IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 6 Ver. IV (Nov. - Dec. 2017), PP 55-60
www.iosrjournals.org

On the Performance ofZedan's Method for the Real Cambered NACA 4412 Airfoil

[1]Manjak N.H, [2]Swem S.Tand [1]Okai J.O
[1]Department of Mathematical Sciences,AbubakarTafawaBalewaUniversity,Bauchi, Nigeria
[2]Department of Mathematics/Statistics/Computer Science,University of Agriculture, Markurdi, Benue, Nigeria.
Corresponding Author:Manjak N.H

Abstact: *In this paper, the methodof Zedanfor computation of the lift coefficientand surface pressure coefficient distribution on arbitrary airfoils in potential flows is applied to the NACA 4412 airfoil and the results compared with NACA experimental data in order to verify its performance for a real cambered airfoil since nosuchairfoilwasconsidered by Zedan. Comparison of results between theory and experiment for both the NACA 4412 and modified NACA 4412 airfoils has shown that the method can be made to agree more closely with experiment when the airfoil is modified by adjusting one of the coefficients in the formula for thickness that results in the least overall change in the airfoil contour when compared with similar modifications.*
Keywords and Phrases:*Airfoil,NACAairfoil, Angle of attack, Lift coefficient, Pressure coefficient*

Date of Submission: 28-12-2017 Date of acceptance: 13-01-2018

I. Introduction

An accurate numerical method for computation ofthelift coefficient (c_l) and surface pressure coefficient distribution (c_p)on arbitraryairfoils in potential flows was presented by Zedan (1990) and applied to the symmetrical NACA 0012 airfoil and three cambered Karman-Trefftzairfoils of widely varying geometrical shapes that have exact solutions. Although the method gives accurate results for these airfoils and out performs the Hess-Smith panel method particularly at and around the leading edge region of the airfoil when compared with NACA experimental data, none of these airfoils is a realcamberedairfoil. Will the performance of the method for real cambered airfoils also be as good as it is for the other cases already considered by Zedan?' This paper investigates this problem by applying the method to the real cambered NACA 4412 airfoiland verifying its performanceby comparing its $c_l$andc_p valueswith those of NACA experimental data.

1.1Analysis of the Method of Zedan for Computation of the Lift and Pressure Distribution on Arbitrary Airfoils

In this method, an airfoil or wing section is first plotted in thez' plane ($z' = x' + y''$)with its leading and trailing edge locations at the point $z' = 0$and$z' = 1$, respectively. It is then translated and positioned with its tail at the point $z = 2c$ in the z plane ($z = x + iy$) using the transformation

$z = z' - (1 - 2c)$(1)

The translated airfoil is now transformed into a pseudo circle in the w plane by the inverse Joukowski map defined by Spiegel (1974) as

$z = w + \frac{c^2}{w}$ or $\frac{z-2c}{z+2c} = \left(\frac{w-c}{w+c}\right)^2$(2)

The real constant c corresponding to the singular point$w = c$ of the transformation (2) in the mapped $w$planeis estimated as $1/4$ of the distance between the trailing edge and a point mid way between the leading edge and the centre of curvature of the nose. The leading edge radius of the NACA 4412 airfoil is given by Abbot and Von Doenhoff (1958)as$1.1019t^2$, where t is the maximum thickness of the airfoil. In order to accomplish the transformation from an airfoil in the z plane to a pseudo circle in the w plane the function (2) is made single-valued and analytic everywhere in the z plane except on the branch line or cut consisting of the line segment $-2c \le x \le 2c$ by letting

$$z - 2c = r_1 e^{i\theta_1}, \quad r_1 = |z - 2c| > 0, \quad 0 \le \theta_1 \le 2\pi$$
$$z + 2c = r_2 e^{i\theta_2}, \quad r_2 = |z + 2c| > 0, \quad 0 \le \theta_2 \le 2\pi$$
$$r_1 + r_2 > 4c$$

so that

$$w = \frac{1}{4}\left(z + \sqrt{z^2 - 4c^2}\right) = \frac{1}{4}\left(\sqrt{r_1}e^{i\frac{\theta_1}{2}} + \sqrt{r_2}e^{i\frac{\theta_2}{2}}\right)^2 \quad (3)$$

