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On Interval Valued Fuzzy Det-Norm Matrix with Partitions

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ABSTRACT : *In this paper, interval valued fuzzy det-norm matrices using the structure of $M_n^*(F)$ and interval valued fuzzy det-norm ordering with interval valued fuzzy matrices are defined with examples. We prove that det-norm ordering is a partition ordering on the set of all idempotent matrices in $M_n^*(F)$. Some theorems of det-norm ordering with interval valued fuzzy matrices are proved.*
Keywords *- det-norm matrix, det-norm ordering, fuzzy matrix, idempotent, interval valued fuzzy matrix.*

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I. Introduction

The concept of fuzzy set was introduced by Zadeh in 1965 [15]. Matrix plays important roles in science and engineering. The classical matrix theory cannot solve the problems involving various types of uncertainties. This type of problems is solved using fuzzy matrix. In [14] Thomason has introduced the concept of fuzzy matrix. After that a lot of works have been done on Fuzzy matrices and its variants [8, 1, 12]. It is well known that the membership value completely depends on the decision makers, its habit, mentality, etc., Sometimes it happens that the membership value cannot be measured as a point, but it can be measured appropriately as an interval. Sometimes the measurement becomes impossible due to the rapid variation of the characteristics of the system whose membership values are to be determined. The concept of interval valued fuzzy matrix (IVFM) as a generalization of fuzzy matrix was introduced and developed by Shyamal and Pal [13] by extending the max–min operation on fuzzy algebra F = [0, 1] for the elements a, b ϵ F, a + b = max {a, b} and a – b = min {a, b}.

Jian. Miao chen [2] introduced the fuzzy matrix partial ordering and generalized inverse. Bertoiuzza [1] introduced the distributivity of t–norms and t–conorms. Meenakshi and Cokilavany [3] introduced the concept of fuzzy 2–normed linear spaces. Nagoorgani and Kalyani [5] introduced the fuzzy matrix m–ordering. Zhou Mina [16] introduced characterization of the minus ordering in fuzzy matrix set. Nagoorgani and Manikandan [6] introduced the properties of fuzzy det–norm matrices. Nagoorgani and Manikandan [7] introduced det–norm ordering with fuzzy matrices and its properties of $M_n(F)$

In this paper, we define interval valued fuzzy det–norm ordering with interval– valued fuzzy matrices. The purpose of the introduction is to explain det–norm ordering with interval-valued fuzzy matrices and partitions of $M_n^*(F)$. In section 2, interval–valued fuzzy det–norm ordering with interval valued fuzzy matrices are introduced in $M_n^*(F)$. In section 3, properties of det-norm ordering with interval valued fuzzy matrices are discussed.

II. Interval Valued Fuzzy Det - Norm Matrices

We consider $F = [0, 1]$ the fuzzy algebra with operation $[+, .]$ and the standard order $\leq$ where $a + b = max\{a . b\}$, $a . b = min\{a , b\}$ for all a, b in F. F is a commutative semi-ring with additive and multiplicative identities 0 and 1 respectively. $M_{mn}^*(F)$ denotes the set of all m×n interval-valued fuzzy matrices over F. In short, $M_n^*(F)$ is the set of all interval valued fuzzy matrices of order n.

Definition 2.1. An m×n matrix $A = [a_{ij}]$ whose components are in the unit interval $[0, 1]$ is called a fuzzy matrix.

Definition 2.2. The determinant $|A|$ of an $m \times n$ fuzzy matrix A is defined as follows;
$|A| = \sum_{\sigma \in S_n} a_{1_{\sigma(1)}} a_{2_{\sigma(2)}} \dots \dots a_{n_{\sigma(n)}}$ where S_n denotes the symmetric group of all permutations of the indices $(1, 2, \dots . n)$

Definition 2.3. An interval-valued fuzzy matrix of order m×n is defined as $A = (a_{ij})_{m\times n}$ where $a_{ij} = [a_{ijL}, a_{ijU}]$ is the ij^{th} element of A represents the membership value. All the elements of an IVFM are intervals and all the intervals are the subintervals of the interval $[0, 1]$.

Definition 2.4. The interval-valued fuzzy determinant (IVFD) of an IVFM A of order n×n is denoted by $|A|$ or $det(A)$ and we defined as

$$|A| = \sum_{\sigma\in S_n} a_{1_{\sigma(1)}} a_{2_{\sigma(2)}} \dots\dots\dots a_{n_{\sigma(n)}}$$
$$= \sum_{\sigma\in S_n} \prod_{i=1}^{n} a_{i_{\sigma(i)}}$$

where $a_{i_{\sigma(i)}} = [a_{i_{\sigma(i)L}}, a_{i_{\sigma(i)U}}]$ and S_n denotes the symmetric group of all permutations of the indices $\{1, 2, \dots n\}$.

The addition and multiplication between two elements a_{ij} and b_{ij} are defined as follows

$$a_{ij} + b_{ij} = [a_{ijL}, a_{ijU}] + [b_{ijL}, b_{ijU}] = [max\{a_{ijL}, b_{ijL}\}, max\{a_{ijU}, b_{ijU}\}]$$
$$a_{ij} \,.\, b_{ij} = [a_{ijL}, a_{ijU}] \,.\, [b_{ijL}, b_{ijU}] = [min\{a_{ijL}, b_{ijL}\}, min\{a_{ijU}, b_{ijU}\}].$$

To analysis more properties of $M_n^*(F)$, we introduce the concept of norm in $M_n^*(F)$ and thus we have defined for every A in $M_n^*(F)$ a non-negative quantity say det-norm is defined in the following way.

Definition 2.5. For every A in $M_n^*(F)$ the interval valued fuzzy det-norm of A is defined as $||A|| = det[A]$, where $A = [a_{ij}] = [a_{ijL}, a_{ijU}]$.

Definition 2.6. An interval valued fuzzy matrix A in $M_n^*(F)$ is called idempotent if $A^2 = A$ or $||A^2|| = det[A]$, where $A = [a_{ij}] = [a_{ijL}, a_{ijU}]$.

Example 2.1.

