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Some Aspects of Mathematical Modeling for Optimization Problem

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Abstract: *Many reasonable optimization problems include mathematical models of complex true wonders. This paper discusses some aspects of modeling that impact the execution of optimization methods. Information and guidance are given concerning the development of smooth models, the change of an optimization problem starting with one class then onto the next, scaling, detailing of imperatives, and techniques for exceptional sorts of models. The improved mathematical models and algorithms of vertical alignment by set forms of the course design are advertised. The issue is settled in a few phases in interrelation with other plan issues. The first algorithm of plunge is given for solving the emerging problem of nonlinear programming. Basic components of limitations are utilized thus it is not required to comprehend any frameworks of linear equations.*

Keywords: *Mathematical Modeling, Optimization Problem, performance, information, models, techniques, algorithms, nonlinear programming, linear equations.*

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I. Introduction

Mathematical models are as often as possible used to consider true wonders that are not defenseless to analytic techniques alone, and to examine the connections among the parameters that influence the working of complex procedures. Models provide a successful, in some cases the main, methods for assessing the consequences of option decisions; for instance, a model is basic in situations where experimentation with this present reality framework is restrictively costly, perilous, or even unthinkable Optimization methods assume an important part in modeling, because a model is not normally created as an end in itself. Or may be, the model is defined so as to decide values of free parameters that create an ideal measure of 'goodness'; for example, the most stable structure, or the best execution on watched information. The connection between the formulation of a model and the related optimization can take a few structures [1-5]. In many examples, virtually all the exertion of model development is pointed towards building a model that mirrors the real world as nearly as could be expected under the circumstances. Simply after the type of the model is basically entire is some idea given to a technique for finding ideal estimations of the parameters. Be that as it may, determination of an off-the-peg algorithm without considering properties of the model regularly prompts superfluous disappointment or gross wastefulness. Then again, we don't advocate over-disentanglement or bending in formulation simply keeping in mind the end goal to have the capacity to illuminate the eventual optimization problem all the more effortlessly. There has been a tendency, especially in the substantial scale zone, to model even exceedingly nonlinear processes as linear programs, in light of the fact that as of not long ago no nonlinear methods were accessible for extremely large problems [6-8]. The push to remove nonlinearities frequently prompts incredibly expanded problem size, A prior variant of this paper was displayed at the meeting 'Software for numerical optimization', London Match 1978 and furthermore essentially influences the idea of the optimal solution (e.g., a linear programming solutions dependably an extraordinary purpose of the achievable district, however the solution of a nonlinear program is typically not). [9]

II. Review Of Literature

A model to be optimized should be produced by striking a sensible harmony between the points of enhanced exactness in the model (which more often than not suggests included multifaceted nature in the formulation) and expanded simplicity of optimization. This may be accomplished by summoning an optimization procedure on progressively more muddled adaptations of the model, in a type of "stepwise" refinement. Along these lines, the impacts of every refinement in the model on the optimization process can be observed, and basic challenges can be found considerably more rapidly than if no optimization were connected until the point when the model was basically total. This is particularly imperative when managing with models that contain many interconnected sub-frameworks, each requiring extensive calculation [10-12]. This paper is not fundamentally worried about how accurately models mirror this present reality, yet rather with angles of modeling that impact the execution of optimization algorithms. Specifically, we might talk about contemplations m formulating models that add to the accomplishment of optimization methods. Our perceptions of down to earth optimization problems have demonstrated that, even with the best available software, the efficient optimization of a model earl be fundamentally reliant on specific properties of the formulation [13]. It is regularly the case that the formulator of the model must make numerous self-assertive choices that don't influence the exactness of the model, yet are vital to whether the model is agreeable to arrangement by an optimization algorithm [14].

Customarily course as three-dimensional bend is spoken to in the frame of two flat curves: horizontal and vertical alignment (later: plan and profile)(Figure 1). The arrangement is a projection of the course to the facilitate XOY plane, and a longitudinal profile is reliance organize z from length of the bend in the arrangement s.

