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Vertex Magic Pyramidal Graphs

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Abstract: *Let G = (V, E) be a graph with p vertices and q edges. The Vertex Magic Pyramidal labeling of a graph G with p vertices and q edges is an assignment of integers from* $\{1,2,3,\ldots,p_q\}$ *to the vertices and edges of G where* p_q *is the* q^{th} *Pyramidal number so that at each vertex the sum of that vertex label and the labels of the edges incident with that vertex is a constant and the constant must be a Pyramidal number. In this paper we prove that the Cycles, Stars, Peterson graph, Complete bipartite graphs are Vertex Magic Pyramidal graphs. By a graph we mean a finite, undirected graph without multiple edges or loops. For graph theoretic terminology, we refer to Harary [2] and Bondy and Murty [4]. For number theoretic terminology, we refer to M. Apostal [1] and Niven and Herbert S. Zuckerman [5].*

Keywords*: Pyramidal number, Vertex magic pyramidal, Magic strength.*

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I. Introduction

A labeling of a graph is an assignment of labels to the vertices or edges or to both the vertices and edges subject to certain conditions. Magic labelings have their origin from magic squares and it was first introduced by Sedlacek. In a Vertex magic pyramidal labeling the weight of a vertex is the sum of the vertex label and the labels of the edges incident with that vertex. In Vertex magic pyramidal labeling the weight of each vertex is a constant and the constant must be a pyramidal number. For a particular graph there are many vertex magic constants. In this paper the range of the Vertex magic constants are determined for certain graphs and their magic strengths are specified. Also Strong, Weak and Ideal magic graphs are identified.

II. Vertex Magic Pyramidal Labeling

Definition 2.1: A Triangular number is a number obtained by adding all positive integers less than or equal to a given positive integer n. If n^{th} Triangular number is denoted by T_n then $T_n = \frac{n(n+1)}{2}$. Triangular numbers are found in the third diagonal of Pascal's Triangle starting at row 3. They are 1, 3, 6, 10, 15, 21...

Definition 2.2: The sum of Consecutive triangular numbers is known as tetrahedral numbers. They are found in the fourth diagonal of Pascal's Triangle. These numbers are 1, 1 + 3, 1 + 3 + 6, 1 + 3 + 6 + 10...
(i.e.) 1, 4, 10, 20, 35…

Definition 2.3: The Pyramidal numbers or Square Pyramidal numbers are the sums of consecutive pairs of tetrahedral numbers. The following are some Pyramidal numbers.1. 1 + 4, 4 + 10, 10 + 20, 20 + 35...
(i.e.) 1, 5, 14, 30, 55...

Remark 2.4: The Pyramidal numbers are also calculated by the following formula:
$p_n = \frac{n(n+1)(2n+1)}{6}$

Definition 2.5**:** The Vertex Magic Pyramidal labeling of a Graph G=(V,E) is defined as a one-to-one function f (we call Vertex Magic Pyramidal function) from V(G)∪E(G) onto the integers$\{1,2,3,\ldots,p_q\}$ with the property that there is a constant λ_f such that f(u) +$\sum f(uv)$ = λ_f where the sum runs over all vertices v adjacent to u and uv is the edge joining the vertices u and v and the constant λ_f must be a Pyramidal number. Here p_q denotes the q^{th} Pyramidal number.The constant λ_f is called the Vertex Magic constant of the given graph. The graph which admits such a labeling is called a Vertex Magic Pyramidal graph.

Remark 2.6: For a graph G, there can be many Vertex Magic Pyramidal functions and for each function f there is a Vertex Magic constant.

Notation: The notation p_i is used for each Pyramidal number where i = 1,2,3...

Theorem 2.7: All Cycles C_n are Vertex Magic Pyramidal with $4n+1 \leq \lambda_f \leq p_{n+1}$ for $3 \leq n \leq 7$ and $5n+5 < \lambda_f \leq p_{n+2}$ $\forall$ $n \geq 8$ where p_n is the n^{th} Pyramidal number.

