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Construction of Character Table of S_6 Using Permutation Module and Semi Standard Young Tableaux.

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Abstract: *In this paper, we construct the irreducible character table for symmetric groupS_6following the same procedure used by Rao and Shankar (2016), in constructing irreducible character of S_5 i.eusing the permutation module and the semi-standard young tableaux.*

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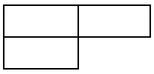
I. Introduction

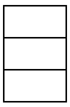
We explore a connection between representations of the symmetric group S_n and Combinatorial objects called Young tableaux. So how are representations of $S_n$related to Young tableau?. It turns out that there is a very elegantdescription of irreducible representations of S_n through Young tableaux. Let us have a glimpse ofthe results. Recall that there are three irreducible representations of S_3. It turns out that they canbe described using the set of Young diagrams with three boxes. The correspondence is illustrated below.

Trivial representation



Sign representation Standard representation

It is true in general that the irreducible representations of S_n can be described using Young diagrams of n boxes! Furthermore, we can describe a basis of each irreducible representation using standard. Young tableaux, which are numberings of the boxes of a Young diagram with {1,2,3,4,…,n} such that the rows and columns are all increasing.

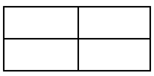


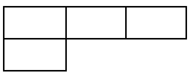
II. Construction of Character Table of S_5 Using Permutation Module And Semi Standard Young Tableaux.

.**Definition 2.1:(Rao and Shankar [2016])** A partition of a positive integer n is a sequence of positive numbers λ=$(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_k)$ satisfying$\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_k > 0$ and n = $\lambda_1 + \lambda_2 + \lambda_3 + \cdots \lambda_k$.We write $\lambda \vdash n$ to denote that λ is a partition of n. For instance the number 4 has five partitions: (4), (3,1), (2, 2), (2, 1, 1), (1, 1,1, 1).

Definition 2.2:(Rao and Shankar [2016]): A Young diagram is a finite collection of boxes (called nodes) arranged in left-justified rows, with the row sizes weakly decreasing.

The Young diagram associated with the partition λ= $(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_l)$ is the one with l rows and λ_i.boxes in the ith row. For instance, the Young diagram corresponding to the partition (2, 2) and (3, 1) of 4 is given below respectively.





Clearly there is a one-to-one correspondence between partitions and Young diagrams, so we shall use these two terms interchangeably.

Definition 2.3 (Rao and Shankar [2016]) A Young tableau t of shape λ, is a Young diagram of λ⊢n with (1,2,3,…,n) filled in the boxes (nodes) of the Young diagram, where each number occurs exactly once. In this case, we say that t is a λ-tableau.
Definition2. 4 (Rao and Shankar [2016]) A standard Young tableau is a Young tableau whose entries are increasing across each row and each column. The standard tableaux for (2,1) are

1	2
3	

1	3
2	

Definition 2.5(Rao and Shankar [2016]) A Young tabloid is an equivalence class of Young tableau under the relation where two tableau are equivalent if each row contains the same elements.
We observe that S_n acts on the set of λ-tableau: if m is any number in a node of the λ tabloid t then mσ is the number in the corresponding node in the new tableau. The new tableau is then tσ. This action gives an n! dimensional representation of S_n where elements of the group act on the right.
Definition2.6(Rao and Shankar [2016]) Let M^{λ} be the representation of S_n whose basis is indexed by the set of Young tabloids and the action on the basis is the action on the tabloids.
Definition 2.7(Rao and Shankar [2016]) Suppose λ⊢n.Let M^{λ} denotes the vector space whose basis is the set of
λ-tabloids. ThenM^{λ} is a representation of S_n known as the permutation module corresponding to λ
Proposition 2.1(Y. Zhao[2008]): If λ=$(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_k)$, then

$$\dim M^{\lambda} = \frac{n!}{\lambda_1! \lambda_2! \lambda_3! \dots \lambda_k!}$$