If $A = \begin{bmatrix} [0.3,\ 0.5] & [0.2,\ 0.4] & [0.6,\ 0.8] \\ [0.2,\ 0.4] & [0.1,\ 0.2] & [0.2,\ 0.4] \\ [0.3,\ 0.5] & [0.2,\ 0.3] & [0.3,\ 0.5] \end{bmatrix}$,

then $\|A\| = [0.3, 0.5]\begin{bmatrix} [0.1,\ 0.2] & [0.2\ , 0.4] \\ [0.2,\ 0.3] & [0.3,\ 0.5] \end{bmatrix} + [0.2,\ 0.4]\begin{bmatrix} [0.2,\ 0.4] & [0.2,\ 0.4] \\ [0.3,\ 0.5] & [0.3,\ 0.5] \end{bmatrix}$
$+ [0.6,\ 0.8]\begin{bmatrix} [0.2,\ 0.4] & [0.1, 0.2] \\ [0.3,\ 0.5] & [0.2,\ 0.3] \end{bmatrix}$
$= [0.3, 0.5]\big[[0.1, 0.2] + [0.2, 0.3]\big] + [0.2, 0.4]\big[[0.2, 0.4] + [0.2, 0.4]\big]$
$+ [0.6,\ 0.8]\big[[0.2, 0.3] + [0.1, 0.2]\big]$
$= [0.3, 0.5][0.2, 0.3] + [0.2, 0.4][0.2, 0.4] + [0.6, 0.8][0.2, 0.3]$
$= [0.2,\ 0.3] + [0.2,\ 0.4] + [0.2,\ 0.3]$
$\|A\| = [0.2, 0.4]$

If $\|A^2\| = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.5] & [0.2, 0.3] & [0.3, 0.5] \end{bmatrix}\begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.5] & [0.2, 0.3] & [0.3, 0.5] \end{bmatrix}$, then

$$\begin{bmatrix} [0.3, 0.5] + [0.2, 0.4] + [0.3, 0.5] & [0.2, 0.4] + [0.1, 0.2] + [0.2, 0.3] & [0.3, 0.5] + [0.2, 0.4] + [0.3, 0.5] \\ [0.2, 0.4] + [0.1, 0.2] + [0.2, 0.4] & [0.2, 0.4] + [0.1, 0.2] + [0.2, 0.3] & [0.2, 0.4] + [0.1, 0.2] + [0.2, 0.4] \\ [0.3, 0.5] + [0.2, 0.3] + [0.3, 0.5] & [0.2, 0.4] + [0.1, 0.2] + [0.2, 0.3] & [0.3, 0.5] + [0.2, 0.3] + [0.3, 0.5] \end{bmatrix}$$

$= \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.3, 0.5] \\ [0.2, 0.4] & [0.2, 0.4] & [0.2, 0.4] \\ [0.3, 0.5] & [0.2, 0.4] & [0.3, 0.5] \end{bmatrix}$
$= [0.3, 0.5]\big[[0.2, 0.4] + [0.2, 0.4]\big] + [0.2, 0.4]\big[[0.2, 0.4] + [0.2, 0.4]\big]$
$+ [0.3, 0.5]\big[[0.2, 0.4] + [0.2, 0.4]\big]$
$= [0.3, 0.5][0.2, 0.4] + [0.2, 0.4][0.2, 0.4] + [0.3, 0.5][0.2, 0.4]$
$= [0.2, 0.4] + [0.2, 0.4] + [0.2, 0.4]$
$= [0.2, 0.4] = \|A\|$

Therefore $\|A^2\| = \|A\| = [0.2, 0.4]$. Hence A is an idempotent

Definition 2.7. For all A in $M_n^*(F)$ define :

$A\{1\} = \{ x \in M_n^*(F) \,/\, \|X\| > \|A\|\}$
$A\{2\} = \{ x \in M_n^*(F) \,/\, \|X\| < \|A\|\}$
$A\{3\} = \{ x \in M_n^*(F) \,/\, \|X\| = \|A\|\}$

$A\{4\} = \{ x \epsilon M_n^*(F) / AXA = A\}$

$A\{5\} = \{x \epsilon M_n^*(F) / XAX = X\}$

Clearly, $M_n^*(F) = A\{1\} \cup A\{2\} \cup A\{3\}$. The set $A\{1\}$ is called as det-superior to A and $A\{2\}$ det-interior to A. Clearly $A\{3\}$ is det-equivalent to A. $A\{4\}$ *and* $A\{5\}$ are known as the sets of inner and outer inverses of A.

Example 2. 2.

$$\text{If } A = \begin{bmatrix} [0.3, 0.5] & [0.2, .04] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.5] & [0.2, 0.3] & [0.3, 0.5] \end{bmatrix}$$

$$\text{and } X = \begin{bmatrix} [0.5, 0.8] & [0.2, .04] & [0.3, 0.6] \\ [0.2, 0.4] & [0.5, 0.9] & [0.2, 0.3] \\ [0.1, 0.2] & [0.4, 0.6] & [0.5, 0.8] \end{bmatrix}$$

$$\text{then } \|A\| = [0.3, 0.5] \begin{bmatrix} [0.1, 0.2] & [0.2, 0.4] \\ [0.2, 0.3] & [0.3, 0.5] \end{bmatrix} + [0.2, 0.4] \begin{bmatrix} [0.2, 0.4] & [0.2, 0.4] \\ [0.3, 0.5] & [0.3, 0.5] \end{bmatrix} + [0.6, 0.8] \begin{bmatrix} [0.2, 0.4] & [0.1, 0.2] \\ [0.3, 0.5] & [0.2, 0.3] \end{bmatrix}$$

$$= [0.3, 0.5]\big[[0.1, 0.2] + [0.2, 0.3]\big] + [0.2, 0.4]\big[[0.2, 0.4] + [0.2, 0.4]\big] + [0.6, 0.8]\big[[0.2, 0.3] + [0.1, 0.2]\big]$$

$$= [0.3, 0.5][0.2, 0.3] + [0.2, 0.4][0.2, 0.4] + [0.6, 0.8][0.2, 0.3]$$

$$= [0.2, 0.3] + [0.2, 0.4] + [0.2, 0.3]$$

$$\|A\| = [0.2, 0.4]$$

$$\|X\| = [0.5, 0.8]\big[[0.5, 0.8] + [0.2, 0.3]\big] + [0.2, 0.4]\big[[0.2, 0.4] + [0.1, 0.2]\big] + [0.3, 0.6]\big[[0.2, 0.4] + [0.1, 0.2]\big]$$

$$= [0.5, 0.8][0.5, 0.8] + [0.2, 0.4][0.2, 0.4] + [0.3, 0.6][0.2, 0.4]$$

$$= [0.5, 0.8] + [0.2, 0.4] + [0.2, 0.4]$$

$\|X\| = [0.5, 0.8]$. Therefore $\|X\| > \|A\|$.

