sigma(w) = y_1\sigma(y_1)$. If $w\sigma(w) = y_j\,\sigma(y_j)$with j > 1, then$\{y_{j-1}\sigma(y_{j-1}), y_r\sigma(y_r)\}$→$y_j\,\sigma(y_j)$.Thus, $y_{j-1}\sigma(y_{j-1})$, and $y_r\sigma(y_r)$ are adjacent by an ordinary arc . But either$y_{j-1}\sigma(y_{j-1})$,j→$y_r\sigma(y_r)$ or $y_r\sigma(y_r)$→$y_{j-1}\sigma(y_{j-1})$, contradicts the fact that$y_j\,\sigma(y_j)$ is the vertex of in-degree zero in the tournament induced by N^+($y_{j-1}\sigma(y_{j-1})$ −N^- ($y_{j-1}\sigma(y_{j-1})$ orN^+($y_r\sigma(y_r)$ −N^- ($y_r\sigma(y_r)$ Thus,w σ(w)=$y_1\sigma(y_1)$ and ℂ =$y_1\sigma(y_1)\; y_2\sigma(y_2)\ldots\ldots\ldots y_r\sigma(y_r) y_1\sigma(y_1)$ is an ordinary cycle. We next show that r = n. Suppose r < n. Then, there is a vertex u $\sigma(u)$, which is not in ℂ and is adjacent to some$y_i\sigma(i)$ of ℂ. Suppose first that u $\sigma(u)$∈ N^+($y_i\sigma(y_i)$)→N^-($y_i\sigma(y_i)$). Then, being out-neighbours of $y_i\sigma(i)$, the vertices$y_{i+1}\sigma(y_{i+1})$ and u $\sigma(u)$ are adjacent. Since$\mathcal{D}$ contains no induced subdigraph isomorphic to the fuzzy digraph and $y_{i+1}\sigma(y_{i+1})$ is the vertex of in-degree zero in the fuzzy subdigraph induced by N^+($y_i\sigma(i)$)−N^-($y_i\sigma(i)$), we have u $\sigma(u)$∈N+(yi+1)−N−(yi+1). This implies that u and yi+2 are adjacent. Similarly, we must have u $\sigma(u)$∈N^+($y_{i+2}\sigma(y_{i+2})$) −N^- ($y_{i+2}\sigma(y_{i+2})$). Continuing this way, we see thatu $\sigma(u)$∈N^+($y_k\sigma(y_k)$) −N^- ($y_k\sigma(y_k)$ for every k = 1,2,...,r. Hence, ℂ is contained in the subdigraph induced by N^-(u $\sigma(u)$)−N^+(u $\sigma(u)$), a contradiction. A similar argument applies for the case ∈$N^-y_i\sigma(i)$−$N^+y_i\sigma(i)$. So, we may assume that u $\sigma(u)$∈N^+($y_i\sigma(i)$)∩N^-($y_i\sigma(i)$) and there is no ordinary arc between u $\sigma(u)$ andℂ. Using the fact that $\mathcal{D}$ is locally semicomplete, it is easy to see thatℂ is contained in the fuzzy subdigraph induced by N^+(u $\sigma(u)$)∩N^-(u $\sigma(u)$), a contradiction. Thus, r = n, i.e., the algorithm labels all vertices of $\mathcal{D}$. To complete Case 1, it suffices to prove that$y_1\sigma(y_1)\; y_2\sigma(y_2)\ldots\ldots\ldots y_n\sigma(y_n)$ is a round labelling. Suppose not. Then, there are three vertices$y_a\sigma(y_a)\; y_b\sigma(y_b) y_c\sigma(y_c)$ listed in the circular order in the labelling such that, without loss of generality,$y_a\sigma(y_a)$→$y_c\sigma(y_c)$ and $y_a\sigma(y_a) \nrightarrow y_b\sigma(y_b)$. Assume that the tree vertices were chosen such that the number of vertices from$y_b\sigma(y_b)$ to $y_c\sigma(y_c)$yc in the circular order is as small as possible. This implies that c = b + 1. Since$y_a\sigma(y_a)$ and$y_b\sigma(y_b)$ are both in-neighbours of$y_c\sigma(y_c)$, they are adjacent. Thus, $y_b\sigma(y_b) \nrightarrow y_a\sigma(y_a)$. Since we also have $y_b\sigma(y_b)$→$y_c\sigma(y_c)$ (recall that $y_c\sigma(y_c)$∈ N^+($y_b\sigma(y_b)$yb)−N^-($y_b\sigma(y_b)$)) by the definition of the labelling) and $\mathcal{D}$ contains no induced subdigraph isomorphic to the fuzzy digraph $y_a\sigma(y_a) \nrightarrow y_c\sigma(y_c)$. So,$y_c\sigma(y_c)$ is not the vertex of in-degree zero in the tournament induced by N^+($y_b\sigma(y_b)$) − N^-($y_b\sigma(y_b)$), contradicting the choice of $y_c\sigma(y_c)$.

