IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 14, Issue 3 Ver. II (May - June 2018), PP 77-85
www.iosrjournals.org

Statistical Analysis of Human Fertility Behavior Using Probability Distributions

H. R. Bakari, M. Mustapha and F. B. Ladan
University of Maiduguri, Maiduguri, Nigeria Department of Mathematical Sciences
Corresponding author: H. R. Bakari

Abstract: The study is aimed at using Gamma and Weibull probability distributions to analyze human fertility behavior in Borno State. Very little insight has been shed on the determinants or consequences of regional differences in fertility regimes on women's lives and fertility transitions. The study therefore employedex-post-factor as research design. This design was appropriately chosen because it is not possible for the researcher to directly manipulate the independent variables (demographic factors). In other words data will be collected after the phenomenon under investigation has taken place.

A sample size of eight hundred and fifty five (855) was selected. Which was believed to be of mix nature as no proper frame could be available to use an adequate sampling scheme (disproportionate stratified sampling). However, every attempt was made to make this sample as a representative for the State.

From the analysis of human fertility it reveals that the socio economic variable is an important factor in human fertility. The age at marriage has a vital role in human fertility. The R^2 value is only 65% in fertility. Other than these variables such as biological, economical and demographic variables put together could contribute only 44% for human fertility. Using the Poisson distribution the age at marriage shows close relation to the human fertility.

Keywords: Demographic factors, Fertility, Gamma and Weibull distributions

---

Date of Submission: 28-05-2018 Date of acceptance: 11-06-2018

I. Introduction

A stochastic process is the mathematical abstraction of an empirical process whose development is governed by probabilistic laws. A stochastic model gives the probability of an event. Stochastic modelling of any physical, biological, social or economic system can give better representation, description and specification than a purely deterministic model. This is because it can take into account the uncertainty on the randomness, which the process may experience and which is not considered in a purely mathematical modelling, without introducing the element of probability of the system. In view of this additional feature, in this study we try to present our results in stochastic models using probability distribution. Very little is known about population determinants in Northern Nigeria, where data on basic demographic indicators show growing differentials across regions in both fertility and health regimes. Very little insight has been shed on the determinants or consequences of regional differences in fertility regimes on women's lives and fertility transitions. It is becoming increasingly clear that women's position in the family and community has an important bearing on fertility regimes in Nigeria but the implications for policies and programmes remain unclear. One task for the research community in the years ahead consists in addressing the issue as to how women's position can be improved in societies that resist educating their daughters and allowing their women to work outside the home.

Nigeria engaged in a national discussion of its population trends, which was followed by the adoption of a national population policy in 1988 and subsequent expansion of government-sponsored family planning services. Not surprisingly, donor assistance for population programs increased in this period, focusing on strengthening national policy and program efforts through technical and financial assistance. The government's objective, shared by donors, was to increase the use of modern contraceptives as a means of improving maternal and child health. As such, the federal government set about building and extending its maternal and child health services under which family planning was one component. Some donors, however, channeled their support largely to the family planning component of those services (e.g. for training family planning personnel, equipping health facilities and supplying contraceptive commodities).

Donors also increased their support to operations and applied research during this period. For instance, studies were supported to improve understanding of: (a) barriers to effective family planning delivery (Aboderin 1986; Akintunde 1986; Coleman 1988; Masha 1986; NTA 1987; Rimon 1986); (b) contraceptive innovators and how their numbers could be increased (Oni 1986; Makinwa-Adebusoye 1991, 1992); and (c) impediments to contraceptive acceptance and how they could be removed (Covington et. al. 1986; Makinwa-Adebusoye 1993).