$$\text{Let } X = \begin{bmatrix} [0.1, 0.3] & [0.5, 0.8] & [0.1, 0.2] \\ [0.4, 0.7] & [0.7, 0.9] & [0.1, 0.3] \\ [0.4, 0.7] & [0.5, 0.8] & [0.1, 0.3] \end{bmatrix}$$

$$\|X\| = [0.1, 0.3]\big[[0.1, 0.3] + [0.1, 0.3]\big] + [0.5, 0.8]\big[[0.1, 0.3] + [0.1, 0.3]\big] + [\ 0.1, 0.2]\big[[0.4, 0.7] + [0.4, 0.7]\big]$$

$$= [0.1, 0.3][0.1, 0.3] + [0.5, 0.8][0.1, 0.3] + [0.1, 0.2][0.4, 0.7]$$

$$= [0.1, 0.3] + [0.1, 0.3] + [0.1, 0.2]$$

$\|X\| = [0.1, 0.3]$. Therefore $\|X\| < \|A\|$

$$\text{Let } X = \begin{bmatrix} [0.2, 0.4] & [0.5, 0.8] & [0.1, 0.2] \\ [0.4, 0.7] & [0.7, 0.9] & [0.1, 0.3] \\ [0.4, 0.7] & [0.5, 0.8] & [0.2, 0.4] \end{bmatrix}$$

$$\|X\| = [0.2, 0.4]\big[[0.2, 0.4] + [0.1, 0.3]\big] + [0.5, 0.8]\big[[0.2, 0.4] + [0.1, 0.3]\big] + [0.1, 0.2]\big[[0.4, 0.7] + [0.4, 0.7]\big]$$

$$= [0.2, 0.4][0.2, 0.4] + [0.5, 0.8][0.2, 0.4] + [0.1, 0.2][0.4, 0.7]$$

$$= [0.2, 0.4] + [0.2, 0.4] + [0.1, 0.2]$$

$\|X\| = [0.2, 0.4]$. Therefore $\|X\| = \|A\|$

Theorem 2.1. For each A in $M_n^*(F)$ the following results hold true:

i) If $X \epsilon A\{i\}$ then X^T is also in $A\{i\}$ for $i = 1, 2, 3$ where X^T is the transpose of X . ie., $\|X\| = \|X^T\|$.

ii) If $A_1 \epsilon A\{1\}$, $A_2 \epsilon A\{2\}$, $A_3 \epsilon A\{3\}$ then $\|A_1 + A_2 + A_3\| = det\ [A_1] = \|A_1\|$.

iii) $\|A_1 A_2 A_3\| = det\ [A_2] = \|A_2\|$

iv) $A^T \epsilon A\{3\}\ for\ all\ A\ in\ M_n^*(F)$

Proof :

i) When $X \epsilon A\{i\}$, $i = 1, 2, 3$. $\|X\| = \|X^T\|$

ii) From definition 2.7, $\|A_1\| > \|A\|$, $\|A_2\| < \|A\|$, $\|A_3\| = \|A\|$

Therefore $\|A_1 + A_2 + A_3\| = det\ [A_1 + A_2 + A_3]$

$= \det[A_1] + \det[A_2] + \det[A_3] = \det[A_1] = \|A_1\|$

iii) $\|A_1 A_2 A_3\| = \det[A_1 A_2 A_3] = \det[A_1] \det[A_2] det[A_3]$

$= \det[A_2] = \|A_2\|$

iv) For all $A\ in\ M_n^*(F)$, $\|A\| = \|A^T\|$. Therefore for all $A\ in\ M_n^*(F)$, $A^T \epsilon A\{3\}$.

Example 2. 3: $\|X\| = \|X^T\|$ for all X in $A\{i\}$, where $i = 1, 2, 3$

Case (i) : A{1}, $X = \begin{bmatrix} [0.5, 0.8] & [0.2, .04] & [0.3, 0.6] \\ [0.2, 0.4] & [0.5, 0.9] & [0.2, 0.3] \\ [0.1, 0.2] & [0.4, 0.6] & [0.5, 0.8] \end{bmatrix}$

$$\|X\| = [0.5, 0.8]\big[[0.5, 0.8] + [0.2, 0.3]\big] + [0.2, 0.4]\big[[0.2, 0.4] + [0.1, 0.2]\big] + [0.3, 0.6]\big[[0.2, 0.4] + [0.1, 0.2]\big]$$

$$= [0.5, 0.8][0.5, 0.8] + [0.2, 0.4][0.2, 0.4] + [0.3, 0.6][0.2, 0.4]$$

$$= [0.5, 0.8] + [0.2, 0.4] + [0.2, 0.4]$$

$$\|X\| = [0.5, 0.8]$$

Now, $X^T = \begin{bmatrix} [0.5, 0.8] & [0.2, .04] & [0.1, 0.2] \\ [0.2, 0.4] & [0.5, 0.9] & [0.4, 0.6] \\ [0.3, 0.6] & [0.2, 0.3] & [0.5, 0.8] \end{bmatrix}$

$$\|X^T\| = [0.5, 0.8]\big[[0.5, 0.8] + [0.2, 0.3]\big] + [0.2, 0.4]\big[[0.2, 0.4] + [0.3, 0.6]\big] + [0.1, 0.2]\big[[0.2, 0.3] + [0.3, 0.6]\big]$$

$$= [0.5, 0.8][0.5, 0.8] + [0.2, 0.4][0.3, 0.6] + [0.1, 0.2][0.3, 0.6]$$

$$= [0.5, 0.8] + [0.2, 0.4] + [0.1, 0.2]$$

$\|X^T\| = [0.5, 0.8]$. Therefore $\|X\| = \|X^T\| = [0.5, 0.8]$

Case (ii) : A{2}, $X = \begin{bmatrix} [0.1, 0.3] & [0.5, 0.8] & [0.1, 0.2] \\ [0.4, 0.7] & [0.7, 0.9] & [0.1, 0.3] \\ [0.1, 0.2] & [0.1, 0.3] & [0.1, 0.3] \end{bmatrix}$

$$\|X\| = [0.1, 0.3]\big[[0.1, 0.3] + [0.1, 0.3]\big] + [0.5, 0.8]\big[[0.1, 0.3] + [0.1, 0.2]\big] + [0.1, 0.2]\big[[0.1, 0.3] + [0.1, 0.2]\big]$$