Proof: Case (i): In this case we prove that the result is true for all Cycles C_n of lenth 3 and 4. Let n = 3.

Define f$(v_1) = \begin{cases} n-1 & for\ \lambda_f = p_3 \\ n+1 & for\ \lambda_f = p_4 \end{cases}$

f$(v_i) = \begin{cases} f(v_{i-1})+1 & for\ i=2 \\ f(v_{i-1})-2 & for\ i=3 \end{cases}$

Define f$(e_1) = \begin{cases} \frac{\lambda_f}{2}-2 & for\ \lambda_f = p_3 \\ \frac{\lambda_f}{2}-3 & for\ \lambda_f = p_4 \end{cases}$

f(e_i) = f(e_{i-1}) +1 for i = 2,3

For n= 4, define f$(v_i) = \begin{cases} 1 & for\ i=1 \\ i+1 & for\ i=2,3 \\ i-2 & for\ i=4 \end{cases}$

Define f$(e_1) = \left[\frac{\lambda_f}{2}\right]$ +1

f$(e_i) = \begin{cases} f(e_{i-1})-4 & for\ i=2,\ \lambda_f\ odd \\ f(e_{i-1})-5 & for\ i=2,\ \lambda_f\ even \\ f(e_{i-2})-1 & for\ i=3 \\ f(e_{i-2})+2 & for\ i=4 \end{cases}$

Case(ii): n is odd. $n \geq 5$

Let $v_1, v_2, \ldots, v_n$ be the vertices of the Cycle C_n and $e_1, e_2, \ldots, e_n$ be its edges.

Subcase 2A: Let λ_f be an odd Pyramidal number.Define f:V∪E→ $\{1,2,3,\ldots,p_q\}$ as follows:

f$(v_i) = \begin{cases} i & for\ 1 \leq i \leq 4 \\ i+3 & for\ i=5 \\ f(v_{i-1})+2 & for\ 6 \leq i \leq n-1 \\ n & for\ i=n \end{cases}$

Define f$(e_1) = \left[\frac{\lambda_f}{2}\right]$ +1

f(e_i) = f(e_1) – k where k = 3,1,4 respectively for $2 \leq i \leq 4$ and

f$(e_i) = \begin{cases} f(e_{i-1})-1 & for\ 5 \leq i \leq n-1 \\ f(e_1)-2 & for\ i=n \end{cases}$

Subcase 2B: Let λ_f be an even Pyramidal number. Define f:V∪E→ $\{1,2,3,\ldots,p_q\}$

as follows: f$(v_i) = \begin{cases} i & for\ 1 \leq i \leq 6 \\ n+2 & for\ i=7 \\ f(v_{i-1})+2 & for\ 8 \leq i \leq n-1 \\ n & except\ for\ n=5 \end{cases}$

For n = 5, define f(v_n) = n+1

f$(e_1) = \left[\frac{\lambda_f}{2}\right]$ +1

f$(e_i) = \begin{cases} f(e_{i-1})-4 & for\ i=2 \\ f(e_{i-2})-1 & for\ 3 \leq i \leq 6 \\ f(e_{i-1})-(n-8) & for\ i=7 \\ f(e_{i-2})-2 & for\ 8 \leq i \leq n-1 \\ f(e_1)-3 & for\ i=n \end{cases}$

Case(iii): Let C_n be a cycle of even length, $n \geq 6$.

Subcase 3A: Let λ_f be an odd Pyramidal number. Define f:V∪E→ $\{1,2,3,\ldots,p_q\}$ as follows:Let C_n be such that $n \equiv 2 \bmod 4$.