Churchill and Brown(1984).If $w = u + iv$ and $z = x + iy$ then seperation of the real and imaginary components of equation (3) yields

$$u = \frac{1}{4}\left\{r_1 \cos\theta_1 + r_2 \cos\theta_2 + 2\sqrt{r_1 r_2}\cos\left(\frac{\theta_1 + \theta_2}{2}\right)\right\} \quad (4)$$

$$v = \frac{1}{4}\left\{r_1 \sin\theta_1 + r_2 \sin\theta_2 + 2\sqrt{r_1 r_2}\sin\left(\frac{\theta_1 + \theta_2}{2}\right)\right\} \quad (5)$$

Notice that the singular point $z = -2c$ of the transformation (2) map onto the singular point $w = -c$. On placing the tail of the airfoil at $z = 2c$ the singular point $z = -2c$ lies inside the airfoil and its image $w = -c$ lies inside the pseudo circle so that the singular points $z = -2c$ and $w = -c$ are not in the flows of the z plane and w plane, respectively (Simakov*et al*. 2000) and hence pose no problem.The centroid of the pseudo circle w_* in the w plane is then determined using the approximation given by Björn (2006) and the axes of the w plane are translated to w_* and rotated by angle α so that the real axis is in the direction of the freestream.The coordinate plane obtained after axes translationand rotation is called the ζ plane. The translation of axes and rotationby angle α is equivalent to the transformation

$$\zeta = (w - w_*)e^{-i\alpha} \quad (7)$$

The relationship between the velocities at points in the plane of the airfoil v_z to the corresponding points in the plane of the pseudo circle v_ζ is derived as

$$v_z = v_\zeta \left|\frac{dw}{dz}\right| \left|\frac{d\zeta}{dw}\right| = v_\zeta \left|\frac{dw}{dz}\right| \quad (8)$$

where

$$\frac{dw}{dz} = \frac{1}{\left(1 - \frac{c^2}{w^2}\right)} \quad (9)$$

and

$$\left|\frac{d\zeta}{dw}\right| = 1 \quad (10)$$

from equations (2) and (7). Equation (8) shows that the velocity field around the airfoil can be computed using the velocity field around the pseudo circle and the derivative of the mapping function. Also notice from equation (8) that the singularity $w = +c$ is a source of error for the computation of the velocity field and hence the pressure distribution.

To compute v_ζ, the method assumes a solution for the complex potential $\Omega(\zeta)$ of the flow past the pseudo circle as

$$\Omega(\zeta) = v_\infty \zeta + \sum_{k=1}^{\infty} \frac{c_k}{\zeta^k} + i\frac{\Gamma}{2\pi}\ln\zeta \quad (11)$$

where the coefficients of the series in the second term $c_k = a_k + ib_k \quad (k = 1,2,3,\dots)$. The first term in equation (11) represents a uniform flow with free stream velocity of magnitude v_∞, the infinite series in the middle represents a doubletat the origin and the higher order terms to account for the deviation from an exact circle. The last term represents a vortex flow with circulation Γ taken clockwise. The complex velocity

$$\frac{d\Omega}{d\varsigma} = v_\infty + \sum_{k=1}^{\infty} \frac{-kc_k}{\zeta^{k+1}} + i\frac{\Gamma}{2\pi\zeta} \quad (12)$$

is analytic everywhere except at the origin; the point $\zeta = 0$. This singular point is within the contour of the pseudo circle and hence posses no problem to the method since the flow under consideration is that which is external to the pseudo circle.The velocity field in the plane of the pseudo circle also satisfies the infinity boundary condition in equation (12); that is,

$$\frac{d\Omega}{d\varsigma} \to v_\infty \, as \, |\zeta| \to \infty$$

If $\Omega(\zeta) = \phi + i\psi$ and $\zeta = re^{i\theta}$, then

$$\psi = v_\infty r\sin\theta + \sum_{k=1}^{\infty}\left\{a_k\left(-\frac{\sin k\theta}{r^k}\right) + b_k\left(\frac{\cos k\theta}{r^k}\right)\right\} + \Gamma\left(\frac{\ln r}{2\pi}\right) \quad (13)$$