$$= [0.1, 0.3][0.1, 0.3] + [0.5, 0.8][0.1, 0.3] + [0.1, 0.2][0.1, 0.2]$$

$$= [0.1, 0.3] + [0.1, 0.3] + [0.1, 0.2]$$

$$\|X\| = [0.1, 0.3]$$

Now, $X^T = \begin{bmatrix} [0.1, 0.3] & [0.4, 0.7] & [0.1, 0.2] \\ [0.5, 0.8] & [0.7, 0.9] & [0.1, 0.3] \\ [0.1, 0.2] & [0.1, 0.3] & [0.1, 0.3] \end{bmatrix}$

$$\|X^T\| = [0.1, 0.3]\big[[0.1, 0.3] + [0.1, 0.3]\big] + [0.4, 0.7]\big[[0.1, 0.3] + [0.1, 0.2]\big] + [0.1, 0.2]\big[[0.1, 0.3] + [0.1, 0.2]\big]$$

$$= [0.1, 0.3][0.1, 0.3] + [0.4, 0.7][0.1, 0.3] + [0.1, 0.2][0.1, 0.3]$$

$$= [0.1, 0.3] + [0.1, 0.3] + [0.1, 0.3]$$

$\|X^T\| = [0.1, 0.3]$, Therefore $\|X\| = \|X^T\| = [0.1, 0.3]$

Case (iii) : A{3}, $X = \begin{bmatrix} [0.2, 0.4] & [0.5, 0.8] & [0.1, 0.2] \\ [0.4, 0.7] & [0.7, 0.9] & [0.1, 0.3] \\ [0.4, 0.7] & [0.5, 0.8] & [0.2, 0.4] \end{bmatrix}$

$$\|X\| = [0.2, 0.4]\big[[0.2, 0.4] + [0.1, 0.3]\big] + [0.5, 0.8]\big[[0.2, 0.4] + [0.1, 0.3]\big] + [0.1, 0.2]\big[[0.4, 0.7] + [0.4, 0.7]\big]$$

$$= [0.2, 0.4][0.2, 0.4] + [0.5, 0.8][0.2, 0.4] + [0.1, 0.2][0.4, 0.7]$$

$$= [0.2, 0.4] + [0.2, 0.4] + [0.1, 0.2]$$

$$\|X\| = [0.2, 0.4]$$

Now, $X^T = \begin{bmatrix} [0.2, 0.4] & [0.4, 0.7] & [0.4, 0.7] \\ [0.5, 0.8] & [0.7, 0.9] & [0.5, 0.8] \\ [0.1, 0.2] & [0.1, 0.3] & [0.2, 0.4] \end{bmatrix}$

$$\|X^T\| = [0.2, 0.4]\big[[0.2, 0.4] + [0.1, 0.3]\big] + [0.4, 0.7]\big[[0.2, 0.4] + [0.1, 0.2]\big] + [0.4, 0.7]\big[[0.1, 0.3] + [0.1, 0.2]\big]$$

$$= [0.2, 0.4][0.2, 0.4] + [0.4, 0.7][0.2, 0.4] + [0.4, 0.7][0.1, 0.3]$$

$$= [0.2, 0.4] + [0.2, 0.4] + [0.1, 0.3]$$

$\|X^T\| = [0.2, 0.4]$. Therefore $\|X\| = \|X^T\| = [0.2, 0.4]$.

Example 2.4.

Let $A = \begin{bmatrix} [0.4, 0.6] & [0.3, 0.5] & [0.5, 0.6] \\ [0.2, 0.4] & [0.1, 0.3] & [0.3, 0.5] \\ [0.4, 0.6] & [0.2, 0.4] & [0.4, 0.6] \end{bmatrix}$,

$A_1 = \begin{bmatrix} [0.1, 0.4] & [0.2, .04] & [0.6, 0.8] \\ [0.6, 0.8] & [0.4, 0.6] & [0.5, 0.6] \\ [0.7, 0.8] & [0.5, 0.6] & [0.6, 0.8] \end{bmatrix}$,

$A_2 = \begin{bmatrix} [0.2, 0.4] & [0.5, 0.6] & [0.6, 0.8] \\ [0.1, 0.3] & [0.4, 0.6] & [0.1, 0.4] \\ [0.1, 0.4] & [0.5, 0.6] & [0.2, 0.4] \end{bmatrix}$,

$A_3 = \begin{bmatrix} [0.3, 0.5] & [0.1, 0.3] & [0.2, 0.4] \\ [0.1, 0.4] & [0.5, 0.6] & [0.3, 0.5] \\ [0.2, 0.4] & [0.4, 0.6] & [0.3, 0.5] \end{bmatrix}$

$\|A\| = [0.4, 0.6]\big[[0.1, 0.3] + [0.2, 0.4]\big] + [0.3, 0.5]\big[[0.2, 0.4] + [0.3, 0.5]\big] + [0.5, 0.6]\big[[0.2, 0.4] + [0.1, 0.3]\big]$

$= [0.4, 0.6][0.2, 0.4] + [0.3, 0.5][0.3, 0.5] + [0.5, 0.6][0.2, 0.4]$

$= [0.2, 0.4] + [0.3, 0.5] + [0.2, 0.4]$

$\|A\| = [0.3, 0.5]$

$\|A_1\| = [0.1, 0.4]\big[[0.4, 0.6] + [0.5, 0.6]\big] + [0.2, 0.4]\big[[0.6, 0.8] + [0.5, 0.6]\big] + [0.6, 0.8]\big[[0.5, 0.6] + [0.4, 0.6]\big]$

$= [0.1, 0.4][0.5, 0.6] + [0.2, 0.4][0.6, 0.8] + [0.6, 0.8][0.5, 0.6]$

$= [0.1, 0.4] + [0.2, 0.4] + [0.5, 0.6]$

$\|A_1\| = [0.5, 0.6]$

$\|A_2\| = [0.2, 0.4]\big[[0.2, 0.4] + [0.1, 0.4]\big] + [0.5, 0.6]\big[[0.1, 0.3] + [0.1, 0.4]\big] + [0.6, 0.8]\big[[0.1, 0.3] + [0.1, 0.4]\big]$