A number of clinic-based studies were undertaken as services expanded (e.g. Kim et. al. 1992). The impact of modernization in reducing the duration of postpartum abstinence and on fertility was also emphasized (e.g. Aborampah 1987).Although no study has evaluated that hypothesis with empirical data, the view continues to persist that men are the dominant decision-makers on fertility matters in Africa (Isiugo-Abanihe 1994; Renne 1993). In recent years, some researchers have started gathering data that allow them to address the issue of the role of husbands in fertility and family planning decisions in Nigeria. Oni and McCarthy (1991),

Bankole (1995) also looked at the importance of spousal agreement for reproductive outcomes. In particular, he examined the effects of joint fertility desires on fertility using panel data from 1984 and 1986 surveys. He found that spousal agreement/disagreement is a significant determinant of subsequent fertility. In cases where the couple disagrees on the desire for more children, his analysis shows that subsequent fertility falls between the fertility of spouses who want more children and those who want to stop having more children. Although he found that the desires of both spouses carry the same weight on subsequent fertility, when he disegregated the analysis by the number of living children, he found that the husband's desires are more important when the family size is small but that the wife's desires become more important when the number of living children is large. He interprets this finding using a life cycle argument, noting that in the Yoruba cultural context, women obtain increased autonomy and status within the household as they secure their position within their natal families.

Matching population growth with development is the real object of global and country action towards improved welfare and human development and economic growth. The puzzling phenomenal difference in levels of welfare and development among the populations of countries are largely explained by the divergence in the nature and magnitude of the dynamics of populations (Ogujiuba, 2006).

It has long been realized that population growth varies over space. Therefore, Borno State is not an exception. All demographic transition models emphasized the synchronization of respective mortality and fertility patterns. Placing mortality decline as a pre-condition for fertility decline formed the cornerstone of the theory. The experience of some African countries also shows that fertility can decline independently of the degree of socio economic development (Kirk, 1996).

According to the theory of demographic transition (ECA 2001 and Cowgil 2002), the shift towards low mortality and fertility rates occurs when there is a process of overall modernization resulting from industrialization, urbanization, education, as well as substantial overall socio-economic development. Such a shift leads initially to a drop in mortality through progress in hygiene and medicine and, subsequently, to a decline in fertility occasioned by economic growth.

Since the last four decades, there has been a remarkable development in the field of mathematical and stochastic models which have found immense application in almost all discipline including the biological system. Mathematics has played a fundamental role in many physical and engineering sciences.

The past century has seen a tremendous growth of physics, astronomy, engineering and space sciences as a result of application of mathematics to these disciplines. In life sciences, however, the application of mathematics has not been made on such a large scale. The lack of this development is partly due to the complicated nature of the biological phenomena themselves. In spite of these difficulties, mathematics has been instrumental in some remarkable discoveries such as those in the study of heredity leading to the development of the science of genetics.

Mathematical models of a natural phenomenon are essential to the development of a theory of the phenomenon. There are large amount of experimental data available in the field of scientific endeavor but unless it is unified by a theory, it is of limited use. Theories are only mnemonic devices to save us from the impossible task of storing all possible information, Bellman (1963). In the development of theories mathematics plays a basic role. In the study of living systems, especially in clinical medicine, there are a large number of problems which need mathematical treatment.

As Sheps and Perin (1963) and Menken (1975), among others, have pointed out, simplified models, unrealistic though they may be, have proved useful in gaining insights such as that a highly effective contraceptive used by a rather small proportion of a population reduces birth rates more than does a less effective contraceptive used by a large proportion of the population. Some fertility researchers have been modeling parts rather than the whole of the reproductive process. The components of birth intervals have been examined, with emphasis on the physiological and behavioral determinants of fertility (see Leridon, 1977). Another focus has been abortions, induced and spontaneous (see: Abramson, 1973; Potter et al., 1975; Michels and Willett, 1996). Fecund- ability investigations have been yet another focus (see Menken, 1975; Wood et al., 1994). Menken (1975) alerts researchers to the impossibility of reliably es- timatingfecundability from survey data. The North Carolina Fertility Study referred to in Dunson and Zhou (2000) is of interest in this connection: In that study couples were followed up from the time they discontinued birth control in order to attempt pregnancy. The enrolled couples provided base-line data and then information regarding ovulation in each menstrual cycle, day-by-day reports on intercourse, first morning urine samples, and the like. Dunson and Zhou present a Bayesian Model and Wood et al. (1994) present a multistate model for the analysis of fecundability and sterility.

1.1 Distributions uses and its properties
1.1.1 Poisson distribution

In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses the probability of a number of events occurring in a fixed of time if these events occur with a known average rate and independently of the time since the last event.

The Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume. The distribution was discovered by Simeon - Denis Poisson (1781-1840) and published, together with his probability theory, in 1838 in his work Recherchessur la probabilite des judgments enmatierescriminelles et matierecivile ("Research on the Probability of Judgements in Criminal and Civil Matters"). The work focused on certain random variables N that count, among other things, a number of discrete occurrences (sometimes called "arrivals") that take place, during a time-interval of given length. If the expected number of occurrences in this interval is λ, then the probability that there are exactly k occurrences (k being a non-negative integer, k = 0,1,2,...) is equal to

$$f(k,\lambda)=\frac{\lambda^k e^{-\lambda}}{k!} \qquad (1.1)$$

The Poisson distribution can be applied to systems with a large number of possible events, each of which is rare. A classic example is the nuclear decay of atoms. The Poisson distribution is sometimes called a Poissonian, analogous to the term Gaussian for a Gauss or normal distribution.

1.1.2 Properties

> The expected value of a Poisson - distributed random variable is equal to and λ so is its variance. The higher moments of the Poisson distribution are Touchard Polynomials in λ, whose coefficients have a combinatorial meaning. In fact when the expected value of the Poisson distribution is 1, then Dobinski's formula says that the nth moment equals the number of partitions of a set of size n.

> The mode of a Poisson - distributed random variable with non - integer λ is equal to $|\lambda|$ which is the largest integer less than or equal to λ. This is also written as floor (λ). When λ is a positive integer, the modes are λ. and λ - 1.

1.1.3 Gamma distribution Characterization

That a random variable X is gamma - distributed with scale 0 and shape k is denoted.
$X \sim \Gamma k,\theta$ or $X - Gamma(k,\theta)$

1.2 Probability density function

The probability density function of the gamma distribution can be expressed in terms of the gamma function parameterized in terms of a shape parameter k and scale parameter θ:

$$f(x,k,\theta)=x^{k-1}\frac{e^{-\frac{x}{\theta}}}{\theta^k\Gamma(k)} \quad \text{for } x>0 \text{ and } k,\theta>0 \qquad (1.2)$$

Alternatively, the gamma distribution can be parameterized in terms of a shape parameter $\alpha = k$ and an inverse scale parameter $\beta=\frac{1}{\theta}$, called a rate parameter:

$$g(x,\alpha,\beta)=x^{\alpha-1}\frac{\beta^\alpha e^{-\beta\alpha}}{\Gamma(\alpha)} \qquad x>0 \qquad (1.3)$$

If α is a positive integer then $\Gamma(\alpha)=(a-1)!$

Both parameterizations are common because either can be more convenient depending on the situation.

1.2.1 Cumulative distribution function.

The cumulative distribution function is the regularized gamma function, which can be expressed in terms of the incomplete gamma function,

$$F(x,k,\theta)=\int_0^c f(u,k,\theta)du=\frac{\gamma(k,\frac{x}{\theta})}{\Gamma(k)} \quad (1.4)$$

1.2.2.Properties
Summation
If X_i has $\Gamma(k,\theta)$ distribution for i=1,2,...,N, then

$$\sum_{i=1}^{N} x_i \sim (\sum_{i=1}^{N} k,\theta) \quad (1.5)$$

For an t> 0 it holds that tx is distributed $\Gamma(k,t\theta)$, demonstrating that θ is a scale parameter.

1.2.3 Exponential family
The Gamma distribution is a two - parameter exponential family with natural parameters k-1 and -1/0 and natural statistics x and In(x).
Information entropyThe information entropy is given by provided all X_1 are independent.
The gamma distribution exhibits infinite divisibility.

$$\frac{-1}{\theta^k\Gamma(k)}\int_0^c \frac{x^{k-1}}{e^{x/\theta}}[(k-1)\ln x - x/\theta - k\ln\theta\Gamma(k)]dx \quad (1.6)$$

$$=-[(k-1)(\ln\theta+\varphi(k))-k-k\ln\theta-\ln\Gamma(k)]$$

$$=k+\ln\theta+\ln\Gamma(k)+(1-k)\varphi(k) \quad (1.7)$$

Where $\varphi(k)$ is the digamma function
Kuliback-Leibler divergence
The directed Kuliback - Leibler divergence between $\Gamma(\alpha_0,\beta_0)$ ('true' distribution and $\Gamma(\alpha,\beta)$ ('approximating' distribution) is given by

$$D_{KL}(\alpha_0,\beta_0 \parallel \alpha,\beta)=Log(\frac{\Gamma(\alpha)\beta_0}{\Gamma(\alpha_0)\beta^{\alpha}}+(\alpha_0-\alpha)\varphi(\alpha_0)+\alpha_0\frac{\beta-\beta_0}{\beta_0} \quad (1.8)$$