Define f$(v_i) = \begin{cases} i & for\ 1 \leq i \leq 4 \\ f(v_{i-1})+4 & for\ i=5, \frac{n}{2}+1 \\ f(v_{i-1})+2 & for\ 6 \leq i \leq \frac{n}{2}\ \ except\ for\ n=10 \\ & and\ for\ \frac{n}{2}+2 \leq i \leq n-1 \\ n & for\ i=n \end{cases}$

f$(e_1) = \left[\frac{\lambda_f}{2}\right]$ +1

Define $f(e_i) = \begin{cases} f(e_1) - k & for\ i = 2,3\ where\ k = i+1, i-2 \\ f(e_{i-2}) - 1 & for\ i = 4 \\ f(e_{i-1}) - 1 & for\ 5 \le i \le \frac{n}{2} \\ f(e_{i-1}) - 3 & for\ i = \frac{n}{2} + 1 \\ f(e_{i-2}) - 2 & for\ \frac{n}{2} + 2 \le i \le n - 1 \\ f(e_1) - 2 & for\ i = n \end{cases}$

Let C_n be such that n ≡ 0 mod 4.

Define $f(v_i) = \begin{cases} i & for\ 1 \le i \le 4 \\ f(v_{i-1}) + 5 & for\ i = 5 \\ f(v_{i-1}) + 1 & for\ i = 6,7 \\ n + 2 & for\ i = 8 \\ f(v_{i-1}) + 2 & for\ 9 \le i \le n - 1 \\ n & for\ i = n \end{cases}$

Define $f(e_1) = \left[\frac{\lambda_f}{2}\right] + 1$

Define $f(e_i) = \begin{cases} f(e_1) - 3 & for\ i = 2 \\ f(e_1) - 1 & for\ i = 3 \\ f(e_{i-2}) - 1 & for\ 4,6 \\ f(e_{i-1}) - 2 & for\ i = 5,7 \\ f(e_{i-1}) - (n - 11) & for\ i = 8 \\ f(e_{i-2}) - 2 & for\ 9 \le i \le n - 1 \\ f(e_1) - 2 & for\ i = n \end{cases}$

Subcase 3B: Let λ_f be an even Pyramidal number.Define f:V∪E→ $\{1,2,3,\dots,p_q\}$ as follows:

Define $f(v_i) = \begin{cases} i & for\ 1 \le i \le 6 \\ f(v_{i-1}) + 5 & for\ i = 7 \\ f(v_{i-1}) + 2 & for\ 8 \le i \le n - 1 \\ n & for\ i = n\ except\ for\ n = 6 \end{cases}$

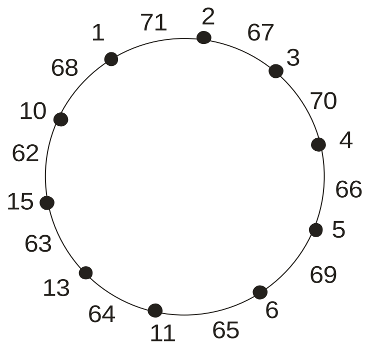
.For n= 6, define $f(v_n)$ = n+1

Define $f(e_1) = \left[\frac{\lambda_f}{2}\right] + 1$

$f(e_i) = \begin{cases} f(e_1) - 4 & for\ i = 2 \\ f(e_{i-2}) - 1 & for\ 3 \le i \le 6 \\ f(e_{i-1}) - 1 & for\ i = 7 \\ f(e_{i-2}) - 2 & for\ 8 \le i \le n - 1 \\ f(e_1) - 3 & for\ i = n \end{cases}$

By the above labeling the Cycles C_n are Vertex Magic Pyramidal.

Example:



Vertex Magic Pyramidal labeling of C_{10} (λ_f = 140)

Remark 2.8 For C_{10}, in this example we have given λ_f the value p_7 = 140, We have 5n+5 ≤ $\lambda_f \le p_{n+2}$ and therefore λ_f can take all the pyramidal numbers lying between 5n+5 and $p_{n+2} = p_{12}$ and hence the possible values of λ_f are 91, 140, 204, 285, 385,506 and 650.