$= [0.2, 0.4][0.2, 0.4] + [0.5, 0.6][0.1, 0.4] + [0.6, 0.8][0.1, 0.4]$

$= [0.2, 0.4] + [0.1, 0.4] + [0.1, 0.4]$

$\|A_2\| = [0.2, 0.4]$

$\|A_3\| = [0.3, 0.5]\big[[0.3, 0.5] + [0.3, 0.5]\big] + [0.1, 0.3]\big[[0.1, 0.4] + [0.2, 0.4]\big] + [0.2, 0.4]\big[[0.1, 0.4] + [0.2, 0.4]\big]$

$= [0.3, 0.5][0.3, 0.5] + [0.1, 0.3][0.2, 0.4] + [0.2, 0.4][0.2, 0.4]$

$= [0.3, 0.5] + [0.1, 0.3] + [0.2, 0.4]$

$\|A_3\| = [0.3, 0.5]$

i) Now, $A_1 + A_2 + A_3 = \begin{bmatrix} [0.3, 0.5] & [0.5, 0.6] & [0.6, 0.8] \\ [0.6, 0.8] & [0.5, 0.6] & [0.5, 0.6] \\ [0.7, 0.8] & [0.5, 0.6] & [0.6, 0.8] \end{bmatrix}$

$\|A_1 + A_2 + A_3\| = [0.3, 0.5][0.5, 0.6] + [0.5, 0.6][0.6, 0.8] + [0.6, 0.8][0.5, 0.6]$

$= [0.3, 0.5] + [0.5, 0.6] + [0.5, 0.6]$

$= [0.5, 0.6] = \|A_1\|$

$\det[A_1] + \det[A_2] + \det[A_3] = [0.5, 0.6] + [0.2, 0.4] + [0.3, 0.5]$

$= [0.5, 0.6] = \|A_1\|$

Therefore $\|A_1 + A_2 + A_3\| = \det[A_1] + \det[A_2] + \det[A_3] = \|A_1\| = [0.5, 0.6]$

ii) Now, $A_1A_2A_3 = \begin{bmatrix} [0.2, 0.4] & [0.5, 0.6] & [0.3, 0.5] \\ [0.2, 0.4] & [0.5, 0.6] & [0.3, 0.5] \\ [0.2, 0.4] & [0.5, 0.6] & [0.3, 0.5] \end{bmatrix}$

$\|A_1A_2A_3\| = [0.2, 0.4][0.3, 0.5] + [0.5, 0.6][0.2, 0.4] + [0.3, 0.5][0.2, 0.4]$

$= [0.2, 0.4] + [0.2, 0.4] + [0.2, 0.4] = [0.2, 0.4] = \|A_2\|$

$\det[A_1]\det[A_2]\det[A_3] = [0.5, 0.6][0.2, 0.4][0.3, 0.5] = [0.2, 0.4] = \|A_2\|$

Therefore $\|A_1A_2A_3\| = \det[A_1]\det[A_2]\det[A_3] = \|A_2\| = [0.2, 0.4]$

iii) Here $\|A\| = \|A^T\| = [0.3, 0.5]$ for all $A \ in \ M_n^*(F)$, $A^T \epsilon A\{3\}$.

Theorem 2.2. Let A and X are two interval valued fuzzy matrices and

i) For all $X \epsilon A\{4\}$ then $\|A\| \leq \|X\|$.

ii) For all $X \epsilon A\{5\}$ then $\|X\| \leq \|A\|$.

iii) For all $X \epsilon A\{4\} \cap A\{5\}$ then the matrices AX and XA are idempotent.

Proof :

i) If $X \epsilon A\{4\}$, then $AXA = A$

Therefore $\|AXA\| = \det[A]$

$\det[A]\det[X]\det[A] = \det[A] = \|A\|$

Hence $\|A\| \leq \|X\|$

ii) If $X \epsilon A\{5\}$, then $XAX = X$

Therefore $\|XAX\| = \det[X]$

$\det[X]\det[A]\det[X] = \det[X] = \|X\|$

Hence $\|X\| \leq \|A\|$

iii) If $X \epsilon A\{4\} \cap A\{5\}$, then $AXA = A$ (2.1)

$XAX = X$ (2.2)

From (2.1), $XAXA = XA$

$(XA)^2 = XA$

From (2.2), $AXAX = AX$

$(AX)^2 = AX$

Hence XA and AX are idempotent.

Example 2.5.

i) If $X\ in\ A\{4\}$ then $\|AXA\| = \|A\| \leq \|X\|$

$$\text{Let } A = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$$

$$\backslash\text{and } X = \begin{bmatrix} [0.5, 0.8] & [0.2, 0.4] & [0.3, 0.6] \\ [0.2, 0.4] & [0.5, 0.9] & [0.2, 0.3] \\ [0.1, 0.2] & [0.4, 0.6] & [0.5, 0.8] \end{bmatrix}$$

$$AXA = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix} \begin{bmatrix} [0.5, 0.8] & [0.2, 0.4] & [0.3, 0.6] \\ [0.2, 0.4] & [0.5, 0.9] & [0.2, 0.3] \\ [0.1, 0.2] & [0.4, 0.6] & [0.5, 0.8] \end{bmatrix}$$

$$\begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$$

$$= \begin{bmatrix} [0.3, 0.7] & [0.2, 0.4] & [0.3, 0.7] \\ [0.2, 0.4] & [0.2, 0.4] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.4] & [0.3, 0.7] \end{bmatrix}$$

$\|AXA\| = [0.2, 0.4]$

$\|A\| = [0.2, 0.4]$

$\|AXA\| = \|A\| = [0.2, 0.4]$ (2.3)

$\|X\| = [0.5, 0.8]$ (2.4)

From (2.3) and (2.4), $\|AXA\| = \|A\| \leq \|X\|$

ii) If $X\ in\ A\{5\}$ then $\|XAX\| = \|X\| \leq \|A\|$

$$\text{Let } A = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix} \text{ and}$$

$$X = \begin{bmatrix} [0.1, 0.3] & [0.5, 0.8] & [0.1, 0.2] \\ [0.4, 0.7] & [0.7, 0.9] & [0.1, 0.3] \\ [0.4, 0.7] & [0.5, 0.8] & [0.1, 0.3] \end{bmatrix}$$

$$XAX = \begin{bmatrix} [0.1, 0.3] & [0.5, 0.8] & [0.1, 0.2] \\ [0.4, 0.7] & [0.7, 0.9] & [0.1, 0.3] \\ [0.4, 0.7] & [0.5, 0.8] & [0.1, 0.3] \end{bmatrix} \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$$

$$\begin{bmatrix} [0.1, 0.3] & [0.5, 0.8] & [0.1, 0.2] \\ [0.4, 0.7] & [0.7, 0.9] & [0.1, 0.3] \\ [0.4, 0.7] & [0.5, 0.8] & [0.1, 0.3] \end{bmatrix}$$

$$= \begin{bmatrix} [0.2, 0.4] & [0.2, 0.4] & [0.1, 0.3] \\ [0.4, 0.7] & [0.4, 0.7] & [0.1, 0.3] \\ [0.4, 0.7] & [0.4, 0.7] & [0.1, 0.3] \end{bmatrix}$$