The function ψ is the stream function of the flow. Setting the stream function to a constant generates the streamlines of flow. Since the flow cannot penetrate the boundary of the pseudo circle, the boundary is also considered a streamline of flow; denote this special streamline by the symbol ψ_0.On applying the condition of constant streamline to equation (13) and noting that since ψ_0 is finite, the infinite series on the right hand side of the equation must converge. Equation (13) then takes the form

$$\sum_{k=1}^{m} a_k \left(\frac{\sin k\theta}{r^k}\right) + \sum_{k=1}^{m} b_k \left(-\frac{\cos k\theta}{r^k}\right) + \Gamma\left(-\frac{\ln r}{2\pi}\right) + \psi_0 = v_\infty r \sin\theta \quad (14)$$

on retaining a limited number of terms , say m, in the series.Since the derivative of the complex potential yields the conjugate of the velocity field, if we let

$$\frac{d\Omega}{d\varsigma} = v_1 - iv_2$$

then we have on retaining the first m terms of the infinite series in equation (14), that

$$v_1 = v_\infty + \Gamma\left(\frac{\sin\theta}{2\pi r}\right) + \sum_{k=1}^{m} a_k \left(\frac{-k\cos(k+1)\theta}{r^{k+1}}\right) + \sum_{k=1}^{m} b_k \left(\frac{-k\sin(k+1)\theta}{r^{k+1}}\right) \quad (15)$$

$$v_2 = \Gamma\left(-\frac{\cos\theta}{2\pi r}\right) + \sum_{k=1}^{m} a_k \left(\frac{-k\sin(k+1)\theta}{r^{k+1}}\right) + \sum_{k=1}^{m} b_k \left(\frac{k\cos(k+1)\theta}{r^{k+1}}\right) \quad (16)$$

The Kutta condition requires that the fluid velocity at the trailing edge T vanishes (Anderson, 1990); that is, $v_{1T} = v_{2T} = 0$.Thus,

$$\sum_{k=1}^{m} a_k \left(\frac{k\cos(k+1)\theta_T}{{r_T}^{k+1}}\right) + \sum_{k=1}^{m} b_k \left(\frac{k\sin(k+1)\theta_T}{{r_T}^{k+1}}\right) + \Gamma\left(\frac{-\sin\theta_T}{2\pi r_T}\right) = v_\infty \quad (17)$$

$$\sum_{k=1}^{m} a_k \left(\frac{-k\sin(k+1)\theta_T}{{r_T}^{k+1}}\right) + \sum_{k=1}^{m} b_k \left(\frac{k\cos(k+1)\theta_T}{{r_T}^{k+1}}\right) + \Gamma\left(\frac{-\cos\theta_T}{2\pi r_T}\right) = 0 \quad (18)$$

Determination of the series coefficients a_k, b_k $(k = 1,2,3, \ldots m)$, circulation Γ, and the pseudo circle streamline ψ_0, is done by taking a number of control points$n > 2m$on the boundary of the pseudo circle and applying the condition of constant streamline given by equation (14) at those points and the Kutta condition represented by equations (17) and (18) to obtain an over determined system of linear equations which is solved using a least square error minimization scheme . These control points are selected by the cosine spacing of the closed interval $0 \leq x \leq 1$to obtain the airfoil coordinates which are then projected onto the pseudo circle by the inverse Joukowskimap.Alternatively, the series coefficients a_k, b_k $(k = 1,2,3, \ldots m)$, circulation Γ, and the pseudo circle streamline ψ_0 can bedetermined by taking $2m$ control points on the boundary of the pseudo circle and applying the condition of constant streamline and the Kutta condition to obtain a closed system of $2m + 2$ equations in $2m + 2$unknowns.The determined values of the coefficients a_k, b_k $(k = 1,2,3, \ldots m)$and circulation Γ are substituted in equations (15) and (16) to obtain the components $v_1 and v_2$ of the velocity vector on the surface of the pseudo circle. The total velocity is then evaluated as

