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Common Fixed Point Theorems In 2-Metric Spaces

MRS. SHWETA SRIVASTAVa
assistant professor
Department of mathematics
RBVR reddy college for women , osmaina University , Hyderabad
Email: Shweta.srivastava.2000@gmail.com

Abstract: *Our aim of this paper is to find some more common fixed point theorems satisfying rational type contractive mappings in 2-metric spaces, which are generalizations of various known results.*
Key words: *fixed point theorem , metric space, contraction*

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I. Introduction

The fixed point theory itself is a beautiful mixture of analysis, topology and geometry. Over the last few decades the theory of fixed points has appeared as a very powerful and important tool in the study of nonlinear phenomena. In particular, fixed point techniques have been applied in a variety of diverse fields as biology, chemistry, economics, engineering, game theory and physics. It is also possible to analyses several concrete problems from science and technology, where one is concerned with a system of differential, integral and functional equations.

The first results regarding fixed point theory given by the Polish mathematician, Banach [13] in 1922. He proved a theorem which ensures; under appropriate conditions the existence and uniqueness of a fixed point. This result is known as 'Banach contraction principle'.

After few years many researchers gave different contraction type mappings. In 1969 Kannan [51] gave a new idea for the contractive type mapping .Chatarjee [16] in 1972 gave a new geometrically concept for contraction type mapping, which has given a new direction to the study of the fixed point theory, There have been lots of generalizations of metric space. One such generalization is Menger space in which, used distribution functions instead of nonnegative real numbers as value of metric.

A Menger space is a space in which the concept of distance is considered to be a probabilistic, rather than deterministic. For detail discussion of Menger spaces and their applications we refer to Schweizer and Sklar [91]. The theory of Menger space is fundamental importance in probabilistic functional analysis.

The present work reported in this thesis has been organized in to seven chapters and covered a wide area of metric space like, complete metric space, cone ball metric space, fuzzy metric space, 2- metric space, Menger space, Intuitionistic fuzzy metric space, and proved some fixed point and common fixed point theorems in this directions .

There are lots of generalizations of metric spaces, 2-metric spaces is one of them. The concept of 2-metric space is a natural generalization of the metric space. Initially, it has been investigated by Gahler [30] and has been developed broadly by Gahler [30, 31] and more. After this number of fixed point theorems have been proved for 2-metric spaces. Our aim of this chapter is to find some more common fixed point theorems satisfying rational type contractive mappings, which are generalization of various known results.
To prove of our results, we need some definitions which are as follows;

II. Preliminaries :

Definition 2.1: A sequence $\{x_n\}$ is said to be a Cauchy sequence in 2-metric space X, if for each $a \in X$,

$$\lim_{m,n\to\infty} d(x_n, x, a) = 0$$

Definition 2.2: A sequence $\{x_n\}$ in 2-metric space X is convergent to an element $x \in X$, if for each $a \in X$,

$$\lim_{n\to\infty} d(x_n, x, a) = 0$$

Definition 2.3: A complete 2-metric space is one in which every Cauchy sequence in X converges to an element of X.

Definition 2.4: Let A and S be mappings from a metric space (X, d) in to itself, A and S are said to be weakly compatible if they commute at their coincidence point. i.e., $Ax = Sx$ for some $x \in X$, then $ASx = Sax$.
Definition 2.5: Two self maps f and g of a metric space (X, d) are called compatible if

$$\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

for some t in X.
Definition 2.6: Two self maps f and g of a metric space (X, d) are called non compatible if there exists at least one sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

for some t in X but

$$\lim_{n\to\infty} d(fgx_n, gfx_n)$$

is either non zero or nonexistent.
Definition2.7: Maps f and g are said to be commuting if $fgx = gfx$ for all $x \in X$
Definition2.8: Let f and g be two self maps on a set X, if $fx = gx$ for some x in X then x is called coincidence point of f and g.
Throughout this chapter X is stand for complete 2-metric space.
3.Main theorem