1.2.4 Laplace transform
The Laplace transformation of the gamma distribution is

$$F(s)=\frac{\beta^{\alpha}}{(s+\beta)^{\alpha}} \quad (1.9)$$

1.3. Weibull distribution
In probability theory and statistics, the Weibull distribution (named after Waloddi Weibull) is a continuous probability distribution. It is often called the Rosin - Rammier distribution when used to describe the size distribution of particles. The distribution was introduced by P. Rosin and E. Rammier in 1933. The probability density function is:

$$f(x,k,\lambda)=\frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}e^{-(x/\lambda)^k} \quad (1.10)$$

For $x>0$ and $f(x,k,\lambda)=0$ for $x\le 0$where $k>0$, $\lambda>0$ is the shape parameter and X>0 is the scale parameter of the distribution. Its complementary cumulative distribution function is a stretched exponential.

The Weibull distribution is often used in the field of life data analysis due to its flexibility - it can mimic the behaviour of other statistical distributions such as the normal and the exponential. It the failure rate decreases over time, then k < 1. If the failure rate is constant over time, then k=1. If the failure rate increases over time then k> 1.

1.3.1 Properties
The nth raw moment is given by:

$$M_n=\lambda^n\Gamma(1+\frac{n}{k}) \quad (1.11)$$

$\mu(x) = \lambda\Gamma(1+\frac{1}{k})$ and (1.12)

$\mathrm{var}(x) = \lambda^2[\Gamma(1+\frac{2}{k}) - \Gamma^2(1+\frac{1}{k})]$ theSkewness is given by (1.13)

$\gamma_1 = \frac{\Gamma(1+\frac{3}{k})\lambda^3 - 3\mu\sigma^2 - \mu^3}{\sigma^3}$ The excess kurtosis is given by: (1.14)

$$\gamma_2 = \frac{-6\Gamma_1^4 + 12\Gamma_1^2\Gamma_2 - 3\Gamma_2^2 - 4\Gamma_1\Gamma_3 + \Gamma_4}{[\Gamma_2 - \Gamma_1^2]^2} \quad (1.15)$$

Where $\Gamma i = \Gamma(1+\frac{i}{k})$. The kurtosis excess may also be written as:

$$\gamma_2 = \frac{\lambda^4\Gamma(1+\frac{4}{k}) - 4\gamma_1\sigma^3\mu - 6\mu^2\sigma^2 - \mu^4}{\sigma^4} \quad (1.16)$$

A generalized, 3-parameter Weibull distribution is also often found in the literature. It has the probability density function.

$$f(x;k,\lambda,\theta) = \frac{k}{\lambda}(\frac{x-\theta}{\lambda})^{k-1} e^{-(\frac{x-\theta}{\lambda})^k} \quad (1.17)$$

For $\geq \theta$ and $f(x,k,\lambda,\theta) = 0$ for $x < \theta$ where $k > 0$, is the shape parameter and λ >0 and θ is the location parameter of the distribution. When θ =0, this reduces to the 2- parameter distribution. Thu cumulative distribution function for the 2 –parameter Weibull is

$$F(x;k,\lambda) = 1 - e^{-(x/\lambda)^k} \quad (1.18)$$

For $x \geq 0$ and $F(x;k,\lambda) = 0$ for $x < 0$

$$F(x;k,\lambda,\theta) = 1 - e^{-(\frac{x-\theta}{\lambda})^k} \quad (1.19)$$

For $x \geq 0$ and $F(x;k,\lambda,\theta) = 0$ for $x < 0$

The failure rate h (or hazard rate) is given by

$$h(x,k,\lambda) = \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1} \quad (1.20)$$

1.4 Model for Human fertility

As defined earlier fertility is the inherent biological capacity to produce births. This capacity involves the genetic inheritance which is beyond our measurement. Hence in this study we have considered the other personal factors which would help women to increase the fertility. Accordingly the factors considered thus are Education of women, Education of husband, Income of women, Income of Husband and Age of the women at the last child birth. For each one of the above variable separate stochastic models are fitted in this.

