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Analysis of the Queue at Neuro-Trauma Centre of National Hospital in Sri Lanka

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Abstract: *This report contains the analysis of queuing systems of Neuro-Trauma Centre at the National Hospital in Sri Lanka. There are several clinics held by Neuro-Trauma Centre and in most of them, a waiting queue may form when the doctors or the serving persons are too busy to meet the requirements of the arriving patients immediately as a result of the demand exceeds the supply. The main purpose of this study is to apply the queuing theory and to evaluate the parameters involved in the service unit for the consultations and pharmacy of Neuro-Trauma Centre at the National Hospital in Sri Lanka. Therefore, a mathematical model is developed based on queuing theory by collecting data through an observational study at the clinic and analyzing them through computations to optimize the queues.*

Keywords: *arrival rate; service rate; number of channels*

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I. Introduction

The Neuro-Trauma Centreat the National Hospital has been expanded in 2011 as a partof Accident and Orthopedic Service of the hospital. It is a well-equipped unit which enablesproviding a better service for a large number of patients within a day. There areseveral clinics that are held by Neuro-Trauma Centreat National Hospital. Usually twoclinics are held per day (morning and afternoon) on weekdays except on public holidays.The clinics are separated for groups of patients with different sicknesses. Thereare four phases (servers) in Neuro-Trauma Centre, which means four servers with fourqueues in terms of Queuing Theory. A queue is formed whenever current demand exceedsthe existing capacity to serve when each phase is so busy that arriving patientscannot receive immediate service facility. So each server process is done as a queuingmodel in this situation.

The Current System At The Clinic Can Be Explained As Follows.

First, the arrived patients are given a token and required to wait in a queue to collecttheir drug card from the record room. Then after collection of the drug card, they haveto wait in a queue at the reception to obtain a number to see the medical officer, surgeonor consultant depending on their sickness. Then again the patients are neededto wait in a queue to get treatment. After getting treated there’s a queue at the pharmacyto take medicine and finally they are needed to wait in a queue at the receptionto obtain an appointment date to revisit the clinic if necessary. There are regular patientswho have been registered before at the clinic as well as new patients who cometo clinic for the first time.

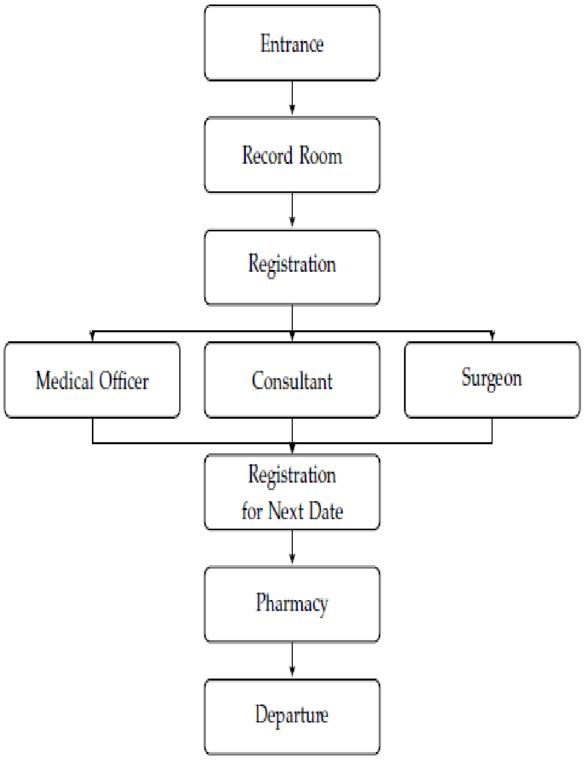

Figure 1: Current system of Neuro-Trauma Centreat National Hospital

II. Materials And Methods

Waiting line models in queuing theory provides the basis for the minimization of longwaiting queues at service counters which is very helpful to eliminate wastage of time. There are three parts of a waiting line; arrival characteristics, waiting line characteristicsand service characteristics.

i. Arrival characteristics

The input source that generates arrivals or customers for a service system hasthree major characteristics:

- *Size of the arrival population*: The number of arrivals of patients varies according to the type of the clinic held. In some clinics, there may be less number of patients while long queues are present in some clinics. The evening clinics on Wednesday and Thursday and the morning clinic on Friday are more crowded with approximately hundred or more number of patients and difficult to manage compared to other clinics.
- *Behavior of arrivals*: Most of the patients are registered patients with the clinic who comes to see the doctor at the clinic which treats their particular sickness. At the same time, there may be few patients who come for the first time. Each patient is bound to follow the system at the clinic while waiting for their turn. Almost all the patients spent at least four hours at the clinic due to the long waiting lines.
- *Pattern of arrivals*: The patients arrive randomly throughout the clinic period. At the beginning of the clinic there is a large bulk of patients arrived at the clinic and with time the number of arrivals decrease gradually. Thenumber of arrivals per unit time can be estimated by a probability distribution known as the Poisson distribution.

ii. Waiting-Line Characteristics

The waiting line is considered to be unlimited at each of the clinic since there isno restriction applied on the number of patients to be treated. The patients inwaiting line are treated according to the first-in, first out (FIFO) rule.

iii. Service characteristics

Two basic properties are important on providing service to the patients:

- *Design of the service system*: Service systems are usually classified in terms of their number of channels (number of servers) and number of phases (number of service stops). They are single-channel queuing system, multichannel queuing system, single-phase system, and multi-phase system. The

3multiple-channel queuing system was applied to this service system since two or more servers or channels are available to handle arriving patients.