Case 2:$\mathcal{D}$ D contains no ordinary cycle. If $\mathcal{D}$ has no ordinary arc, $\mathcal{D}$ is complete. Thus, any labelling of V($\mathcal{D}$) is round. So assume that $\mathcal{D}$ has an ordinary arc. Since $\mathcal{D}$ has an ordinary arc, but has no ordinary cycle, we claim that there is a vertex $z_1\sigma(z_1)$ with N^-($z_1\sigma(z_1)$)−N^+($z_1\sigma(z_1)$) =∅ and N^+($z_1\sigma(z_1)$)−N^-($z_1\sigma(z_1)$)≠∅. Indeed, let $w_2\sigma(w_2) w_i\sigma(w_1)$ be an ordinary arc in $\mathcal{D}$. We may set $z_1\sigma(z_1)$ =$w_2\sigma(w_2)$ unless N^-($w_2\sigma(w_2)$)−$N^+w_2\sigma(w_2)$ ≠∅. In the last case there is an ordinary arc whose head is $w_2\sigma(w_2)$. Let $w_3\sigma(w_3) w_2\sigma(w_2)$ be such an arc. Again, either we may set$z_1\sigma(z_1)$ = $w_3\sigma(w_3)$ or there is an ordinary arc $w_4\sigma(w_4) w_3\sigma(w_3)$. Since $\mathcal{D}$ is finite and contains no ordinary cycle, the above process cannot repeat vertices and hence terminates at some vertex $w_i\sigma(w_i)$ such that we may set $z_1\sigma(z_1)$ = $w_j\sigma(w_j)$. We apply the following algorithm to find a path in $\mathcal{D}$. Begin with $z_1\sigma(z_1)$ and, for each i = 1,2,..., let $w_{i+1}\sigma(w_{i+1})$ be the vertex of in-degree zero in the (transitive) tournament induced by $N^+(z_i\sigma(z_i))-N^-(z_i\sigma(z_i))$ unless this set is empty. Since $\mathcal{D}$ has no ordinary cycle, this produces a path P $=z_1\sigma(z_1), z_2\sigma(z_2),\ldots\ldots z_s\sigma(z_s)$ with $N^+(z_s\sigma(z_s))-N^-(z_s\sigma(z_s))$ $=\emptyset$. Applying an argument similar to that used above, we can show that $z_1\sigma(z_1), z_2\sigma(z_2),\ldots\ldots z_s\sigma(z_s)$ is a round labelling of the fuzzysubdigraph induced by V(P). Thus, if P contains all vertices of $\mathcal{D}$, then a round labelling of $\mathcal{D}$ is established. So assume that there is a vertex v not in P, which is adjacent to some vertex of P. It is easy to see that there is no ordinary arc between v and P. This implies that v $\in N^+(z_i\sigma(z_i))\cap N^-(z_i\sigma(z_i))$ for each i = 1,2,...,s. In fact, it is not hard to see that the same is true for every vertex v $\in V(\mathcal{D})-V(P)$. Therefore, if we apply the above algorithm starting from an appropriate vertexnotinP,weobtainanew ordinary pathQandV(Q)∩V(P) =∅. By applying the above algorithm as many times as possible, we obtain a collection of vertex-disjoint ordinary paths P^k = $z_1^1\sigma(z_1^1), z_2^2\sigma(z_2^2)\ldots\ldots z_k^k\sigma(z_k^k))$ k = 1,2,...,t. Let$z_1^{t+1}\sigma(z_1^{t+1})$,...,$z_{m_{t+1}}^{t+1}\sigma(z_{m_{t+1}}^{t+1})$ betheremainingvertices. It is easy to verify that labelling the vertices according to the ordering

$$(z_1^1\sigma(z_1^1), z_2^1\sigma(z_2^1)\ldots\ldots z_m^1\sigma(z_m^1)\ldots\ldots z_1^{t+1}\sigma(z_1^{t+1}),\ldots, z_{m_{t+1}}^{t+1}\sigma(z_{m_{t+1}}^{t+1}))$$

results in a round labelling of $\mathcal{D}$.

IV. Conclusion

Finally we have analyzed about the fuzzy round digraph and the conditions for the fuzzy locally semicomplete digraph being a fuzzy round digraph and their properties.This study will help full in system where fuzzy digraphs are applied.

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