III. Common Fixed Point Theorem for Four Self Mapping

Theorem 3.1: Let S, T be any two self mappings of a 2- metric space X satisfying the condition

$$\begin{aligned} d(Su, Tv, a) \le\ & \alpha_1 \left[\frac{d^2(u,Sw,a)+d^2(u,v,a)}{1+d(u,Sw,a)+d(u,v,a)}\right] \\ & +\alpha_2 \left[\frac{d^2(v,Tt,a)+d^2(Sw,Tt,a)}{1+d(v,Tt,a)+d(Sw,Tt,a)}\right] \\ & +\alpha_3\sqrt{d(v, Sw, a).\, d(u, Tt, a)} + \alpha_4[d(sw, Tt, a)] \\ & +\alpha_5[d(u, v, a)] \qquad 3.1(i) \end{aligned}$$

for all $u, v, w, t \in X$ where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are non negative reals such that $2\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 < 1$, then S, T have a unique common fixed point.
Proof: Let x_0 be an arbitrary element of X and we construct a sequence $\{x_n\}$ defined as follows
$Sx_{n-1} = x_n, Tx_n = x_{n+1}, Sx_{n+1} = x_{n+2}, Tx_{n+2} = x_{n+3}$
and $TSx_{n-1} = x_{n+1}, STx_n = x_{n+2}, TSx_{n+1} = x_{n+3}, STx_{n+2} = x_{n+4}$
where $n = 1, 2, 3, \ldots$
Now putting $u = Ty$, $v = Sx$, $w = x$ and $t = y$ in 5.2.1(i)then we have

$$\begin{aligned} d(STy, TSx, a) \le\ & \alpha_1 \left[\frac{d^2(Ty,Sx,a)+d^2(Ty,Sx,a)}{1+d(Ty,Sx,a)+d(Ty,Sx,a)}\right] \\ & +\alpha_2 \left[\frac{d^2(Sx,Ty,a)+d^2(Sx,Ty,a)}{1+d(Sx,Ty,a)+d(Sx,Ty,a)}\right] \\ & +\alpha_3\sqrt{d(Sx, Sx, a).\, d(Ty, Ty, a)} + \alpha_4[d(Sx, Ty, a)] \\ & +\alpha_5[d(Ty, Sx, a)] \end{aligned}$$

$$\begin{aligned} d(STy, TSx, a) \le\ & 2\alpha_1 d(Sx, Ty, a) \\ & +2\alpha_2 d(Sx, Ty, a) + \alpha_4 d(Sx, Ty, a). \end{aligned}$$

$+\alpha_5 d(Sx, Ty, a).$ 3.1(ii)
Now putting $x = x_{n-1}$ and $y = x_n$ in 5.2.1(ii)then we have

$$\begin{aligned} d(STx_n, TSx_{n-1}, a) \le\ & 2\alpha_1 d(Sx_{n-1}, Tx_n, a) \\ & +2\alpha_2 d(Sx_{n-1}, Tx_n, a) + \alpha_4\, d(x_{n+2}, x_{n+1}, a) \\ & +\alpha_5 d(Sx_{n-1}, Tx_n, a) \end{aligned}$$

$$\begin{aligned} d(x_{n+2}, x_{n+1}, a) \le\ & 2\alpha_1 d(x_n, x_{n+1}, a) \\ & +2\alpha_2 d(x_n, x_{n+1}, a) + \alpha_4 d(x_n, x_{n+1}, a) \\ & +\alpha_5 d(x_n, x_{n+1}, a) \qquad 3.1(iii) \end{aligned}$$

from 2.1(iii)we conclude that $d(x_{n-1}, x_n, a)$ decreases with n.
i.e., $d(x_{n-1}, x_n, a) \to d(x_0, x_1, a)$ when $n \to \infty$
If possible let $d(x_0, x_1, a) > 0$ and taking limit $n \to \infty$ on 3.1(iii) then we have
$d(x_0, x_1, a) \le 2\alpha_1 d(x_0, x_1, a) + 2\alpha_2 d(x_0, x_1, a) + \alpha_4 d(x_0, x_1, a) + \alpha_5 d(x_0, x_1, a)$
$= (2\alpha_1 + 2\alpha_2 + \alpha_4 + \alpha_5) d(x_0, x_1, a)$

$< d(x_0, x_1, a)$

Since $2\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 < 1$. Which is not possible hence

$d(x_0, x_1, a) = 0$.

Next we shall show that $\{x_n\}$ is Cauchy sequence.