Proposition 2.2(Y. Zhao [2008]) Supposeλ=$(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_k)$ be a partition of n, let $\mu = (\mu_1, \mu_2, \dots, \mu_r)$ be the cycle type of $g \in S_n$.The character M^{λ}evaluated at an element of S_n with μ cycle type μ is equal to the coefficient of$x_1^{\lambda_1} x_2^{\lambda_2} \dots x_k^{\lambda_k}$ in

$$\prod_{i=1}^{r} x_1^{\mu_1} + x_2^{\mu_2} + \dots + x_k^{\mu_i}$$

Example 2.1(Rao and Shankar [2016])
let us compute the full list of the characters of the permutation modules for S_5. The partitions of S_5 are given as follows, (5), (4,1),(3,2),(3,1,1),(2,2,1), (2,1,1,1,), (1,1,1,1,1).
The character at the identity element is equal to the dimension,and it can be found through proposition 7 For instance, the character of $M^{(3,1,1)} = \frac{5!}{3!1!1!} = 20$
Say we want to compute the character of$M^{(4,1)}$ at the permutation which has cycle type (3,1,1),by using proposition 8 ,we see that the character is equal to the coefficient of $x_1^4 x_2^2$
in $(x_1^3 + x_2^3)(x_1 + x_2)^2$. Other characters can be similarly computed, and the result is shown in the following table.

Table 2.1: (some irreducible characters of S_5)(Rao and Shankar [2016])

S_5	(1,1,1,1,1)	(2,1,1,1)	(2,2,1)	(3,1,1)	(3,2)	(4,1)	(5)
$M^{(5)}$	1	1	1	1	1	1	1
$M^{(4,1)}$	5	3	1	2	0	1	0
$M^{(3,2)}$	10	4	2	1	1	0	0
$M^{(3,1,1)}$	20	6	0	2	0	0	0
$M^{(2,2,1)}$	30	6	2	0	0	0	0
$M^{2,1,1,1)}$	60	6	0	0	0	0	0
$M^{(1,1,1,1)}$	120	0	0	0	0	0	0

Note that in the above table , we did not construct the character table S_5, as all the M^{λ} are infact reducible with the exception of . in the next facts , we take a step further and construct the irreducible representation of S_5. This method depended on the inner product formula, to inference irreducible character indicator in the example below. The trivial representation is already irreducible, so the top row is an irreducible character; let's call it $S_5 = M^{(5)}$ .We can figure out how many copies of $\chi_5$each of the lower characters contains by taking inner products.
Now, we have

$$\langle \chi_{(4,1)}, M^{(3,2)} \rangle = 1$$

$$\langle \chi_{(4,1)}, M^{(3,1,1)} \rangle = 2$$

$$\langle \chi_{(4,1)}, M^{(2,2,1)} \rangle = 2$$
$$\langle \chi_{(4,1)}, M^{(2,1,1,1)} \rangle = 3$$
$$\langle \chi_{(4,1)}, M^{(1,1,1,1,1)} \rangle = 4$$

Also

$$\langle \chi_{(3,2)}, M^{(3,1,1)} \rangle = 1$$
$$\langle \chi_{(3,2)}, M^{(2,2,1)} \rangle = 2$$
$$\langle \chi_{(3,2)}, M^{(2,1,1,1)} \rangle = 3$$
$$\langle \chi_{(3,2)}, M^{(1,1,1,1,1)} \rangle = 5$$

Thus the character $\boldsymbol{\chi_{(3,2)} = M^{(3,2)} - \chi_{(4,1)}}$ **and**

$$\boldsymbol{\chi_{(3,1,1)} = \chi_{(3,2)} - 2\chi_{(4,1)}}$$

Further,

$$\langle \chi_{(3,1,1)}, M^{(3,1,1)} \rangle = 1$$
$$\langle \chi_{(3,1,1)}, M^{(2,2,1)} \rangle = 3$$
$$\langle \chi_{(3,1,1)}, M^{(2,1,1,1)} \rangle = 6$$