Theorem 2.9: All Stars $K_{1,n}$ are Vertex Magic Pyramidal for n ≥ 3 with the Magic constants λ_f range from $\frac{n^2+3n}{2} < \lambda_f \le p_n$.

Proof: Let v_o be the root vertex of the Star $K_{1,n}$. Let v_i, i = 1 to n be the pendent vertices and e_i, i = 1 to n be the edges.

Define f(v_1) = $\lambda_f - 1$,

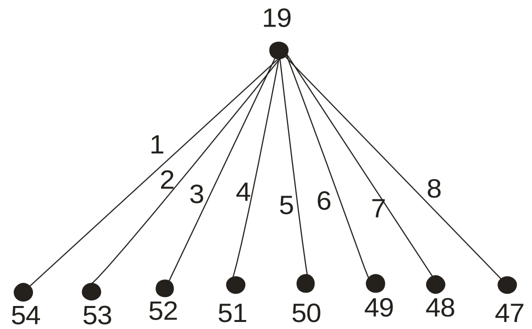
f(v_i) = f$(v_{i-1}) - 1$ for $1 \leq i \leq n$

$f(e_i)$ = i for $1 \leq i \leq n$

f$(v_o) = \lambda_f - \sum_{i=1}^{n} f(e_i)$

By the above labeling all Stars $K_{1,n}$ are Vertex Magic Pyramidal for n ≥ 3.

Example:



Vertex Magic Pyramidal labeling of $K_{1,8}$ (λ_f = 55)

Remark 2.10: For $K_{1,8}$ in this example we have given λ_f the value p_5 = 55, We have $\frac{n^2+3n}{2} < \lambda_f \leq p_n$ and therefore λ_f can take all the pyramidal numbers lying between $\frac{n^2+3n}{2}$ and $p_n = p_8$ and hence the possible values of λ_f are 55, 91, 140, 204.

Theorem 2.11: The Peterson graph is Vertex Magic Pyramidal with $p_{m-3} \leq \lambda_f \leq p_{n+2}$ where m,n are the vertices and edges in the graph.

Proof: Let v_i, 1 ≤ i ≤ 10 be the vertices in the clockwise direction. Let e_i= $v_i v_{i+1}$, 1≤ i ≤ 4, $e_5 = v_5 v_1$, e_i = $v_i v_{i-5}$, 6 ≤ i ≤ 10 be 15 be the edges in the clockwise direction. Let e_i, 11 ≤ i ≤ 15 be the edges of the Star shaped Cycle of the Peterson graph where, $e_{11} = v_9 v_6$, in the clockwise direction. Therefore m = 10, n = 15.

Define f(v_1) = $\begin{cases} \left[\frac{\lambda_f}{2}\right] - 1 \; if \; \lambda_f \equiv 0mod2 \,, \; \lambda_f \equiv 1mod6, \quad \lambda_f \neq p_n \\ \left[\frac{\lambda_f}{2}\right] \; if \; \lambda_f \equiv 3mod6 \; and \quad \lambda_f = p_n \end{cases}$

f(v_i) = $\begin{cases} f(v_{i-1}) - 3 \quad for \; 2 \leq i \leq 4 \\ f(v_{i-1}) + 2 \qquad for \; i = 5 \end{cases}$