Now

$d(x_m, x_n, a) \le d(x_m, x_{m+1}, a) + d(x_{m+1}, x_{n+1}, a) + d(x_{n+1}, x_n, a)d(x_{n+1}, x_n, a)$

$d(x_m, x_n, a) \le d(x_m, x_{m+1}, a)$

$+d(x_n, x_{n+1}, a) + d(Sx_n, Tx_m, a)$ 3.1(iv)

On putting $u = x_n, v = x_m, w = x_{m-1}, t = x_{n-1}$ in 3.1(i) then we have

$$d(Sx_n, Tx_m, a) \le \alpha_1 \left[\frac{d^2(x_n,Sx_{m-1},a)+d^2(x_n,x_m,a)}{1+d(x_n,Sx_{m-1},a)+d(x_n,x_m,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(x_m,Tx_{n-1},a)+d^2(Sx_{m-1},Tx_{n-1},a)}{1+d(x_m,Tx_{n-1},a)+d(Sx_{m-1},Tx_{n-1},a)}\right]$$

$$+\alpha_3 \sqrt{d(x_m, Sx_{m-1}, a). d(x_n, Tx_{n-1}, a)} +\alpha_4[d(x_n, x_m, a)]$$

$$+ \alpha_5[d(x_n, x_m, a)]$$

$$= \alpha_1 \left[\frac{d^2(x_n,x_m,a)+d^2(x_n,x_m,a)}{1+d(x_n,x_m,a)+d(x_n,x_m,a)}\right]$$

$$+\alpha_2 \left[\frac{d(x_m,x_n,a)+d(x_m,x_n,a)}{1+d^2(x_m,x_n,a)+d^2(x_m,x_n,a)}\right]$$

$$+\alpha_3 \sqrt{d(x_m, x_m, a). d(x_n, x_n, a)} + \alpha_4[d(x_n, x_m, a)]\ \alpha_5[d(x_n, x_m, a)]$$

$$= 2\alpha_1 d(x_n, x_m, a) + 2\alpha_2 d(x_n, x_m, a) + \alpha_4 d(x_n, x_m, a) + \alpha_5 d(x_n, x_m, a)$$

$$d(Sx_n, Tx_m, a) \le (2\alpha_1 + 2\alpha_2 + \alpha_4 + \alpha_5)d(x_n, x_m, a) \qquad 3.1(v)$$

from 3.1(iv) and 3.1(v)we have

$$d(x_m, x_n, a) \le d(x_m, x_{m+1}, a) + d(x_n, x_{n+1}, a)$$

$$+(2\alpha_1 + 2\alpha_2 + \alpha_4 + \alpha_5)d(x_n, x_m, a)$$

Letting $m, n \to \infty$ then $d(x_n, x_m, a) \to 0$, as $2\alpha_1 + 2\alpha_2 + \alpha_4 + \alpha_5 < 1$

Hence $\{x_n\}$ is a Cauchy sequence.

Now we prove z is a common fixed point of S, T.

On putting $u = z, v = x_{n-1}, w = z$ and $t = x_{n-2}$ in 3.1(i) we have

$$d(Sz, Tx_{n-1}, a) \le \alpha_1 \left[\frac{d^2(z,Sz,a)+d^2(z,x_{n-1},a)}{1+d(z,Sz,a)+d(z,x_{n-1},a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(x_{n-1},Tx_{n-2},a)+d^2(Sz,Tx_{n-2},a)}{1+d(x_{n-1},Tx_{n-2},a)+d(Sz,Tx_{n-2},a)}\right]$$

$$+\alpha_3 \sqrt{d(x_{n-1}, Sz, a). d(z, Tx_{n-2}, a)} +\alpha_4 d(Sz, Tx_{n-2}, a)$$

$$+\alpha_5[d(z, x_{n-1}, a)]$$

$$d(Sz, x_n, a) \le \alpha_1 \left[\frac{d^2(z,Sz,a)+d^2(z,x_{n-1},a)}{1+d(z,Sz,a)+d(z,x_{n-1},a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(x_{n-1},x_{n-1},a)+d^2(Sz,x_{n-1},a)}{1+d(x_{n-1},x_{n-1},a)+d(Sz,x_{n-1},a)}\right]$$

$$+\alpha_3 \sqrt{d(x_{n-1}, Sz, a). d(z, x_{n-1}, a)} +\alpha_4[d(sz, x_{n-1}, a)]. +\alpha_5[d(z, x_{n-1}, a)].$$