$$v_\zeta = |v_1 - iv_2| = \sqrt{u^2 + v^2} \quad (19)$$

The velocity on the surface of the airfoilv_z can now be computed in terms of v_ζ using equation (8). Finally the pressure coefficient distributionc_p is obtained using the formula given by Anderson (1991) and Deglaier*et al*. (2008) as

$$c_p = 1 - \left(\frac{v}{v_\infty}\right)^2 \quad (20)$$

The lift coefficient is computed using the formula given by Anderson (1991)andKaramcheti (1966) as

$$c_l = \frac{2\Gamma}{v_\infty l} \quad (21)$$

where$l$ is the airfoil length and Γ is the value of circulation computed from the system of equations.

1.2 Discussion of Zedan's Method for theNACA 4412 and Modified NACA 4412 Airfoils.

The method of Zedan (1990) is now verified by comparing its lift coefficient and pressure coefficient distribution values for the NACA 4412 and modified NACA 4412 airfoils usingNACA experimental data as yardstick.

1.3 Comparison of the Lift Coefficient from the Method of Zedan ($m = 12,\ n = 30$) with Experimental Results

When the angle of attack $\alpha = 2.9^0$and6.4^0 the computed lift coefficients on the NACA 4412 airfoilby the method of Zedan (1990) for the case $m = 12$ and $n = 30$ are 1.151 and 1.567 , respectively. The experimental lift coefficients for the same angles of attack are given by Pinkerton (1936)as 0.667 and 1.024, respectively. This marked variation in c_l between the theory here and experiment is due to the fact that the inviscid

flow theory does not account for the effects of the viscous boundary layer (pinkerton (1936) and Anderson (1990)).
However, when the airfoil is modified the predicted values of c_l agree better with experiment and are calculated as0.865 and 1.283, respectively.

1.4 Comparison of the Pressure CoefficientDistribution from the Method of Zedan ($m = 12,\ n = 30$) with Experimental Results

Figures 1 and 2 show comparisons of the pressure coefficient distribution from the method of Zedan (1990)with experiment for the NACA 4412 airfoil when the flow angles of attack are respectively, $\alpha = 2.9^0$and6.4^0while figures 4 and 5 are comparisons between the two methods for the modified NACA 4412 airfoil for the same flow angles of attack, respectively.

Figure 1: Pressure Distribution on the NACA 4412 Airfoil at 2.9^0 Angle of Attack($m = 12, n = 30$)