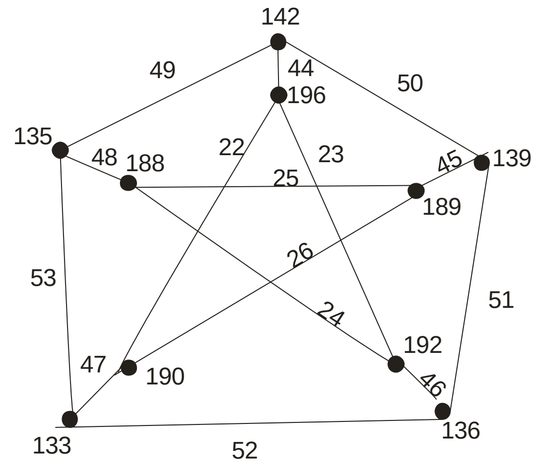
Define f(e_1) = $\begin{cases} \left[\frac{f(v_1)}{3}\right] + 3 \; for \; p_{m-3} \leq \lambda_f \leq p_{n+2}, \; \lambda_f \neq p_m, p_{m-2}, p_{m+4} \\ \left[\frac{f(v_1)}{3}\right] + 4 \; for \; \lambda_f = p_m, p_{m-2}, p_{m+4} \end{cases}$

f(e_i) = $\begin{cases} f(e_{i-1}) + 1 \quad for \; 2 \leq i \leq 15, i \neq 5,6,11 \\ f(e_{i-1}) - 4 \qquad for \; i = 5 \\ \left[\frac{f(e_{i-5})}{2}\right] \quad for \; i = 11 \end{cases}$

For i = 6, $f(e_i)$ = $\begin{cases} \left[\frac{f(v_1)}{3}\right] - 3 \; for \; p_{m-3} \leq \lambda_f \leq p_{n+2}, \; \lambda_f \neq p_m, \; p_{m+4} \\ \left[\frac{f(v_1)}{3}\right] - 2 \; for \; \lambda_f = p_m, \; p_{m+4} \end{cases}$

f(v_i) = $\lambda_f - \sum f(e)$ for $6 \leq i \leq 10$ where $\sum f(e)$ denote the sum of the labels of the edges incident with v_i. Hence Peterson graph is Vertex Magic Pyramidal.

Example:



Vertex Magic Pyramidal labeling of Peterson graph (λ_f = 285)

Remark 2.12: For the above Peterson graph we have given λ_f the value p_9 = 285, We have $p_{m-3} \leq \lambda_f \leq p_{n+2}$ and therefore λ_f can take all the pyramidal numbers lying between p_{m-3} and $p_{n+2} = p_{17}$ and hence the possible values of λ_f are 140, 204, 285, 385, 506, 650, 819, 1015, 1240, 1496 and 1785.

Theorem 2.13: All Complete bipartite graphs $K_{m,n}$, are Vertex Magic Pyramidal graphs. with $p_{m+1} \leq \lambda_f \leq p_{mn}$ for $m \neq n, m > n$ and $p_{n+1} \leq \lambda_f \leq p_{mn}$ for $m \neq n, n > m$. $For\ m = n$ λ_f ranges from $p_{m+2} \leq \lambda_f \leq p_{mn}$.

Proof: Let G be a Complete bipartite graph $K_{m,n}$. Let G = (V(G), E(G). Then V can be partitioned into two subsets V_1 and V_2 such that every line joins a point of V_1 to a point of V_2. Let $v_1, v_2, \ldots, v_m$ be the vertices of V_1 and $u_1, u_2, \ldots, u_n$ be the vertices of V_2. Let $e_1, e_2, \ldots, e_{mn}$ be the edges of $K_{m,n}$. Therefore we have V = $V_1 \cup V_2$. Let $|V_1(G)|$ = m and $|V_2(G)|$ = n. Hence $|V(G)|$ = m+n and $|E(G)|$ = mn. Define f: V(G)∪E(G).→ $\{1,2,3, \ldots, p_q\}$ as follows:

f(e_1) = m+n

f(e_i) = $f(e_{i-1})$+1 for $2 \leq i \leq mn$ except for m = n = 3

For m = n = 3, define

f(e_i) = $\begin{cases} f(e_{i-1}) + 1 & for\ 2 \leq i \leq mn-2, i = mn \\ f(e_{i-1}) + 2 & for\ i = mn-1 \end{cases}$