- *Distribution of service time*: Service time at the clinic is considered to be random service time due to the difference in time periods for the patients to get treated. Therefore it can be described by the negative exponential probability distribution.

The queuing equations for the multiple-channel queuing system are as follows.
Average number of channels open M, Average arrival rate *l*, Average service rate at each channel μ,

Server utilization: $\rho = \frac{\lambda}{\mu M}$

The probability that there are no patients in the system: $P_0 = \frac{1}{\sum_{n=0}^{M-1}\frac{1}{n!}(\frac{\lambda}{\mu})^n + \frac{1}{M!}(\frac{\lambda}{\mu})^M(\frac{M\mu}{M\mu-\lambda})}$ for $M\mu > \lambda$

The average number of patients in the system: $L_s = \frac{\lambda\mu(\frac{\lambda}{\mu})^M}{(M-1)!(M\mu-\lambda)^2}P_0 + \frac{\lambda}{\mu}$

The average time a patient spends in the waiting line and being served: $W_s = \frac{\mu(\frac{\lambda}{\mu})^M}{(M-1)!(M\mu-\lambda)^2}P_0 + \frac{1}{\mu} = \frac{L_s}{\lambda}$

The average number of patients in line waiting for service: $L_q = L_s - \frac{\lambda}{\mu}$

The average time a patient spends in the queue waiting for service: $W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda}$

According to the current system at the Neuro-Trauma Center of National Hospital,the evening clinics on Wednesday and Thursday and the morning clinic on Friday aremore crowded compared to other clinics. Therefore, only those three clinics were takeninto account in this research. And four phases (Medical officer, Consultant, Surgeonand Pharmacy) were chosen out of several phases in the current procedure (others arenegligible).
Two solutions are recommended using the queuing model as follows.
Part 1- Withoutcontrolling the arrivals.
Part 2-With controlling the arrivals.

2.1 Part I - Without controlling the arrivals

In this section, the collected data on number of channels, service rate at each channeland arrival rate at each phase (medical officer, consultant, surgeon and pharmacy) atthe three clinics was analyzed without doing any changes to raw data observed at theclinic.
According to the collected data, average number of channels, arrival rate per minute,service rate per minute and value at each phase for several time intervals of 30 minuteswere calculated for all four phases. Such values in the case of the phase 'MedicalOfficer' is given below.

Table 1: The server utilization for the Medical Officer

Day	Time	M	λ/min	μ/min	ρ=λ/(μM)
Wednesday	1.15-1.45	2	2.283333333	0.688888889	1.657258
	1.45-2.15	3	1.344444444	1.366666667	0.327913
	2.15-2.45	1	0	0.811111111	0
Thursday	12.45-1.15	1	2.077777778	0.066666667	31.16667
	1.15-1.45	3	0.522222222	0.688888889	0.252688
	1.45-2.15	5	0.266666667	1.111111111	0.048
	2.15-2.45	4	0.166666667	0.9	0.046296
	2.45-3.15	1	0	0.266666667	0
Friday	8.45-9.15	2	2.244444444	0.711111111	1.578125
	9.15-9.45	3	0.222222222	0.888888889	0.083333
	9.45-10.15	1	0	0.177777778	0
	10.15-10.45	0	0	0	0
	10.45-11.15	1	0	0.444444444	0
	11.15-11.45	1	0	0.244444444	0

In the queuing model, if the utilization value is greater than 1 it is considered that thesystem is ineffective and there is a long waiting queue and if value is less than 0.5, thesystem is unnecessarily effective. Therefore, in order to recommend a new optimizedstrategy, the time intervals in which value is greater than 1 and less than 0.5 were consideredand average number of channels was increased if the value is greater than 1and it was reduced if the value is less than 0.5. So the modifications were done only inrelevant time intervals considering the calculated values for those time intervals. Anynumber of channels more than that of suggested is considered to be not cost effectiveand lesser number of channels is considered as inefficient when suggesting the modelsince unnecessary improvements may not be realistic and sometimes difficult to implementin practice.

The calculated parameters of the queuing model in the case of the phase 'Medical Officer' is given below.