The data pertaining to the above are completely taken from three Senatorial Districts of Borno State. With each having two local government areas. They are,

1. Maiduguri (MMC)
2. Jere LGA
3. Monguno LGA
4. Magumeri LGA
5. Biu LGA
6. Askira/Uba LGA

The scatter diagram for each one of the independent variable showed an increase initially and then a continuous decrease till the end and hence the gamma distribution and it's generalized form Weibull distribution are considered for representation. Below we give the methods adopted in fitting the above two models.

1.4.1 Gamma or Type III distribution

The general form of the frequency function of the gamma distribution with parameters a and n is

$$f(x) = \frac{a^n}{\Gamma n} e^{-nx} x^{n-1} \qquad \begin{matrix} x > 0 \\ n > 0 \end{matrix} \qquad (1.21)$$

i.e of the form

$$y = ke^{-ax} x^{n-1} \qquad (1.22)$$

This when transferred into natural logarithmic form gives

$$\log y = \log k - ax + (n-1)\log x \qquad (1.23)$$

$$y = A + Bx + C\log x \qquad (1.24)$$

Where

$$y = \log y \qquad (1.25)$$

$$A = \log k$$

$$B = -a$$

And $C = n-1$

The three constant A, B, and C are estimated from 1.24 by using OLS

(b) Weibull distribution:

The general form of the frequency function is

$$Y = ae^{-bx} x^{n-1} \qquad (1.26)$$

When $\beta = 1$ this reduces to the gamma distribution

Transformation with natural logarithm

$\log Y = \log a - bx^{\beta} + (n-1)\log x$ i.e. of the form $y = A + Bx^{\beta} + C\log x$

Where

$$y = \log Y$$

$$A = \log a$$

$$B = -b$$

And $C = n-1$

There are 4 constants A, B, C and β . To estimate this we have only 3 normal equations. Hence for β values are oscillated above $\beta = 1$ and below $\beta = 1$. That is repeated estimations are of performed from β = .999, .998 ....downwards and β = 1.001, 1.002 .... upwards and the criteria for the best β is that which makes R^2 – high and Chi-Square between the observed and expected as minimum.

1.5 Estimation and inference for gamma distribution

Using the methods described in the previous section, the estimated parameter values A, B, C of equation (1.24) along with level of significance are presented in table 2

Table 2 Parameter values for the gamma distribution

S/No	Variable Name	Parameter Values			R-Square
		a	b	c	
1.	Education of women	0.78	-0.52	0.11	0.51
2.	Education of husband	0.71	-0.47	0.08	0.47
3.	Income of women	0.67	-0.63	0.05	0.44
4.	Income of husband	0.55	-0.40	0.06	0.41
5.	Age of women at last birth	0.67	-0.57	0.04	0.39

n = 855

The details presented in the table 2 above shows that R-Square is significant at one Percentage level of probability for all the five equations. This shows the goodness of the fit and the explanatory power of the choice variable for the Gamma distribution function fitted therein. In the case of the education of women through the type III (Gamma) frequency function, the variable education of women was able to express 51 percentage of the variations in the fertility of women, whereas the variable education of husband was able to express 47 Percentage of the variations in the fertility of women. The variable income of women was able to express 44 percentage of the variations in the fertility and age of the women at the last child birth was able to express only 39 percentage of the variations in the fertility of women. Thus the gamma distribution is able to represent the fertility position of women with respect to the variables considered. Comparison of R - square value also shows that the explanatory power is high for the education of women. That is of all the variables considered here education might make her think about her off-springs much more than anything else.

4.4 Weibull distribution

As in the previous case here also the method of estimation is as discussed earlier and the estimated parameter values along with R-square and the level of significance are presented below in Table 3.