Letting $n \to \infty$ then we have

$$d(Sz, z, a) \le \alpha_1 \left[\frac{d^2(z,Sz,a)+d^2(z,z,a)}{1+d(z,Sz,a)+d(z,z,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(z,z,a)+d^2(Sz,z,a)}{1+d(z,z,a)+d(Sz,z,a)}\right]$$

$$+\alpha_3 \sqrt{d(z, Sz, a). d(z, z, a)} + \alpha_4[d(sz, z, a)] + \alpha_5[d(z, z, a)]$$

$\Rightarrow d(Sz, z, a) \le (\alpha_1 + \alpha_2)d(Sz, z, a)$.

$\Rightarrow d(Sz, z, a) < d(Sz, z, a)$ Since $2\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 < 1$.

Which gives $d(Sz, z, a) = 0 \Rightarrow Sz = z$.

Thus z is a fixed point of S.

Similarly we can show that z is a fixed point of T.

Hence z is a common fixed point of S, T.

We are taking one another point q which is not equal to z such that

$Sq = q = Tq$.

On putting $u = z, v = q, w = q, t = z$ in 3.1(i)then we have

$$d(Sz, Tq, a) \le \alpha_1 \left[\frac{d^2(z,Sq,a)+d^2(z,q,a)}{1+d(z,Sq,a)+d(z,q,a)}\right]$$

$$+\alpha_2 \left[\frac{d^2(q,Tz,a)+d^2(Sq,Tz,a)}{1+d(q,Tz,a)+d(Sq,Tz,a)}\right]$$

$$+\alpha_3 \sqrt{d(q, Sq, a). d(z, Tz, a) +} \alpha_5[d(sq, Tz, a)]$$

$+\alpha_4[d(z,q,a)]$

$$d(z,q,a) \le \alpha_1\left[\frac{d^2(z,q,a)+d^2(z,q,a)}{1+d(z,q,a)+d(z,q,a)}\right]$$

$$+\alpha_2\left[\frac{d^2(q,z,a)+d^2(q,z,a)}{1+d(q,z,a)+d(q,z,a)}\right]$$

$$+\alpha_3\sqrt{d(q,q,a).d(z,z,a)}+\alpha_4[d(q,z,a)]$$

$$+\alpha_5[d(z,q,a)]$$

$$d(z,q,a) \le (2\alpha_1+2\alpha_2+\alpha_4+\alpha_5)d(z,q,a)$$

$$d(z,q,a) < d(z,q,a)$$

Since $2\alpha_1+2\alpha_2+\alpha_3+\alpha_4 < 1$.

Which gives $d(z,q,a)=0 \Rightarrow z=q$.

Hence z is unique. This completes the proof of the theorem.

Corollary 3.2: Let S, T, R be any three self mappings of a 2- metric space X satisfying the condition

$$d(SRu,TRv,a) \le \alpha_1\left[\frac{d^2(u,SRw,a)+d^2(u,TRt,a)+d^2(u,SRw,a)}{1+d(u,SRw,a)+d(u,TRt,a)+d(u,SRw,a)}\right]$$

$$+\alpha_2\left[\frac{d^2(v,SRw,a)+d^2(u,TRt,a)+d^2(v,TRt,a)}{1+d(v,SRw,a)+d(u,TRt,a)+d(v,TRt,a)}\right]$$

$$+\alpha_3\sqrt{d(v,SRw,a)d(u,TRt,a)}$$

$$+\alpha_4[d(SRw,TRt,a)]$$

$$+\alpha_5[d(u,v,a)].$$

for $u,v,w,t \in X$ where $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5$ are non negative reals such that $\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5 < 1$ then SR, TR have a unique common fixed point.

References

[1]. Aage, C. T. and Salunke, J. N., "On Fixed Point Theorems in Fuzzy Metric Spaces." Int. J. Open Problems Compt. Math., 3 (2), **(2010)**, 123-131.

[2]. Banach, S. "Surles Operation Dansles Ensembles Abstraites Etleur Application Integrals." Fund. Math., 3, **(1922)**, 133-181.