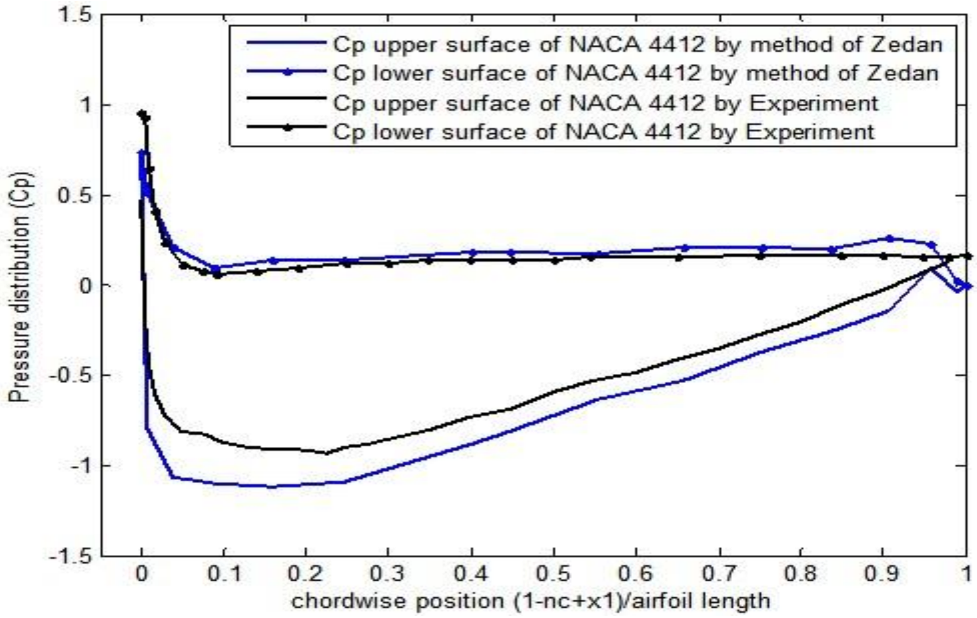


Figure 2: Pressure Distribution on the NACA 4412 Airfoil at 6.4^0 Angle of Attack ($m = 12, n = 30$)

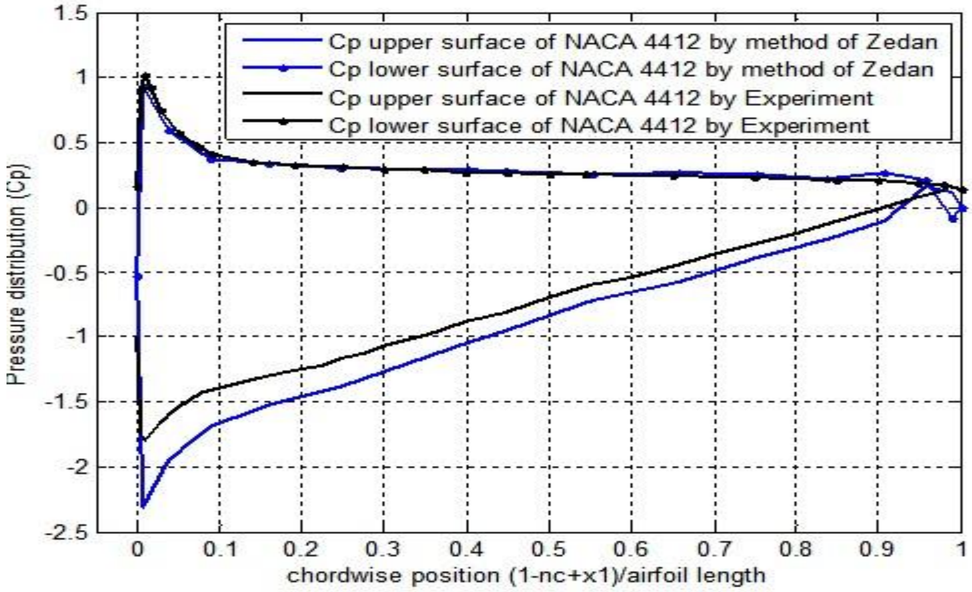


Figure 3: Pressure Distribution on the modified NACA 4412 Airfoilat2.9^0 Angle of Attack ($m = 12, n = 30$**)**