Therefore, $\boldsymbol{\chi_{(2,2,1)} = M^{(2,2,1)} - \chi_{(3,1,1)} - 2\chi_{(3,2)} - 2\chi_{(4,1)}}$

$$\langle \chi_{(2,2,1)}, M^{(2,1,1,1)} \rangle = 2$$
$$\langle \chi_{(2,2,1)}, M^{(1,1,1,1,1)} \rangle = 5$$
$$\langle \chi_{(2,1,1,1)}, M^{(1,1,1,1,1)} \rangle = 4$$

Therefore, $\boldsymbol{\chi_{(2,1,1,1)} = M^{(2,1,1,1)} - 3\chi_{(3,1,1)} - 3\chi_{(4,1)} - 3\chi_{(3,2)} - 2\chi_{(2,2,1)}}$, **and,**

$$\boldsymbol{\chi_{(1,1,1,1,1)} = M^{(1,1,1,1,1)} - 5\chi_{(2,2,1)} - 4\chi_{(2,1,1,1)} - 6\chi_{(3,1,1)} - 5\chi_{(3,2)} - 4\chi_{(4,1)}}$$

The complete character table for S_5 is given below

Table 2.2: (Character table of S_5) (Rao and Shankar [2016])

S_5	(1,1,1,1,1)	(2,1,1,1)	(2,2,1)	(3,1,1)	(3,2)	(4,1)	(5)
χ_5	1	1	1	1	1	1	1
$\chi_{(4,1)}$	4	2	0	1	-1	0	-1
$\chi_{(3,2)}$	5	1	1	-1	1	-1	0
$\chi_{(3,1,1)}$	6	0	-2	0	0	0	1
$\chi_{(2,2,1)}$	5	-1	1	-1	-1	1	0
$\chi_{(2,1,1,1)}$	4	-2	0	1	1	0	-1
$\chi_{(1,1,1,1,1)}$	1	-1	1	1	-1	-1	1

III. Construction of Character Table of S_6 Using Permutation Module And Semi Standard Young Tableaux.

Analogously, we shall now use similar example to construct character table for S_6

Example 3.1

Now, we construct the character table for S_6.The partitions of S_6 are given as follows
(6),(5,1),(4,2),(4,1,1),(3,3),(3,2,1),(3,1,1),(2,2,2),(2,2,1,1),(2,2,1,1),(2,1,1,1,1),(1,1,1,1,1,1)

The character at the identity element can be found through proposition 1 For instance, the character of $M^{(2,2,1,1)} = \frac{6!}{2!2!1!1!} = 180$

Say we want to compute the character of$M^{(2,2,2)}$at the permutation which has cycle type(3,2,1)Using proposition 1, we see that the character is equal to the coefficient of $x_1^2 x_2^2 x_3^2$in $(x_1^3 + x_2^3 + x_3^3)(x_2^2 + x_2^2 +$ *x32x1+x2+x3.*

Table 3.1:(Some irreducible characters ofS_6)

S_6	(1,1,1,1,1,1)	(2,1,1,1,1)	(2,2,1,1)	(2,2,2)	(3,1,1,1)	(3,2,1)	(3,3)	(4,1,1)	(4,2)	(5,1)	(6)
$M^{(6)}$	1	1	1	1	1	1	1	1	1	1	1
$M^{(5,1)}$	6	4	2	0	3	1	0	2	0	1	0
$M^{(4,2)}$	15	7	3	3	3	1	0	1	1	0	0
$M^{(4,1,1)}$	30	12	2	0	6	0	0	2	0	0	0
$M^{(3,3)}$	20	8	4	0	2	2	2	0	0	0	0
$M^{(3,2,1)}$	60	16	4	0	3	1	0	0	0	0	0