Table 3 Estimated parameter of the Weibull distribution

S/No	Variable Name	Parameter Values				R-Square
		a	b	c	β	
1.	Education of women	0.68	-0.48	0.14	0.02	0.70
2.	Education of husband	0.63	-0.40	0.12	0.01	0.65
3.	Income of women	0.57	-0.53	0.11	0.02	0.61
4.	Income of husband	0.52	-0.40	0.10	0.01	0.54
5.	Age of women at last birth	0.67	-0.57	0.07	0.07	0.51

The details presented in Table 3 shows that as expected initially the Weibull distribution is able to express significantly higher percent of variations in the fertility of women with respect to each one of the variables under consideration and also with the adjustment factor β. As seen form the column for 3 the adjustment parameter value are different for different variables. It is 0.02 for education of women and income of women where as it is 0.01 for education and income of husband, however it is 0.03 for the variables age of women at the last childbirth. Regarding the R-square it is uniformly significant at 1 percentage level of probability for all five independent variables taken for the study. The R square is 0.75 for the variable, education of women. This implies that 75 percentage of the variation in the fertility is being explained by the variable education of women. Similarly the variable education of husband is able to express 67 percentage of the variations in the fertility, 63 percentage of variations in Fertility is being explained by the variable Income of women, 56 percentage by the income of husband and 53 percentage by the age of women at the last child birth. The following inferences are made.

1. The explanatory power is higher for all the variables in the Weibull distribution. It is 20 percentage higher for education for women. 18 percentage higher for education of husband and income of women 13 percentage higher for income of husband and 15 percentage higher age of the women at the last childbirth.

2. In the same function for the variable education and income the explanatory power is higher for women than it is for their men counter parts.

3. As indicated in the beginning the negative effects of all the variables are evident from the negative sign for the exponential function for all the variables in both the functions. Thus the study reveal that Weibull distribution is the best in cases wherein the choice variables have only negative effect on the dependent variable.

4.5 Poisson Distribution

The first phase of the South India Fertility Project has shown that fertility variations are very common across South Indian regions. It also tells that these variations are readily linked to specific feature of the regions and that on the whole is declining very fast. It has been found that the Socio - economic condition is a crucial factor. This is substantiated by the studies conducted by Murthi et.al (1995). This has created a curiosity in us to know its present status in Southern Senatorial District, it is a highly educated district in Borno.

II. Results and Discussion

Random samples of 855 households were selected. In each, one married couple was selected and from each selected couple the information pertaining to fertility (Y), Education of Wife (Xi), Education of Husband (X2), Occupation of Wife (Xa) Occupation of Husband (X4), Place of residence (X5), Religion (X6), Age at Marriage (X7) and Type of house (X8) were obtained. The data were initially subjected to multiple regression analysis. The Step-wise regression analysis revealed that the maximum Contribution for Fertility is the age at marriage. The highly significant negative association of age at marriage with fertility was reasonable to isolate it and fits the Poisson distribution.

Table 4 Statistical details pertaining to the estimated fertility equation

S/No	Variable	Symbol	Regression coefficient
1.	Education of Wife	X_1	-0.123
2.	Education of husband	X_2	-0.093
3.	Occupation of wife	X_3	-0.123
4.	Occupation of husband	X_4	-0.088

5.	Place of residence	X_5	-0.156
6.	Religion	X_6	0.138
7.	Age at marriage	X_7	-0.342
8.	Type of house	X_8	-0.083
9.	Regression (constant)	B_0	0.325

In this study fertility is related to all possible Social – economic factors. Random samples of 855 households were selected. In each, one married couple was selected and from each selected couple the information pertaining to fertility (Y), Education of Wife (Xi), Education of Husband (X2), Occupation of Wife (Xa) Occupation of Husband (X4), Place of residence (X5), Religion (X6), Age at Marriage (X7) and Type of house (X8) were obtained. The data were initially subjected to multiple regression analysis. The Step-wise regression analysis revealed that the maximum Contribution for Fertility is the age at marriage. The highly significant negative association of age at marriage with fertility was reasonable to isolate it and fits the Poisson distribution.