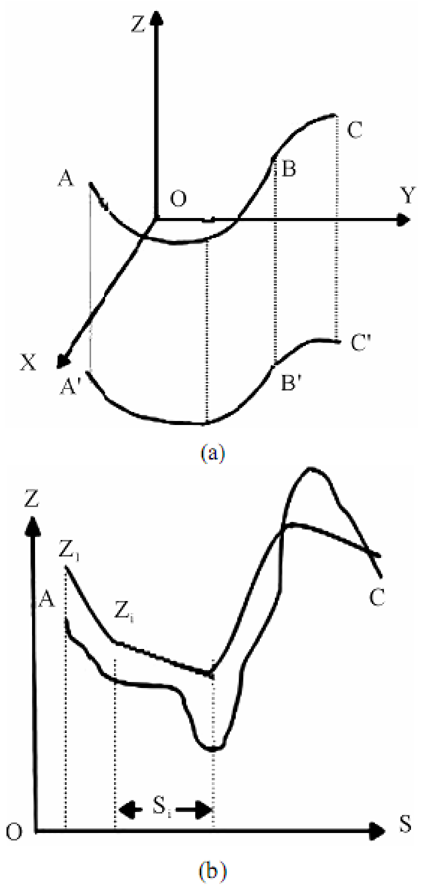

Figure 1- Route representation

Position of the route is impacted by earth help, geological, hydrological, and climatic and different conditions. Hence, the course is an external of some functional, and seeking of the optimum route ought to be considered as a problem of variation calculus [15]. Most importantly, we will take note of that it is impractical to express the practical in an unequivocal frame, or to record the route equation, that is to formalize the problem.

Classification of Optimization problems: The broadest type of an optimization problem is that of minimizing a scalar capacity of the independent variables (the objective function), subject to confinements or requirements on worthy values of the variables [16]. We should fundamentally be concerned with problems in which the arrangement of acceptable variables is characterized by relations including persistent elements of the variables:

$$\text{NLP} \quad \min_{x \in \mathbb{R}^n} F(x)$$

$$\text{subject to } c_i(x) = 0 \quad i = 1, 2, \dots, m_1$$
$$c_i(x) \geqslant 0 \quad i = m_1 + 1, \dots, m_2$$

In this formulation, the functions F and (c_i) are named the problem functions. Imperatives on the parameters may take different structures, e.g., some of the variables might be limited to a limited arrangement of values only. Problems of this sort are for the most part substantially more troublesome to solve than those of the

frame NLP; some possible approaches to models with such limitations are noted later in the paper. An important point to be considered indwelling is whether the formulation has highlights that upgrade the simplicity of optimization, since a general algorithm for NLP will by and large be wasteful if connected to an issue with special features [17]. For motivations behind picking an algorithm, optimization problems are normally separated into class's determined by properties of the problem functions, where problems in every classification are best tackled by a different algorithm.
The following table gives a common characterization conspire, where noteworthy favorable circumstances can be taken of each characteristic:

Properties of $F(x)$	Properties of $\{c_i(x)\}$
Linear	None
Sums of squares of linear functions	Simple bounds
Quadratic	Linear
Sums of squares of nonlinear functions	Sparse linear
Nonlinear	Nonlinear

Certain problem qualities have a substantially more prominent effect on the simplicity of optimization that others; for example, problem size. Past one-dimensional problems (which are perpetually regarded as an extraordinary case), the following partitioning line happens when the problem measure turns out to be large to the point that: (a) the data can't be put away in the working memory of the computer; (b) abusing the sparsity (extent of zeros) in the problem data prompts a noteworthy change in productivity. Prior to that point, nonetheless, the effort required to comprehend a run of the mill problem is, generally, limited by a sensibly carried on polynomial capacity of problem size Therefore, expanding the number of parameters in an unconstrained problem from, say is normally not critical. General directly constrained problems are detectably harder to understand than those with bound limitations just, and the nearness of nonlinear constraints presents an even larger increase in trouble. Hence, it is now and then prudent to reformulate a model in order to wipe out nonlinear constraints; this paper will be talked about further later in the paper. Probably the most crucial property of the problem functions effortlessly of optimization is differentiability, which is critical on the grounds that algorithms are in view of utilizing available information about a capacity at one point to reason its conduct at different focuses. On the off chance that the problem functions are twice consistently differentiable, say, the capacity of an algorithm to find the solution is incredibly upgraded contrasted with the situation when the problem functions are non-differentiable. In this way, most optimization software is intended to solve smooth problems, and there is an awesome impetus to formulate differentiable model capacities. For a smooth problem inside a particular classification, there still remains a lot of decision in algorithm choice, depending, for instance, on how much subsidiary data is accessible, the relative cost of computing certain amounts, et cetera. As a general rule, algorithms have a tendency to end up noticeably more fruitful and strong as more data is provided.