$\|XAX\| = [0.1, 0.3]$

$\|X\| = [0.1, 0.3]$

$\|XAX\| = \|X\| = [0.1, 0.3]$ (2.5)

$\|A\| = [0.2, 0.4]$ (2.6)

From (2.5) & (2.6), $\|XAX\| = \|X\| \leq \|A\|$.

iii) If X $in\ A\{5\}$, then $||XAXA|| = ||XA||$

$$||(XA)^2|| = ||XA||$$

$$XAXA = \begin{bmatrix} [0.1, 0.3] & [0.5, 0.8] & [0.1, 0.2] \\ [0.4, 0.7] & [0.7, 0.9] & [0.1, 0.3] \\ [0.4, 0.7] & [0.5, 0.8] & [0.1, 0.3] \end{bmatrix} \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$$
$$\begin{bmatrix} [0.1, 0.3] & [0.5, 0.8] & [0.1, 0.2] \\ [0.4, 0.7] & [0.7, 0.9] & [0.1, 0.3] \\ [0.4, 0.7] & [0.5, 0.8] & [0.1, 0.3] \end{bmatrix} \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$$

$\|XAXA\| = [0.2, 0.4]$

$$XA = \begin{bmatrix} [0.1, 0.3] & [0.5, 0.8] & [0.1, 0.2] \\ [0.4, 0.7] & [0.7, 0.9] & [0.1, 0.3] \\ [0.4, 0.7] & [0.5, 0.8] & [0.1, 0.3] \end{bmatrix} \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$$

$\|XA\| = [0.2, 0.4]$

$||(XA)^2|| = \|XA\| = [0.2, 0.4]$.

If $X \epsilon\ A\{4\}$, then $||AXAX|| = ||AX||$

$$||(AX)^2|| = ||AX||$$

$$AXAX = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix} \begin{bmatrix} [0.5, 0.8] & [0.2, 0.4] & [0.3, 0.6] \\ [0.2, 0.4] & [0.5, 0.9] & [0.2, 0.3] \\ [0.1, 0.2] & [0.4, 0.6] & [0.5, 0.8] \end{bmatrix}$$

$$\begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix} \begin{bmatrix} [0.5, 0.8] & [0.2, 0.4] & [0.3, 0.6] \\ [0.2, 0.4] & [0.5, 0.9] & [0.2, 0.3] \\ [0.1, 0.2] & [0.4, 0.6] & [0.5, 0.8] \end{bmatrix}$$

$\|AXAX\| = [0.2, 0.4]$

$$AX = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix} \begin{bmatrix} [0.5, 0.8] & [0.2, 0.4] & [0.3, 0.6] \\ [0.2, 0.4] & [0.5, 0.9] & [0.2, 0.3] \\ [0.1, 0.2] & [0.4, 0.6] & [0.5, 0.8] \end{bmatrix}$$

$\|AX\| = [0.2, 0.4]$

$||(AX)^2|| = \|AX\| = [0.2, 0.4]$.

Therefore XA and AX are idempotent.

III. Interval Valued Fuzzy Matrices with Det-Norm Ordering

Definition 3.1. The det-norm ordering $A \leq B$ in $M_n^*(F)$ is defined as $A \leq B \Leftrightarrow \|A\| \leq \|B\|$ (or) $A \leq B \Leftrightarrow \det[A] \leq \det[B]$, where $A = [a_{ij}] = [a_{ijL}, a_{ijU}]$, and $B = [b_{ij}] = [b_{ijL}, b_{ijU}]$.

Example 3.1.

Let $A = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$ and

$$B = \begin{bmatrix} [0.4, 0.7] & [0.3, 0.5] & [0.7, 0.9] \\ [0.3, 0.6] & [0.3, 0.4] & [0.4, 0.6] \\ [0.5, 0.9] & [0.4, 0.8] & [0.5, 0.9] \end{bmatrix}$$

$\|A\| = [0.2, 0.4]$

$$\|B\| = [0.4, 0.7]\big[[0.3, 0.4] + [0.4, 0.6]\big] + [0.3, 0.5]\big[[0.3, 0.6] + [0.4, 0.6]\big] + [0.7, 0.9]\big[[0.3, 0.6] + [0.3, 0.4]\big]$$
$$= [0.4, 0.6] + [0.3, 0.5] + [0.3, 0.6]$$

$\|B\| = [0.4, 0.6]$. Therefore $\|A\| \leq \|B\|$

Theorem 3.1. The det - ordering is not a partial ordering in IVFM

Proof:

i) $\det[A] \leq \det[B]$ $for\ all\ A\ \epsilon\ M_n^*(F)$

Therefore $A \leq B$.

Hence reflexivity is true.

ii) $A \leq B => \|A\| \leq \|B\|$ (3.1)

$B \leq A => \|B\| \leq \|A\|$ (3.2)

From (3.1) and (3.2) $\|A\| = \|B\|$

But $\|A\| = \|B\|$ does not imply that $A = B$.
Therefore the anti-symmetry is not true.
(iii) $A \leq B,\ B \leq C \Rightarrow A \leq C\ for\ all\ A,\ B,\ C\ \epsilon\ M_n^*(F)$
when $A \leq B \Rightarrow \|A\| \leq \|B\|$
$B \leq C \Rightarrow \|B\| \leq \|C\|$
$A \leq C \Rightarrow \|A\| \leq \|C\|$
Therefore transitive is true.
Hence the det-ordering is not a partial ordering in $M_n^*(F)$.

Example 3.2.