[3]. Cho, Y.J, "Fixed Points for Compatible Mappings of Type (A)." Math. Japonica, 38(3), **(1993)**, 497-508.

[4]. Cho Y.J, Khan M.S. and Singh S.L., Common Fixed Points of Weakly Commutating Mappings." Univ. Novom Sadu, Sb.Rd. Prirod-Mat. Fak. Ser. Mat., 18(1), **(1988)**, 129-142.

[5]. Grabiec, M., "Fixed Points in Fuzzy Metric Spces." Fuzzy Sets and Systems, 27, **(1988)**, 385-389.

[6]. Gregori, V., Romaguera, S. and Veeramani, P., "A Note on Intuitionistic Fuzzy Metric Spaces." Chaos Solitons & Fractals, 28(4), **(2006)**, 902-905.

[7]. George, A., Veeramani P., "On Some Results in Fuzzy Metric Spaces." Fuzzy Sets and Systems, 64, **(1994)**, 395-399.

[8]. Hadzic, O. and Pap, E., "Fixed point theory in PM- spaces." Dordrecht Kluwer Academic Publishers, **(2001).**

[9]. an, M.D., "A Common Fixed Point Theorem in 2-Metric Spaces." Math. Japonica, 36(5), **(1991)**, 907-914.

[10]. Kannan, R., "Some Results on Fixed Point Theorems." Bull. Cat. Math. Soc., 60, **(1968)**, 71-78.

[11]. Kannan, R., "Some Results on Fixed Points- II." Amer. Bull. Maths. Monthly, 76, **(1969)**, 405 -408.

[12]. Kramosil, O. and Michalek, J., "Fuzzy Metrics and Statistical Metric Spaces." Kybernetika, 15, **(1975)**, 326-334.

[13]. Lakshmikantham, V. and Ciric, L., "Coupled Fixed Point Theorems for Nonlinear Contractions in Partially Ordered Metric Spaces." Nonlinear Analysis, 70(12), **(2009)**, 4341-4349.

[14]. Maiti, M. and Pal, T.K., "Generalization of Two Fixed Point Theorems." Bull. Calcutta Math. Soc., 70, **(1978)**, 57- 61.

[15]. Meir, A. and Keeler E., "A Theorem on Contraction Mappings." J. Math. Anal. Appl., 28, **(1969)**, 326-329.

[16]. Menger, K., "Probabilistic Geometry." Proc. Nat. Acad. Sci. U.S.A., 37, **(1951)**, 226–229.

[17]. Menger, K., "Statistical Metric." Proc. Nat. Acad. Sci. U.S.A., 28(12), **(1942)**, 535-537.

[18]. Mihet, D., "Fixed Point Theorem in Fuzzy Metric Spaces Using Property E. A." Nonlinear Anal., 73, **(2010)**, 2184-2188.

[19]. Mustafa, Z. and Sims B., "A New Approach to Generalized Metric Spaces." J. Nonlinear and Convex Anal., 7(2), **(2006)**, 289-297.

[20]. Muthuy, P.P, Chang, S.S, Cho Y.J. and Sharma B.K., "Compatible Mappings of Type (A) and Common Fixed Point Theorems." Kyungpook Math, J., 32(2), **(1992)**, 203-216.

[21]. Naschie, M.S. El., "On the Uncertainty of Cantorian Geometry and the Two Slit Experiment." Chaos, Solitons & Fractals, 9(3), **(1998)**, 517-529.

[22]. Naschie, M.S. El., "Quantum Gravity, Clifford Algebras, Fuzzy Set Theory and the Fundamental Constants of Nature." Chaos, Solitons & Fractals, 20(3), **(2004)**, 437- 450.

[23]. Nigam, P. and Malviya, N., "Some Fixed Point Theorems for Occasionally Weakly Compatible Mappings in Fuzzy Metric Spaces." International Journal of Theoretical and Applied Science, 3(1), **(2011)**, 13-15.

[24]. Pant, R.P., Jha K., "A Remark On Common Fixed Points Of Four Mappings In A Fuzzy Metric Space." J. Fuzzy Math, 12(2), **(2004)**, 433-437.

[25]. Pant R.P., "Common Fixed Points of Four Mappings." Bull. Cal. Math. Soc., 90, **(1998)**, 281-286.