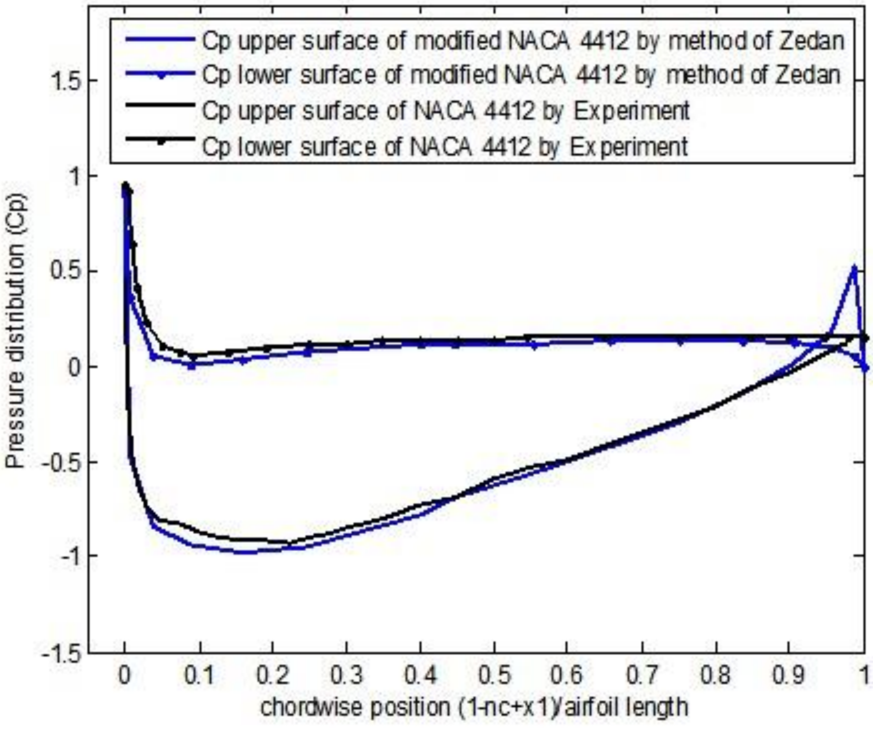
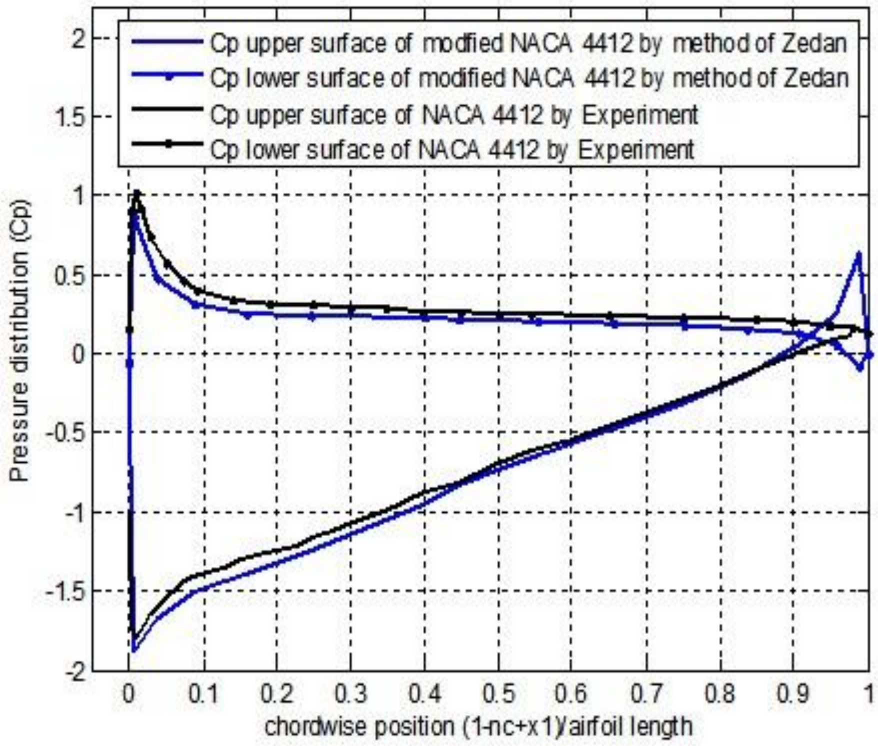


Figure 4: Pressure Distribution on the modified NACA 4412 Airfoil at 6.4^0 Angle of Attack ($m = 12, n = 30$)