f(v_1) = $\lambda_f - \sum_{i=1}^{n} f(e_i)$

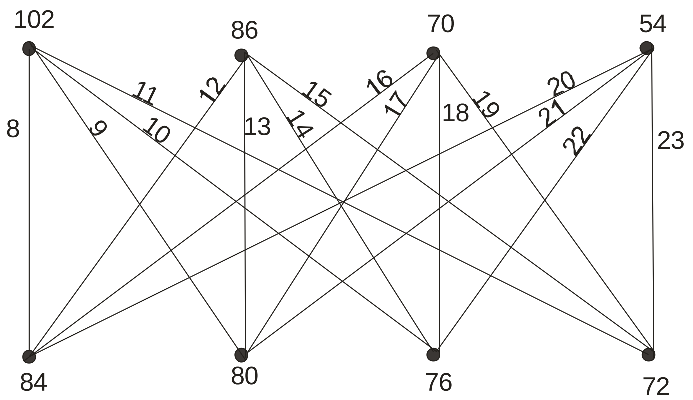
f(v_i) = $f(v_{i-1}) - n^2 \forall\ v_i \in V_1, 2 \leq i \leq m$

f(u_1) = $\lambda_f - \sum f(e_i)$ where i = 1,n+1,2n+1,3n+1…

f(u_i) = $f(u_{i-1}) - m \forall\ u_i \in V_2, 2 \leq i \leq n$

Hence $K_{m,n}$ is Vertex Magic Pyramidal with λ_f in the above range.

Example:



Vertex Magic Pyramidal labeling of $K_{4,4}$ (λ_f = 140)

Remark 2.14: For $K_{4,4}$ we have given λ_f the value p_7 = 140, We have $p_{m+2} \leq \lambda_f \leq p_{mn}$ and therefore λ_f can take all the pyramidal numbers lying between p_{m+2} and $p_{mn} = p_{16}$ and hence the possible values of λ_f are 91,140, 204, 285, 385, 506, 650, 819, 1015, 1240, 1496.

III. Vertex Magic Strength

Definition 3.1: The Vertex Magic Strength m(G) of a graph G is defined as the minimum of λ_f, where the minimum is taken over all Magic Pyramidal labelings of G. Analogous to the minimum magic strength, the maximum magic strength M(G) is defined as the maximum of all λ_f .

Definition 3.2: A Vertex Magic Pyramidal graph G is said to be Strong vertex magic if m(G)=M(G), Ideal vertex magic if M(G) – m(G) $< p_q$, Weak vertex magic if M(G) – m(G) $> p_q$ where p_q is the q^{th} Pyramidal number.

Lemma 3.3: The Stars $K_{1,n}$ are Strong vertex magic for n = 3 ,4 and Ideal vertex magic for all n ≥ 5.

Proof: The Magic constant for all Stars ranges from $\frac{n^2+3n}{2} < \lambda_f \leq p_n$.

When n= 3 we have $\frac{3^2+9}{2} < \lambda_f \leq p_3$

$\therefore 9 < \lambda_f \leq p_3 = 14$. Under this range 14 is the only Pyramidal number.

Therefore m(G) = M(G) = 14. Hence $K_{1,n}$ is Strong vertex magic for n = 3.

When n = 4 we have $\frac{4^2+12}{2} < \lambda_f \leq p_4$. Therefore $14 < \lambda_f \leq p_4 = 30$. Under this range 30 is the only Pyramidal number. Therefore we have m(G) = M(G) = 30. Hence $K_{1,n}$ is Strong vertex magic for n = 4. For all n ≥ 5, m(G) is approximately equivalent to $\frac{n^2+3n}{2}$ and M(G) = p_n. Clearly M(G)$-m(G) = p_n - \frac{n^2+5n}{2} < p_n = p_q$. Hence the Stars $K_{1,n}$ are Ideal vertex magic for all n ≥ 5.