The above analysis shows that the major factor affecting the fertility of women is the age at marriage. As the raw data indicates the marriages between 19 and 23 years of age yields better results. Next in the order is the religion. This might be due to the fact that even now among the Muslim Community in Northern Senatorial District education is not given any importance and their marriages take place at a very early age. The step-wise multiple regressions performed admitted the variable age at marriage as the first inside the function with co-efficient of determination equal to 0.387 and the first regression equation is

$$Y = 0.325 - 0.123\, X, \tag{4.27}$$

This helped in isolating the variable Women's education separately. The graph of fertility with age at the marriage showed a sudden downward trend and in the raw data it was observed that mean variance are almost equal and hence the Poisson distribution suits it and the equation is

$$Y = \frac{e^{-1.68(1.68)^n}}{n!} \tag{4.28}$$

III. Conclusion

From the analysis of human fertility it reveals that the socio economic variable is an important factor in human fertility. The age at marriage has a vital role in human fertility. The R^2 value is only 65% in fertility. Other than these variables such as biological, economical and demographic variables put together could contribute only 44% for human fertility. Using the Poisson distribution the age at marriage shows close relation to the human fertility.

References

[1]. Aboderin, M.A. (1986). An overview of a community-based distribution project in Oyo State of Nigeria. Pp 50-54 in Proceedings of a conference on community-based distribution and alternative delivery systems in Africa, Harare, Zimbabwe, November 3-7, 1986.

[2]. Aborampah, O. (1987). Plural marriage and fertility differentials: a study of the Yoruba ofWestern Nigeria. Human Organization; 46(1):29.38.

[3]. Abramson, F.D. (1973) High foetal mortality and birth intervals. Popul. Stud. 27:235-242.

[4]. Akintunde, F.O. (1986). Committee on Women and Development (COWAD) as participating agents for market-based distribution of public health facilities: the Nigerian experience. Pp. 59-63 in Proceedings of a conference on community-based distribution and alternative deliverysystems in Africa, Harare, Zimbabwe, November 3-7.

[5]. Asika, N. (1991). Research Methodology in the behavioral sciences. Longman Nigeria Plc.pp35

[6]. Bankole, A. (1995). Desired fertility and fertility behaviour among the Yoruba of Nigeria: A

[7]. study of couple preferences and subsequent fertility. Population Studies, 49,2(July): 317-328.

[8]. Bellman, R. (1963). Mathematical Model Making as an Adaptive Process. Mathematical Optimization Techniques. University of California Press pp121-133

[9]. Brooks- Gunn, J. and Reita, E. O.(1992). The role of pubertal processes. In. S.S.Felman as G.R. Eliot at the threshold, the Developing, Adolescent 16-53 Massachusaetts, Harvard University Press.

[10]. Coleman, P.L. (1988). Enter-educate: new word from John Hopkins. JOICFP REVIEW; 152:28-31.

[11]. Covington, D.L., Otolorin, E.O., Janowitz, B., Gates, D.S., Lamptey, P., Ladipo, O.A. (1986).

[12]. Physician attitudes and family planning in Nigeria. Studies in Family Planning 17,4(July- August):172-180.

[13]. Dunson, D.B. and Zhou, H. (2000) A Bayesian model for fecundability and sterility. J. Am. Stat. Assoc. 95(452):1054-1062.

[14]. Leridon, H. (1977) Human Fertility: The Basic Components (trans: Helzner, J.F.). University of Chicago Press, Chicago

[15]. Majumdar, H and Sheps, M.C. (1970). Estimators of type I Geometric Distrbution from observation of Conception time. Demography **7**: 349-360

[16]. Michels, K.B. and Willett, W.C. (1996) Does induced or spontaneous abortion affect the risk of cancer? Epidemiology 7:521-528.

[17]. Misra, K.N.(1982).A note on the distribution of conception times using an alternative distribution for fecundability parameter 'p' University of Jodhpur, Unpublished Msc Statistics, Indian.

[18]. Menken, J. (1975) Biometric models of fertility. Soc. Forces 54:52-65.

[19]. Murthi, M. Guio, A.C. and Dreze, J. (1995) Mortality. Fertility and Gender Bias in India: A District-Level Analysis. Population & Development Review.21:745-782

[20]. Potter ,W.R.G. and Parker, M.P.(1964). Predicting the time required to conceived. Population Studies**18**:99 116.