$M^{(3,1,1,1)}$	120	24	0	0	6	0	0	0	0	0	0
$M^{(2,2,2)}$	90	18	6	6	0	0	0	0	0	0	0
$M^{(2,2,1,1)}$	180	24	4	0	0	0	0	0	0	0	0
$M^{(2,1,1,1,1)}$	360	24	0	0	0	0	0	0	0	0	0
$M^{(1,1,1,1,1,1)}$	720	0	0	0	0	0	0	0	0	0	0

From the above table we observe that only $M^{(6)}$ is irreducible and let $M^{(6)} = \chi_6$.The remaining irreducible characters are obtained as follows.

We have .

$\langle \chi_6, M^{(5,1)} \rangle = 1$ $\langle \chi_6, M^{(3,1,1,1)} \rangle = 1$

$\langle \chi_6, M^{(4,2)} \rangle = 1$ $\langle \chi_6, M^{(2,2,2)} \rangle = 1$

$\langle \chi_6, M^{(4,1,1)} \rangle = 1$ $\langle \chi_6, M^{(2,2,1,1)} \rangle = 1$

$\langle \chi_6, M^{(3,3)} \rangle = 1$ $\langle \chi_6, M^{(2,1,1,1,1)} \rangle = 1$

$\langle \chi_6, M^{(3,2,1)} \rangle = 1$ $\langle \chi_6, M^{(1,1,1,1,1,1)} \rangle = 1$

Since we know how many copies of χ_6 occur in the lower representations, we can subtract them of and get a new table.

Table3.2 : (Some irreducible characters of S_6)

S_6	(1,1,1,1,1,1)	(2,1,1,1,1)	(2,2,1,1)	(2,2,2)	(3,1,1,1)	(3,2,1)	(3,3)	(4,1,1)	(4,2)	(5,1)	(6)
χ_6	1	1	1	1	1	1	1	1	1	1	1
$\chi_{(5,1)}$	5	3	1	-1	2	0	-1	1	-1	0	-1
$M^{(4,2)}$	14	6	2	2	2	0	-1	0	0	-1	-1
$M^{(4,1,1)}$	29	11	1	-1	5	-1	-1	1	-1	-1	-1
$M^{(3,3)}$	19	7	3	-1	1	1	1	-1	-1	-1	-1
$M^{(3,2,1)}$	59	15	3	-1	2	0	-1	-1	-1	-1	-1
$M^{(3,1,1,1)}$	119	24	0	0	6	-1	-1	-1	-1	-1	-1
$M^{(2,2,2)}$	89	18	6	6	-1	-1	-1	-1	-1	-1	-1
$M^{(2,2,1,1)}$	179	24	4	-1	-1	-1	-1	-1	-1	-1	-1
$M^{(2,1,1,1,1)}$	359	24	-1	-1	-1	-1	-1	-1	-1	-1	-1
$M^{(1,1,1,1,1,1)}$	719	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Now row 2 is an irreducible character $\chi_{(5,1)}$, we can now repeat by taking the inner product of $\chi_{(5,1)}$ with the M characters and subtractingthem off .

We have.

$\langle \chi_{(5,1)}, M^{(4,2)} \rangle = 1$ $\langle \chi_{(5,1)}, M^{(2,2,2)} \rangle = 2$

$\langle \chi_{(5,1)}, M^{(4,1,1)} \rangle = 2$ $\langle \chi_{(5,1)}, M^{(2,2,1,1)} \rangle = 3$

$\langle \chi_{(5,1)}, M^{(3,3)} \rangle = 1$ $\langle \chi_{(5,1)}, M^{(2,1,1,1,1)} \rangle = 4$

$\langle \chi_{(5,1)}, M^{(3,2,1)} \rangle = 2$ $\langle \chi_{(5,1)}, M^{(1,1,1,1,1,1)} \rangle = 5$