[26]. Pant, R. P., "R-Weak Commutatibility and Common Fixed Points." Soochow Journal Math, 25, **(1999)**, 37-42.

[27]. Pathak, H.K., Kang, S.M. and Back, J.H., "Weak Compatible Mappings of Type (A) and Common Fixed Points." Kyungbook Math. J., 35, **(1995)**, 345- 359.

[28]. Pathak, H.K. and Singh, P.," Common Fixed Point Theorem for Weakly Compatible Mapping." International Mathematical Forum, 2(57), **(2007)**, 2831-2839.

[29]. Park, J. H., "Intuitionistic Fuzzy Metric Spaces." Chaos Solitons & Fractals, 22, **(2004)**, 1039-1046.

[30]. Park, J. H., Park, J. S. and Kwun, Y. C., "A common fixed point theorem in intuitionstic fuzzy metric spaces." Advance in Natural Computing Data Mining (Proceeding 2nd ICNC and 3rd FSKK), **(2005)**, 293-300.

[31]. Park, J. S. and Kwun, Y. C. and Park, J.H., "Compatible Mapping and Compatible Mapping of Type α and β in Intuitionistic Fuzzy Metric Spaces." Demonstration Mathematica, 39(3), **(2006)**, 671-684.

[32]. Popa, V., "Fixed Points for Non-Surjective Expansion Mappings Satisfying an Implicit Relation." Bull. Stiint. Univ. Baia Mare Ser. B. Fasc. Mat. Inform. 18(1), **(2002)**, 105-108.
[33]. Rhoades, B.E., "Some Theorems on Weakly Contractive Maps." Nonlinear Anal, 47, **(2001)**, 2683-2693.
[34]. Rhoades, B.E., "Contractive Definitions and Continuity." Contemporary Math., 72, **(1988)**, 233-245.
[35]. Romaguera, S. and Tirado, P., "Contraction Maps on Ifqm-Spaces with Application to Recurrence Equation of Quicksort." Electronics Notes in Theoretical Computer, 225, **(2009)**, 269-279.
[36]. Saadati, R. and Park, J. H., "On the Intuitionistic Fuzzy Topological Spaces." Chaos Solitons & Fractals, 27(2), **(2006)**, 331-344.
[37]. Sabetghadam, F., Masiha, H. P. and Sanatpour, A. H., "Some Coupled Fixed Point Theorem in Cone Metric Spaces." Fixed Point Theory Appl. **(2009)**, Art. ID 125426.
[38]. Samet, B., "Coupled Fixed Point Theorem for Generalized Meir-Keeler Contaction in Partially Ordered Metric Space." Nonlinear Anal., 72 (12), **(2010)**, 4508-4517.
[39]. Schweizer, B. and Sklar, A., "Probabilistic Metric Spaces." Elsevier North Holland New York, 15, **(1983).**
[40]. Sehgal, V. M. and Bharucha-Reid, A. T., "Fixed Point of Contraction Mappings on PM Spaces." Math. Syst. Theory, 6, **(1972)**, 97-100.
[41]. Sessa S., "On a Weak Commutability Condition in Fixed Point Considerations." Publ. Inst. Math. (Beograd), 32(46), **(1982)**, 146-153.
[42]. Singh, B. and Jain, S., "Semi Compatibility and Fixed Point Theorem in Menger Space." J. Chungcheong Math Soc., 17(1), **(2004)**, 1-17.
[43]. Turkoglu, D., Alaca, C., Cho, Y. J. and Yildiz, C., "Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Spaces." Journal of Applied Mathematics and Computing, 22(12), **(2006)**, 411-424.
[44]. Vasuki, R., "Common Fixed Points for R-Weakly Commuting Maps in Fuzzy Metric Spaces." Indian J. Pure Appl. Math, 30, **(1999)**, 419-423.
[45]. Wang, W.Z., "Common Fixed Points for Compatible Mappings of Type (A) in 2-Metric Spaces." Honam Math. J., 22, (2000), 91-97.
[46]. Zadeh, L.A., "Fuzzy sets." Inform and Control, 8, **(1965)**, 338-353.
[47]. Zhang, Q, Song, Y., "Fixed Point Theory for Generalized Φ-Weakly Contraction." Appl. Math Lett., 22, **(2009)**, 75-78.

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