Observe from figures 1 and 2 that the prediction of pressure coefficient distribution along the lower airfoil surface is fairly good except at and around the trailing edge region where the method fails at the intersection of the two curves.Unfortunately, the prediction of pressure coefficient distribution on the upper airfoil surface is not so good.However, when the airfoil is modified the shape of the pressure distribution curves changes and the prediction of pressure coefficient distribution along the upper airfoil surface fortunately improves while that on the lower surface sadly worsens when the flow angle of attack is 6.4^0. A careful observation of the pressure distribution curves between the NACA 4412 with experiment and modified NACA 4412 with experiment and the lift coefficient predicted in both cases, it is reasonable to consider the results on the modified airfoil as a fair approximation of the experimental values and reality, moreso that the modification does not significantly alter the shape of the original airfoil with the trailing edge region been most influenced (Nico, 2010).

II. Conclusion

In this paper, the method of Zedan (1990) for computation of the lift coefficientand surface pressure coefficient distribution on arbitrary airfoils in potential flows is applied to the NACA 4412 and modified NACA 4412 airfoils and both results were compared with NACA experimental data in order to verify its performance for a real cambered airfoil since no such airfoil was considered in Zedan (1990). Results from the computations showed marked variations inc_l and c_p between the theory and experiment for the NACA 4412 airfoil. This is of course due to the fact that the theory neglects the frictional force of the viscous fluidacting on the airfoil. However, the c_l andc_p predicted by themethod of Zedan (1990) on the modified NACA 4412 airfoil were closer to experiment than that for the actual case. Since modification of the airfoil as done in this paperdoes not significantly alter its shape, we conclude that the predictions of c_l and c_p by the method for this case can be taken as its prediction for the actual case.

References

[1]. Abbot, I.H. and Von Doenhoff,A.E. (1959).**Theory of Wing Sections Including a Summary of Airfoil Data**. Dover Publications Inc, New York, pp. 31-123

[2]. Anderson, J. K. (1991). **Fundamentals of Aerodynamics**. McGraw-Hill, New York,pp. 112-229

[3]. Björn, R. (2006).Conformal Mapping Potential Flow around a Wing SectionUsed as a Test Case for the Inviscid Part of Rans Solvers. Paper presented at the European Conference on Computational Fluid Dynamics, TU Delft, The Netherlands.

[4]. Churchill, R.V. and Brown, J. W.(1984). **Complex Variables and Applications**.McGraw-Hills International Editions, New York, pp.100-220
[5]. Deglaire, P., Ågren, O., Bernhoff, H. and Leijon, M. (2008). Conformal Mapping and Efficient Boundary Element Method without Boundary Elements for Fast Vortex Particle Simulations.*European Journal of Mechanics B/Fluids* **27**: 150-176.
[6]. Karamcheti, K. (1966). **Principles of Ideal-Fluid Aerodynamics**. John Wiley and Sons, New York, pp. 312-627
[7]. Nico, C. A. (2012).Studying the Effect of Boundary Layer Suction.Unpublished master's thesis, Delft University of Technology, The Netherlands.
[8]. Pinkerton, R. M. (1936). Calculated and Measured Pressure Distributions Over the Midspan Section of the NACA 4412 Airfoil, NACA Report No. 563.
[9]. Simakov, S. T.,Dostovalova, A. S., and Tuck, E.O. (2000).A GUI for Computing Flows Past General Airfoils. Paper presented at the Australasian MATLAB User Conference, Melbourne.
[10]. Spiegel, M. R. (1974).**Complex Variables (Schaum's Outline Series)**. McGraw-Hill Book Company, Singapore, pp. 200-231
[11]. Zedan, M.F. (1990), Series-Complex Potential Solution of Flow around Arbitrary airfoils. *Journal of Aircraft*.**27**: 936-940.

ManjakN.H."on The Performance of Zedan's Method For The Real Camberednaca 4412 Airfoil ." IOSR Journal of Mathematics (IOSR-JM) 13.6 (2017): 55-60.