[21]. Singh, S.N. (1968) Some probability distribution utilized in human fertility,seminar volume Statistics B.H.U. Veransy (INDIA) pp74-84
[22]. Singh, S. N. (1963) Probability models for the variation in the number of births per couples, Journal of American Statistical Association.**58**:721-727.
[23]. Potter, R.G., Ford, K., and Boots, B. (1975) Competition between spontaneous and induced abortions. Demography 12:129-141.
[24]. Bongaarts, J. (1975) A method for estimation of fecundability: Demography,**12**:645-660.
[25]. Brass, W. (1958) The distribution of births in human population, Population Studies,**12**:59-72.
[26]. Cowgill, K. (2002). Urban growth in developing Countries: A review of projection prediction. Population and Development review 25(4): 757-778.
[27]. Cox, D.R. and Oakes, D (1984) Analysis of Survival Data, London: Chapman and Hall 4-5.
[28]. Economic Commission for Africa (2001) The State of Demographic Transition in Africa, Food Security and Sustainable Development Division (FSSDD)
[29]. Goldman, N.,Westoff, C.F and Paul, L.E. (1985) Estimation of fecundability from survey data, Studies in Family planning.**16**:232-259.
[30]. Henry, L.(1957) Fecondite it Famille: Models Mathematiques, Population**12**:413-444.
[31]. Kirk, D. (1996). "Demographic Transition Theory". In Population Studies. 50(3): 223-240
[32]. Krishnan Namboodiri (2015) Demographic Analysis: Stochastic Approach. In Population Studies. 50(3): 240-248
[33]. Lanchenbruch, P. A.(1967) Frequency and timing of intercourse: Its relation to the probability of conception, Population Studies,**21**:23-31.
[34]. Leridon, H.(1977) Human fertility: The basic components, translated in English by Judith F Helzner, The University of Chicago and London.
[35]. Makinwa-Adebusoye, P. (1991). Contraception Among Urban Youth in Nigeria. Pp. 2155-2172
[36]. in IRD/Macro International, Inc. Proceedings of the Demographic and Health Surveys World Conference, Washington, D.C. 1991. vol. 3. Columbia, Md.
[37]. l ------(1992). Sexual Behaviour, Reproductive Knowledge and Contraceptive Use Among Urban
[38]. Youth in Nigeria. International Family Planning Perspectives, 18(2):66-70.
[39]. l ------(1993). A report on the agenda to improve population program implementation in Nigeria:
[40]. Lagos and Zone B report. Paper presented at the joint meeting of the Africa Population
[41]. Advisory Committee and the Global Coalition for Africa, Cotonou, Benin Republic, June.
[42]. Masha, A. (1986). Marketing to market women in Nigeria. Pp. 64-65 in Proceedings of a Conference on Community-based Distribution and Alternative Delivery Systems in AfricaHarare, Zimbabwe, November 3-7, 1986.
[43]. Ogujiuba, K (2006): Economic Growth and Population Dynamics in Nigeria: Simulating Alternative Scenarios within A Macro-Econometric Model. Ph.D Thesis University of Nigeria Nsukka. Oni, G.A. (1986). Contraceptive knowledge and attitudes in urban Ilorin, Nigeria. Journal of Biosocial Science, 18(3):273)283.
[44]. Preston, S. H., Heuveline, P. and Guillot, M. (2001) Demography: Measuring and modeling population processes. Blackwell Publishing.
[45]. Renne, E.P. (1993). Gender ideology and fertility strategies in an Ekiti Yoruba village. Studies in Family Planning 24,6(Nov/Dec):343-353. l
[46]. Perrin,E.B.andSheps,M.C.(1964)Human reproductive: A Stochastic process Biometrics,20:28-45.
[47]. Sheps,M.C. and Menken, J.A. (1973) Mathematical models of Conception and Birth, University of Chicago Press, Chicago.
[48]. Wood, J.W., Holman, D.J., Yashin, A.I., Peterson, R.J., Weinstein, M.,and Chang, M.C. (1994) A multistate model of fecund ability and sterility.

H. R. Bakari "Statistical Analysis of Human Fertility Behavior Using Probability Distributions. "IOSR Journal of Mathematics (IOSR-JM) 14.3 (2018): 77-85.