$\langle \chi_{(5,1)}, M^{(3,1,1)} \rangle = 3$

Therefore,

$$\boldsymbol{\chi_{(4,2)} = M^{(4,2)} - \chi_{(5,1)}}$$

Also

$\langle \chi_{(4,2)}, M^{(4,1,1)} \rangle = 1$ $\langle \chi_{(4,2)}, M^{(2,2,2)} \rangle = 3$

$\langle \chi_{(4,2)}, M^{(3,3)} \rangle =1$ $\langle \chi_{(4,2)}, M^{(2,2,1,1)} \rangle = 4$

$\langle \chi_{(4,2)}, M^{(3,2,1)} \rangle = 2$ $\langle \chi_{(4,2)}, M^{(2,1,1,1,1)} \rangle = 6$

$\langle \chi_{(4,2)}, M^{(3,1,1,1)} \rangle = 3$ $\langle \chi_{(4,2)}, M^{(1,1,1,1,1,1)} \rangle = 9$

Therefore

$$\boldsymbol{\chi_{(3,3)} = M^{(3,3)} - \chi_{(5,1)} - \chi_{(4,2)}}$$

Further,

$\langle \chi_{(4,1,1)}, M^{(3,3)} \rangle = 1$ $\langle \chi_{(4,1,1)}, M^{(2,2,1,1)} \rangle = 3$

$\langle \chi_{(4,1,1)}, M^{(3,1,1,1)} \rangle = 3$ $\langle \chi_{(4,1,1)}, M^{(2,1,1,1,1)} \rangle = 6$

$\langle \chi_{(4,1,1)}, M^{(2,2,2)} \rangle = 1$ $\langle \chi_{(4,2)}, M^{(1,1,1,1,1,1)} \rangle = 10$

$$\boldsymbol{\chi_{(4,1,1)} = M^{(4,1,1)} - 2\chi_{(5,1)} - \chi_{(4,2)}}$$

$\langle \chi_{(3,3)}, M^{(3,2,1)} \rangle = 1$ $\quad\quad\quad\quad \langle \chi_{(3,3)}, M^{(2,2,1,1)} \rangle = 2$
$\langle \chi_{(3,3)}, M^{(3,1,1,1)} \rangle = 1$ $\quad\quad \langle \chi_{(3,3)}, M^{(2,1,1,1,1)} \rangle = 3$
$\langle \chi_{(3,3)}, M^{(2,2,2)} \rangle = 1$ $\quad\quad \langle \chi_{(3,3)}, M^{(1,1,1,1,1,1)} \rangle = 5$

And

$\langle \chi_{(3,2,1)}, M^{(2,2,1,1)} \rangle = 4$

$\langle \chi_{(3,2,1)}, M^{(3,1,1,1)} \rangle = 2$ $\quad\quad \langle \chi_{(3,2,1)}, M^{(2,1,1,1,1)} \rangle = 8$

$\langle \chi_{(3,2,1)}, M^{(2,2,2)} \rangle = 2$ $\langle \chi_{(3,2,1)}, M^{(1,1,1,1,1,1)} \rangle = 16$

Therefore

$\boldsymbol{\chi_{(3,2,1)}} = \quad \boldsymbol{M^{(3,2,1)} - 2\chi_{(5,1)} - 2\chi_{(4,2)} - \chi_{(4,1,1)} - \chi_{(3,3)}}$
$\boldsymbol{\chi_{(3,1,1)}} = \quad \boldsymbol{M^{(3,1,1,1)} - 3\chi_{(5,1)} - 3\chi_{(4,2)} - 3\chi_{(4,1,1)} - \chi_{(3,3)} - 2\chi_{(3,2,1)}}$
$\boldsymbol{\chi_{(2,2,2)}} = \quad \boldsymbol{M^{(2,2,2,)} - 2\chi_{(5,1)} - 3\chi_{(4,2)} - \chi_{(4,1,1)} - \chi_{(3,3)} - 2\chi_{(3,2,1)}}$