Lemma 3.4: All Complete bipartitite graphs $K_{m,n}$ are Ideal Vertex Magic Pyramidal for any m,n.

Proof: For a Complete bipartitite graph $p_{m+1} \leq \lambda_f \leq p_{mn}$ for $m \neq n, m > n$ and $p_{n+1} \leq \lambda_f \leq p_{mn}$ for $m \neq n, n > m$. If $m = n$ $p_{m+2} \leq \lambda_f \leq p_{mn}$. Clearly $p_{mn} - p_{m+1} < p_{mn}$ for any m,n. Similar condition also holds good for other ranges of λ_f. Hence M(G) – m(G) $< p_{mn}$ for any m,n where mn is the number of edges in the graph. Therefore all Complete bipartitite graphs $K_{m,n}$ are Ideal Vertex Magic Pyramidal for any m,n.

Lemma 3.5: The Cycle C_n is Ideal vertex magic pyramidal for n = 4 ,Weak vertex magic pyramidal for n= 3 and for all n ≥ 5.

Proof: For n = 4 the vertex magic constants λ_f range from $4n+1 \leq \lambda_f \leq p_{n+1}$. Therefore we have $17 \leq \lambda_f \leq p_5 = 55$. The pyramidal numbers in this range are 30 and 55. Hence m(G) = 30, M(G) = 55. Now M(G) $- m(G) = 55 - 30 = 25 < p_4 = 30$. Hence C_n is Ideal vertex magic pyramidal for n = 4. For n = 3, $13 \leq \lambda_f \leq p_4 = 30$. The pyramidal numbers in this range are 14 and 30. Hence m(G) = 14, M(G) = 30. Now M(G) $- m(G) = 30 - 14 = 16 > p_3 = 14$. Hence C_n is Weak vertex magic pyramidal for n = 3.

For all n ≥ 5, we have $5n+5 < \lambda_f \leq p_{n+2}$. Now m(G) is approximately equivalent to 5n+5 and M(G) = p_{n+2} .

Clearly

$$
\begin{aligned}
M(G) - m(G) &= p_{n+2} - 5n - 5 \\
&= p_{n+1} + (n+2)^2 - 5n - 5 \\
&= p_n + (n+1)^2 + (n+2)^2 - 5n - 5 \\
&= p_n + n^2 + 1 + 2n + n^2 + 4 + 4n - 5n - 5 \\
&= p_n + (2n^2 + n) > p_n \text{ for any n.}
\end{aligned}
$$

Hence C_n is Weak vertex magic pyramidal for all n ≥ 5.

Remark 3.6: The Peterson graph is Weak Vertex Magic Pyramidal. In the Vertex magic labeling of Peterson graph $p_{m-3} \leq \lambda_f \leq p_{n+2}$. For m = 10 , n = 15 we have $p_7 \leq \lambda_f \leq p_{17}$. Hence $140 \leq \lambda_f \leq 1785$. Now M(G) = 1785 and m(G) = 140. M(G) – m(G) = 1645 $> p_{15}$ = 1240 which implies that Peterson graph is Weak Magic Pyramidal.

III. Conclusion

If a graph has atleast three cycles with d(v) ≥ 6 for some vertex v then G fails to be a Vertex magic pyramidal graph. Also if G has four or more cycles with d(v) ≥ 3 for atleast three vertices then G is not Vertex magic pyramidal. Such graphs may be investigated.Analogous to the Vertex magic pyramidal graph Edge magic pyramidal graph is defined with the edge weights as magic constants which are pyramidal numbers. In the above labeling the pyramidal numbers are brought into existence. As the difference between any two pyramidal numbers is a perfect square and the difference is sufficiently large, pyramidal numbers can be used as frequencies in distance labelings such as L(3,2,1), L(4,3,2,1) and Radio labelings to make the frequencies of the transmitters sufficiently large for better transmission.

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