Vertical Alignment as the Problem of Nonlinear Programming: In the event that we mean the longitudinal ground profile H(s), and the projected line of Z(s), at that point the problem is the accompanying. For a given H (s) to locate a broken Z (s) that fulfills all the constraints and gives:

$$\min \int_0^{s_0} F\left(Z(s), H(s), s\right) \mathrm{d}s$$

Realistic models must consider the structure of the transverse profiles of roadbed, the nearness of courses, small bridges, balance of banks and cuttings, techniques for exhuming, and so forth. The issue of variational math we diminish to an issue of nonlinear programming, which has many fascinating elements, is not reliant on the particular type of capacity F. The quantity of components required broken is obscure. Hence, we trust that its hubs and hubs of the ground profile have a similar abscissa. The ground profile is constantly spoken to as a broken with uneven stride, and this presumption makes it conceivable to settle the quantity of components n (the measurement of the issue) and s_i - the length of the elements (Figure 1).

Problems with discrete or integer variables: Numerous practical problems happen in which some of the variables are limited to be members of a limited set of values. These variables are named discrete or whole number. Cases of such variables are: things that are possible or made in specific sizes just, for example, the yield rating of pumps or brace sizes; or the number of ventures made by a voyager. Such restrictions imply that the standard meanings of differentiability and congruity are not material, and therefore numerical methods for differentiable problems must be utilized in a roundabout way (aside from a specific number of special cases where the solution of the continuous problem is known to fulfill the discrete/whole number requirements naturally). In the event that the target and limitation capacities are linear, numerous exceptional number linear programming methods have been produced, prominently variations of 'branch and bound'; in some other extraordinary cases, dynamic programming methods can be connected. Be that as it may, we might be worried about blended whole number nonlinear problems, i.e. nonlinear problems with a blend of discrete and continuous variables.

Pseudo-discrete variables: The first problem concerns the design of a network of urban sewer or seepage channels. Inside a given territory, the position of an arrangement of sewer vents depends on requirements of get to and the geology of the road format. It is required to interconnect the sewer vents with straight pipes so the fluid from a wide catchment region enters the sewer vents and streams under gravity down the framework and out of the territory. Every sewer vent has a few information funnels and a solitary yield pipe. To encourage the stream, the channels are set at a point to the horizontal.

In a general problem with pseudo-discrete variables, suppose that x_1 must assume one of the values d_1, d_2...., dr. Let x^c denote the value of x at the solution of the continuous problem, which is assumed to be unique. Suppose that x_1^c satisfies:

$$d_s < x_1^c < d_{s+1}$$

The value of the objective function F at x^c is a lower bound on the value of F at any solution of the discrete problem, since if X_1 is restricted to be any value other than x_1^c, the objective function for such a value must be larger than $F(x^c)$, irrespective of the values of $x_2 ...... x_n$.

It is important in such problems to take note of that the solution of the continuous problem can simply be utilized as an underlying assessment for the solution of a restricted problem In numerous practical problems, only two or three solutions of a confined problem are expected to decide a worthy solution of a discrete problem. The additional computing cost in fathoming the extra confined problems associated with the discrete variables is prone to be a small amount of the cost to understand the first full continuous problem; if not, this suggests the discrete solution differs considerably from the continuous solution.

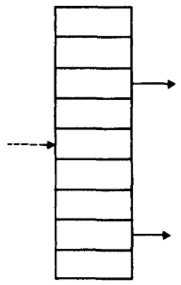

Figure 2- Diagram of distillation column

In such conditions, it might be beneficial to adjust the unessential conditions with the goal that the two arrangements are nearer. For instance, in problems of supply, for example, the urban sewer problem, alternative supplies might be looked for whose determinations are nearer to the comparing components of the continuous problem, since by definition this change would yield a huge lessening in construction costs.

Integer variables: The second type of discrete variable is one for which there is no sensible elucidation of a non-integer value, for instance, when the variable speaks to a number of things, or a change from one fundamentally unrelated procedure or asset to another (e.g., the change between covering materials in focal point outline). This sort of discrete variable problem is a great deal harder to explain than the first. On the off chance that the number of such variables is little, say under five, and the quantity of distinct values that each variable can take is additionally little, a combinatorial approach is conceivable. In this setting a combinatorial technique is one in which the target work is limited for each conceivable mix of values that the discrete variables can accept. It might occur by and by that a few blends are viewed as far-fetched, thus not all cases should be attempted. A combinatorial approach is frequently sensible for constrained problems on the grounds that numerous infeasible mixes can be dispensed with before any computation takes put. Likewise, with a combinatorial approach there is a usable arrangement regardless of where the algorithm is ended. At first sight it may be suspected that the variable associated with the nourish level has no ceaseless simple. Nonetheless, Sargent and Gamini presented an arrangement of new variables; where each new variable corresponds to a phase of the column, and speaks to the rate of the aggregate sustain to be contribution at that particular level. The new model is indicated in Figure 3. The problem is then re-detailed and solved, treating the new variables as ceaseless, and its solution is taken to demonstrate properties of the solution of the original problem. For instance, if the solution of the continuous problem demonstrates that 90% of the nourish ought to go to a specific stage, this stage is probably going to be the one at which to include the sustain in the discrete model.