Let $A = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$,

$B = \begin{bmatrix} [0.4, 0.7] & [0.3, 0.5] & [0.7, 0.9] \\ [0.3, 0.6] & [0.3, 0.4] & [0.4, 0.6] \\ [0.5, 0.9] & [0.4, 0.8] & [0.5, 0.9] \end{bmatrix}$ and

$C = \begin{bmatrix} [0.5, 0.8] & [0.4, 0.6] & [0.8, 0.9] \\ [0.4, 0.7] & [0.4, 0.6] & [0.5, 0.7] \\ [0.6, 0.9] & [0.5, 0.9] & [0.7, 0.9] \end{bmatrix}$

$\|A\| = [0.2, 0.4]$, $\|B\| = [0.4, 0.6]$, $\|C\| = [0.5, 0.7]$

i) $\|A\| \leq \|A\|\ for\ all\ A\ \epsilon\ M_n^*(F)$, $A \leq A$. Therefore reflexivity is true.
ii) $A \leq B \Rightarrow \|A\| \leq \|B\|$
$[0.2, 0.4] \leq [0.4, 0.6]$
$B \leq A \Rightarrow \|B\| \leq \|A\|$
But $[0.4, 0.6] \geq [0.2, 0.4]$
Here $\|A\| = \|B\|$ does not imply A = B. Therefore anti symmetry is not true.
iii) $A \leq B \Rightarrow \|A\| \leq \|B\|$
$[0.2, 0.4] \leq [0.4, 0.6]$
$B \leq C \Rightarrow \|B\| \leq \|C\|$
$[0.4, 0.6] \leq [0.5, 0.7]$
A $\leq C \Rightarrow \|A\| \leq \|C\|$
$[0.2, 0.4] \leq [0.5, 0.7]$. Therefore transitive is true.
Hence the det-ordering is not a partial ordering in $M_n^*(F)$.

Theorem 3.2. If A and B are two IVFMs and $A \leq B$, then (i) $A^T \leq B^T$ (ii) $AB^T + B^TA \leq BB^T + B^TB$ (iii) $A^TA + AA^T \leq B^TB + BB^T$ (iv) $\det[A^n] \leq \det[B^n]$.

Proof:

(i) $\|A\| = det[A^T]$, $\|B\| = det[B^T]$
Therefore $\|A\| \leq \|B\| \Rightarrow \det[A^T] \leq \det[B^T]$
i.e., $A \leq B \Rightarrow A^T \leq B^T$

(ii) $\det[AB^T + B^TA] \leq \det[A]\det[B^T] + \det[B^T]\det[A]$
$= det[A]\,det[B] + det[B]\,det[A]$
$= det[A] + det[A] = \det[A]$ (since $A \leq B$) (3.3)
$\det[BB^T + B^TB\ ] \leq \det[B]\det[B^T] + \det[B^T]\det[B]$
$= det[B]\,det[B] + det[B]\,det[B]$
$= det[B] + det[B] = \det[B]$ (3.4)
$A \leq B \Rightarrow \det[A] \leq \det[B]$
From (3.3) and (3.4) $\det[AB^T + B^TA] \leq \det[BB^T + B^TB]$
Therefore $AB^T + B^TA \leq BB^T + B^TB$.

(iii) $\det[A^TA + AA^T] = \det[A^T]\det[A] + \det[A]\det[A^T]$
$= \det[A]\det[A] + \det[A]\det[A]$
$= \det[A]$
$\det[B^TB + BB^T] = \det[B^T]\det[B] + \det[B]\det[B^T]$
$= det[B]$
$when\ A \leq B \Rightarrow \det[A] \leq \det[B]$
$\det[A^TA + AA^T] \leq \det[B^TB + BB^T]$
Therefore $A^TA + AA^T \leq B^TB + BB^T$.

(iv) $\det[A^n] = \det[A.A \ldots n\ times] = \det[A]$
$\det[A].\det[A] \ldots$ n times $= \det[A]$

$\det[B^n] = \det[B.B \ldots n\ times] = \det[B]$
$\det[B].\det[B] \ldots$ n times $= \det[B]$
$when \quad A \leq B => \det[A] \leq \det[B].$
Hence $\det[A^n] \leq \det[B^n]$.

Example 3.3.

Let $A = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$ and

$$B = \begin{bmatrix} [0.4, 0.7] & [0.3, 0.5] & [0.7, 0.9] \\ [0.3, 0.6] & [0.3, 0.4] & [0.4, 0.6] \\ [0.5, 0.9] & [0.4, 0.8] & [0.5, 0.9] \end{bmatrix}$$

$\|A\| = [0.2, 0.4], \qquad \|B\| = [0.4, 0.6]$

Now, $A^T = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.3, 0.7] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.3] \\ [0.6, 0.8] & [0.2, 0.4] & [0.3, 0.7] \end{bmatrix}$

$$\begin{aligned} \|A^T\| &= [0.3, 0.5]\big[[0.1, 0.2] + [0.2, 0.3]\big] + [0.2, 0.4]\big[[0.2, 0.4] + [0.2, 0.3]\big] \\ &\quad + [0.3, 0.7]\big[[0.2, 0.4] + [0.1, 0.2]\big] \\ &= [0.3, 0.5][0.2, 0.3] + [0.2, 0.4][0.2, 0.4] + [0.3, 0.7][0.2, 0.4] \\ &= [0.2, 0.3] + [0.2, 0.4] + [0.2, 0.4] \end{aligned}$$

$\|A^T\| = [0.2, 0.4]$

$$B^T = \begin{bmatrix} [0.4, 0.7] & [0.3, 0.6] & [0.5, 0.9] \\ [0.3, 0.5] & [0.3, 0.4] & [0.4, 0.8] \\ [0.7, 0.9] & [0.4, 0.6] & [0.5, 0.9] \end{bmatrix}$$

$\|B^T\| = [0.4, 0.7][0.4, 0.6] + [0.3, 0.6][0.4, 0.8] + [0.5, 0.9][0.3, 0.5]$

$\|B^T\| = [0.4, 0.6]$

(i) $\|A\| = det[A^T] = [0.2, 0.4]$

$\|B\| = det[B^T] = [0.4, 0.6]$

Therefore $\|A\| \leq \|B\| => \det[A^T] \leq \det[B^T]$

$[0.2, 0.4] \leq [0.4, 0.6]$

i.e., $A \leq B => A^T \leq B^T$

(ii) Now, $AB^T = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix} \begin{bmatrix} [0.4, 0.7] & [0.3, 0.6] & [0.5, 0.9] \\ [0.3, 0.5] & [0.3, 0.4] & [0.4, 0.8] \\ [0.7, 0.9] & [0.4, 0.6] & [0.5, 0.9] \end{bmatrix}$

$$= \begin{bmatrix} [0.6, 0.8] & [0.4, 0.6] & [0.5, 0.9] \\ [0.2, 0.4] & [0.2, 0.4] & [0.2, 0.4] \\ [0.3, 0.7] & [0.3, 0.6] & [0.3, 0.7] \end{bmatrix}$$