And

$\langle \chi_{(3,1,1)}, M^{(2,2,1,1)} \rangle = 1$
$\langle \chi_{(3,1,1)}, M^{(2,1,1,1,1)} \rangle = 4$
$\langle \chi_{(3,1,1)}, M^{(1,1,1,1,1,1)} \rangle = 10$

And$\langle \chi_{(2,2,2)}, M^{(2,2,1,1)} \rangle = 1$
$\langle \chi_{(2,2,2)}, M^{(2,1,1,1,1)} \rangle = 2$
$\langle \chi_{(2,2,2)}, M^{(1,1,1,1,1,1)} \rangle = 5$

$\langle \chi_{(2,2,1,1)}, M^{(2,1,1,1,1)} \rangle = 3$
$\langle \chi_{(2,2,1,1)}, M^{(1,1,1,1,1,1)} \rangle = 9$
$\langle \chi_{(2,1,1,1,1)}, M^{(1,1,1,1,1,1)} \rangle = 5$

$\boldsymbol{\chi_{(2,2,1,1)}} = \quad \boldsymbol{M^{(2,2,1,1)} - 3\chi_{(5,1)} - 4\chi_{(4,2)} - 3\chi_{(4,1,1)} - 2\chi_{(3,3)} - \chi_{(2,2,2)} - 9\chi_{(2,2,1,1)} - 4\chi_{(3,2,1)} - \chi_{(3,1,1,1)} - \chi_{(2,2,2)}}$

$\boldsymbol{\chi_{(2,1,1,1,1)}} = \quad \boldsymbol{M^{(2,1,1,1,1)} - 4\chi_{(5,1)} - 6\chi_{(4,2)} - 6\chi_{(4,1,1)} - 3\chi_{(3,3)} - 2\chi_{(2,2,2)} - 3\chi_{(2,2,1,1)} - 8\chi_{(3,2,1)} - 4\chi_{(3,1,1,1)}}$
$\boldsymbol{\chi_{(1,1,1,1,1,1)}} = \quad \boldsymbol{M^{(1,1,1,1,1,1)} - 5\chi_{(5,1)} - 9\chi_{(4,2)} - 10\chi_{(4,1,1)} - 5\chi_{(3,3)} - 5\chi_{(2,2,2)} - 9\chi_{(2,2,1,1)} - 16\chi_{(3,2,1)} - 10\chi_{(3,1,1,1)} - 5\chi_{(2,1,1,1,1)}}$

The complete character table for S_6 is given below.

Table3.3 : (Character Table of S_6)

S_6	(1,1,1,1,1,1)	(2,1,1,1,1)	(2,2,1,1)	(2,2,2)	(3,1,1,1)	(3,2,1)	(3,3)	(4,1,1)	(4,2)	(5,1)	(6)
χ_6	1	1	1	1	1	1	1	1	1	1	1
$\chi_{(5,1)}$	5	3	1	-1	2	0	-1	1	-1	0	-1
$\chi_{(4,2)}$	9	3	1	3	0	0	0	-1	1	-1	0
$\chi_{(4,1,1)}$	10	2	-2	-2	1	-1	1	0	0	0	1
$\chi_{(3,3)}$	5	1	1	-3	-1	1	2	-1	-1	0	0
$\chi_{(3,2,1)}$	16	0	0	0	-2	0	-2	0	0	1	0
$\chi_{(3,1,1,1)}$	10	-2	-2	2	1	1	1	0	0	0	-1
$\chi_{(2,2,2)}$	5	-1	1	3	-1	-1	2	1	-1	0	0
$\chi_{(2,2,1,1)}$	9	-3	1	-3	0	0	0	1	1	-1	0
$\chi_{(2,1,1,1,1)}$	5	-3	1	1	2	0	-1	-1	-1	0	1
$\chi_{(1,1,1,1,1,1)}$	1	-1	1	-1	1	-1	1	-1	1	1	-1

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