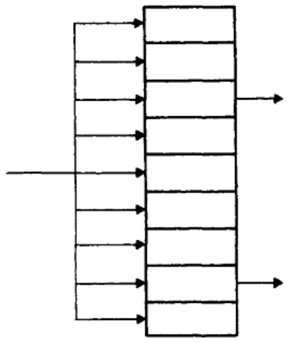

Figure-3 New model of distillation column

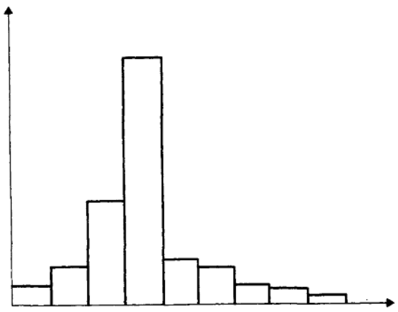

Figure 4- Typical optimal percentage feed levels

Figure 5 shows some typical percentage feed levels for a nine-stage continuous model; stage four appears to be the most likely candidate for the value of the discrete variable.

Formulation of constraints:

Indeterminacy in constraint formulation: A difficulty in formulating model with constraints on the factors is the likelihood of making an ineffectively postured optimization problem, despite the fact that the fundamental model has a well-defined solution. This circumstance can exist for some reasons, which are excessively numerous to list here. For example, excess constraints may be incorporated that are just direct mixes of other constraints, keeping in mind the end goal to give a rundown of specific exercises. Such components may fill a helpful need inside the model, and the modeler realizes that they ought to have no impact on the optimal values of the model parameters. Shockingly, the execution of optimization algorithms may in this manner be antagonistically influenced.

The use of tolerance constraints: Equality constraints occur in problem formulations for a variety of reasons. Often the very nature of the variables imposes an equality constraint, for example, if the variables $\{x_i\}$ represent proportions or probabilities, this gives rise to the constraint $\sum_{i=1}^{n} x_i = 1$ (as well as non-negativity restrictions). Constraints of this sort are 'genuine 'equalities, as in the computed solution must fulfill them precisely (where precisely implies inside working accuracy). In any case, it is not surprising in modeling that requirement that may appear to be at first to be firm equality constraints ought to be dealt with rather as limitations that need not be happy with most extreme conceivable precision. For example, this situation occurs when the hidden model is known to contain inaccuracies. The term tolerance constraint alludes to a range constraint with an extremely limit run, which gives the impact of fulfilling an equality constraint just to inside a recommended resilience. Consequently, the linear constraint:

$$a^T x = b$$

Would be replaced by:

$$b - \epsilon_2 \leqslant a^T x \leqslant b + \epsilon_1$$

The variables $\{x_1, \ldots, x_8\}$ represent the partial pressures of the following species, in the order given: propane, carbon monoxide, nitrogen oxide, carbon dioxide, oxygen, water, nitrogen, and hydrogen. Clearly it is required that $x_i \geqslant 0, i = 1, \ldots, 8,$ since negative values would have no physical meaning. The eight nonlinear reaction equations are as follows, where the constants $\{K_1, \ldots, K_8\}$ are the reaction constants whose logarithms are defined by logarithms in the temperature:

$$\frac{x_4^3 x_6^4 x_7^5}{x_1 x_3^{10}} - K_1 = f_1(x) = 0$$

$$\frac{x_4^3 x_6^4}{x_1 x_5^5} - K_2 = f_2(x) = 0$$

$$\frac{x_4^3 x_8^{10}}{x_1 x_6^6} - K_3 = f_3(x) = 0$$

$$\frac{x_4 \sqrt{x_7}}{x_2 x_3} - K_4 = f_4(x) = 0$$

$$\frac{x_4 x_8}{x_2 x_6} - K_5 = f_5(x) = 0$$

$$\frac{x_6 \sqrt{x_7}}{x_3 x_8} - K_6 = f_6(x) = 0$$

$$\frac{x_6}{x_8 \sqrt{x_5}} - K_7 = f_7(x) = 0$$

$$\frac{x_4}{x_2 \sqrt{x_5}} - K_8 = f_8(x) = 0$$

The linear equations derived from conservation of elements are the following, where the constants $\{a_1, \ldots, a_8\}$ represent the initial partial pressure of the various species:

Oxygen balance

$$x_2 + x_3 + 2x_4 + 2x_5 + x_6 - a_2 - a_3 - 2a_4 - 2a_5 - a_6 = f_9(x) = 0$$

Carbon balance

$$3x_1 + x_2 + x_4 - 3a_1 - a_2 - a_4 = f_{10}(x) = 0$$

Hydrogen balance

$$8x_1 + 2x_6 + x_8 - 8a_1 - 2a_6 - a_8 = f_{11}(x) = 0$$

Nitrogen balance

$$x_3 + 2x_7 - a_3 - 2a_7 = f_{12}(x) = 0$$

Total balance

$$\sum_{i=1}^{8} x_i - \sum_{i=1}^{8} a_i = f_{13}(x) = 0$$

The usual method for solving a set of over determined equations is to minimize the sum of squares of the residuals, i.e.:

$$\text{minimize} \sum_{i=1}^{13} f_i^2(x)$$

One difficulty with this approach is that the equations are of two distinct types, and it is desirable to preserve the natural separation of linear and nonlinear equations during the process of solution.

Scaling nonlinear least-squares problems: Nonlinear least-squares problems most commonly arise when a model function, say ***y(x, t)***, needs to be fitted as closely as possible to the set of observations ***{Yj}*** at the points ***{tj}***. The important feature of nonlinear data-fitting problems is that the variables to be estimated can sometimes be scaled automatically by scaling the independent variable ***t***.

To see how this may happen, we consider the following example. The formulation is a simplified version of a real problem, but the original names of the variables have been retained. The function to be minimized is:

$$\sum_{j=1}^{m} \left(\frac{y(p_j) - Y_j}{\Delta y_j} \right)^2$$

The functional form assumed for ***y (p)*** is:

$$y(p) = \sum_{j=0}^{J} A_j p^j + \sum_{k=1}^{K} B_k \exp\left(- \frac{(p - \bar{p}_k)^2}{2\sigma_k^2} \right)$$

Where the parameters to be estimated are $\{B_k, \bar{p}_k, \sigma_k\}, k = 1, \ldots, K$. A typical data set and fitting function ***y (p)*** are shown in Figure 5.The problem can be interpreted as fitting a Gaussian curve to each of the K peaks, together with a background function (the first term on the right-hand side of the example shown in Figure 5, K is four, and $\{\bar{p}_k\}, k = 1, \ldots, 4,$ are estimates of the corresponding peak positions; clearly each p~ lies in the range.

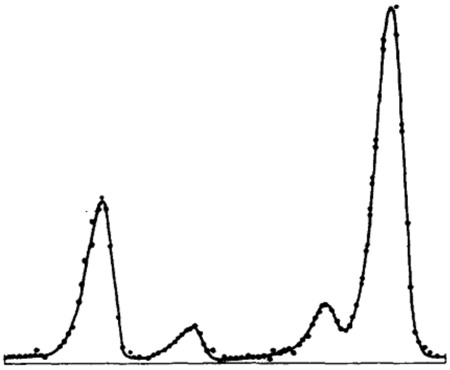

Figure 5- Typical data set and fitting function

$$\Phi(z) \equiv \sum_{j=1}^{J} \bar{A}_j z^j + \sum_{k=1}^{K} B_k \exp\left(\frac{-(z - \bar{z}_k)^2}{2\bar{\sigma}_k^2}\right)$$

Often it is not necessary to re compute $\{A_j\}, \{\bar{p}_k\}$ and $\{\sigma_k\}$ from $\{\bar{A}_j\}, \{\bar{z}_k\},$ and $\{\bar{\sigma}_k\}$. For example, we may wish to compute the area under the $\Phi(z)$ curve, or to compute values of ***y (p)*** at values other than $\{p_j\}$. In such cases the transformed function is just as useful as the original (and often much better).

III. Conclusion

The developed mathematical models and optimization algorithms permit illuminating the problem comprehensively, in the nearness information with different fulfillment and detail, utilizing different criteria. We have described some standard (and, generally, clear) techniques that can make optimization problems emerging from modeling more agreeable to arrangement by standard algorithms and software. All of the strategies given here have been utilized effectively in our own particular encounters with real problems. Certain preventative rules have additionally been recommended in the expectation of staying away from visit traps. Obviously, the nature of conceivable models varies so much that it is difficult to treat every single important perspective of modeling. The primary purpose of this paper is that designers of models ought to consider in the underlying stages a definitive need to explain an optimization problem, since it is far-fetched that optimization software will ever achieve the state where a general routine can be utilized with exemption.

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