Now, $B^TA = \begin{bmatrix} [0.4, 0.7] & [0.3, 0.6] & [0.5, 0.9] \\ [0.3, 0.5] & [0.3, 0.4] & [0.4, 0.8] \\ [0.7, 0.9] & [0.4, 0.6] & [0.5, 0.9] \end{bmatrix} \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$

$$= \begin{bmatrix} [0.3, 0.7] & [0.2, 0.4] & [0.4, 0.7] \\ [0.3, 0.7] & [0.2, 0.4] & [0.3, 0.7] \\ [0.3, 0.7] & [0.2, 0.4] & [0.6, 0.8] \end{bmatrix}$$

$$AB^T + B^TA = \begin{bmatrix} [0.6, 0.8] & [0.4, 0.6] & [0.5, 0.9] \\ [0.3, 0.7] & [0.2, 0.4] & [0.3, 0.7] \\ [0.3, 0.7] & [0.3, 0.6] & [0.6, 0.8] \end{bmatrix}$$

$$\|AB^T + B^TA\| = [0.3, 0.7] \tag{3.5}$$

Now, $BB^T = \begin{bmatrix} [0.4, 0.7] & [0.3, 0.5] & [0.7, 0.9] \\ [0.3, 0.6] & [0.3, 0.4] & [0.4, 0.6] \\ [0.5, 0.9] & [0.4, 0.8] & [0.5, 0.9] \end{bmatrix} \begin{bmatrix} [0.4, 0.7] & [0.3, 0.6] & [0.5, 0.9] \\ [0.3, 0.5] & [0.3, 0.4] & [0.4, 0.8] \\ [0.7, 0.9] & [0.4, 0.6] & [0.5, 0.9] \end{bmatrix}$

$$= \begin{bmatrix} [0.7, 0.9] & [0.4, 0.6] & [0.5, 0.9] \\ [0.4, 0.6] & [0.4, 0.6] & [0.4, 0.6] \\ [0.5, 0.9] & [0.4, 0.6] & [0.5, 0.9] \end{bmatrix}$$

$$B^T B = \begin{bmatrix} [0.4, 0.7] & [0.3, 0.6] & [0.5, 0.9] \\ [0.3, 0.5] & [0.3, 0.4] & [0.4, 0.8] \\ [0.7, 0.9] & [0.4, 0.6] & [0.5, 0.9] \end{bmatrix} \begin{bmatrix} [0.4, 0.7] & [0.3, 0.5] & [0.7, 0.9] \\ [0.3, 0.6] & [0.3, 0.4] & [0.4, 0.6] \\ [0.5, 0.9] & [0.4, 0.8] & [0.5, 0.9] \end{bmatrix}$$

$$= \begin{bmatrix} [0.5, 0.9] & [0.4, 0.8] & [0.5, 0.9] \\ [0.4, 0.8] & [0.4, 0.8] & [0.4, 0.8] \\ [0.5, 0.9] & [0.4, 0.8] & [0.7, 0.9] \end{bmatrix}$$

$$BB^T + B^T B = \begin{bmatrix} [0.7, 0.9] & [0.4, 0.8] & [0.5, 0.9] \\ [0.4, 0.8] & [0.4, 0.8] & [0.4, 0.8] \\ [0.5, 0.9] & [0.4, 0.8] & [0.7, 0.9] \end{bmatrix}$$

$$\|BB^T + B^T B\| = [0.7, 0.9][0.4, 0.8] + [0.4, 0.8][0.4, 0.8] + [0.5, 0.9][0.4, 0.8]$$

$$= [0.4, 0.8] \quad (3.6)$$

From equation (3.5) and (3.6)

$$AB^T + B^T A \leq BB^T + B^T B$$

iii) Now,

$$AA^T = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix} \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.3, 0.7] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.3] \\ [0.6, 0.8] & [0.2, 0.4] & [0.3, 0.7] \end{bmatrix}$$

$$= \begin{bmatrix} [0.6, 0.8] & [0.2, 0.4] & [0.3, 0.7] \\ [0.2, 0.4] & [0.2, 0.4] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.4] & [0.3, 0.7] \end{bmatrix}$$

$$A^T \mathrm{A} = \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.3, 0.7] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.3] \\ [0.6, 0.8] & [0.2, 0.4] & [0.3, 0.7] \end{bmatrix} \begin{bmatrix} [0.3, 0.5] & [0.2, 0.4] & [0.6, 0.8] \\ [0.2, 0.4] & [0.1, 0.2] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.3] & [0.3, 0.7] \end{bmatrix}$$

$$= \begin{bmatrix} [0.3, 0.7] & [0.2, 0.4] & [0.3, 0.7] \\ [0.2, 0.4] & [0.2, 0.4] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.4] & [0.6, 0.8] \end{bmatrix}$$

$$A^T \mathrm{A} + \mathrm{A}A^T = \begin{bmatrix} [0.6, 0.8] & [0.2, 0.4] & [0.3, 0.7] \\ [0.2, 0.4] & [0.2, 0.4] & [0.2, 0.4] \\ [0.3, 0.7] & [0.2, 0.4] & [0.6, 0.8] \end{bmatrix}$$

$$\| A^T \mathrm{A} + \mathrm{A}A^T \| = [0.6, 0.8][0.2, 0.4] + [0.2, 0.4][0.2, 0.4] + [0.3, 0.7][0.2, 0.4]$$

$$= [0.2, 0.4] \quad (3.7)$$

$$\|B^T B + BB^T\| = [0.4, 0.8] \quad (3.8)$$

From equation (3.7) and (3.8)

$$A^T A + AA^T \leq B^T B + BB^T$$

IV. Conclusion

In this paper, the concept of interval valued fuzzy det-norm matrices and interval valued fuzzy matrices with det-norm ordering are discussed. Numerical examples are given to clarify the developed theory of the proposed det-norm ordering with interval valued Fuzzy matrix.

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