Table 2: The increased average number of channels for Medical officer

Day	Time	M	λ/min	μ/min	ρ=λ/(μM)	Variable	Value
Wednesday	1.15-1.45	3	2.8333	0.6889	0.7366	P_0	0.0801
						L_s	3.7392
						W_s	2.4564
						L_q	1.5295
						W_q	1.0048
Thursday	12.45-1.15	32	2.0778	0.0667	0.9739	P_0	0
						L_s	62.3128
						W_s	29.9901
						L_q	31.1462
						W_q	14.9901
Friday	8.45-9.15	4	2.2444	0.7111	0.7890	P_0	0.0294
						L_s	5.3142
						W_s	2.3677
						L_q	2.1579
						W_q	0.9614

2.2 Part II - With controlling the arrivals

In this section the arrivals were controlled on each day at the clinic by giving priorappointments to the patients. An average number of patients per minutes and anaverage service rate per minute were allocated by giving appointments at each phasefor the three clinics and possible number of channels was computed in each case tooptimize the waiting time by using the queuing model.

Table 3: Suggested arrival and service rates

Phase	Day	λ/min	μ/min
Medical Officer	Wednesday	1.2092	0.9556
	Thursday	0.6067	0.6067
	Friday	0.4111	0.4111
Consultant	Thursday	0.0778	0.0778
	Friday	0.2032	0.2032
Surgeon	Wednesday	0.1356	0.1356
	Friday	0.1370	0.1370
Pharmacy	Wednesday	0.6302	0.6302
	Friday	0.7422	0.7422

Here, average arrival rate per minute and average service rate per minute are the sameexcept for the phase medical officer on Wednesday clinic. Therefore, only one channelis required at each phase on all three days to optimize the waiting line except for phasemedical officer on Wednesday clinic where two channels are required to optimize thewaiting line according to queuing theory computations.

III. Results and Discussion

3.1 Part I - Without controlling the arrivals

Table 4: The current and suggested model on average number of channels

Phase	Wednesday			Thursday			Friday		
	Time	C	S	Time	C	S	Time	C	S
Medical Officer	1.15-1.45	2	3	12.45-1.15	1	32	8.45-9.15	2	4
	1.45-2.15	3	1	1.15-1.45	3	1	9.15-9.45	3	1
	2.15-2.45	1	1	1.45-2.15	5	1	9.45-10.15	1	1
				2.15-2.45	4	1	10.15-10.45	0	0
				2.45-3.15	1	1	10.45-11.15	1	1
							11.15-11.45	1	1
Surgeon	1.00-1.30	1	5	Not available			7.45-8.15	1	6
	1.30-2.00	2	1				8.15-8.45	1	2
	2.00-2.30	2	1				8.45-9.15	1	1
	2.30-3.00	2	1				9.15-9.45	1	1
	3.00-3.30	1	1				9.45-10.15	1	1
							10.15-10.45	1	1
Consultant	Not available			12.45-1.15	1	6	7.45-8.15	1	9
				1.15-1.45	1	5	8.15-8.45	1	2
				1.45-2.15	1	2	8.45-9.15	1	1
				2.15-2.45	1	1	9.15-9.45	1	1
				2.45-3.15	1	1	9.45-10.15	1	1
				3.15-3.45	1	1	10.15-10.45	1	1
				3.45-4.15	1	1	10.45-11.15	1	1
Pharmacy	1.15-1.45	1	3	Crowd is manageable			9.00-9.30	1	3
	1.45-2.15	1	2				9.30-10.00	1	2
	2.15-2.45	1	2				10.00-10.30	1	1
	2.45-3.15	1	1				10.30-11.00	1	1
	3.15-3.45	1	1				11.00-11.30	1	1
	3.45-4.15	1	1						
	4.15-4.45	1	1						

C- Current average number of arrivals, S- Suggested average number of arrivals

The proposed model for Part 1 contains the suggested number of channels which isrequired to optimize the current system at the clinic. The arrival rate of patients at thebeginning of the clinic is uncontrollably high. The number of channels at each phaseat the clinic was increased and reduced where it was necessary to smooth the queueof patients. And the solutions are given for 30 minutes of time intervals in which inefficiency exists. The suggested number of channels varies with the time intervals.In most of the time intervals at each phase, current and recommended average numberof channels is different. Therefore a comparison between them would be helpful tounderstand the improvements which are required to be done. It can be concluded thatthe suggested number of channels for the time intervals at the beginning of the clinicis much higher than the current number of channels.

3.2 Part II - With controlling the arrivals

The proposed model for Part 2 contains the suggested arrival rate and the suggestednumber of channels for 30 minutes of time intervals by giving appointments to patients.Therefore, a fixed number of channels and an arrival rate can be controlled bythis model.From the comparison of current and average number of channels, it can be concludedthat in most of the time intervals the suggested number of channels are less than thecurrent number of channels. And a fixed number of channels can be allocated at eachphase throughout the clinic time. Therefore, the suggested system will be more costeffective than the current system.

Table 5: The suggested number of appointments for the clinic

	Wednesday		Thursday		Friday	
Phase	Channels	Patients/ 30 min	Channels	Patients/ 30 min	Channels	Patients/ 30 min
Medical Officer	2	36	1	18	1	12
Consultant	Not Available		1	2	1	6
Surgeon	1	4	Not Available		1	4
Pharmacy	1	19	Crowd is Manageable		1	22

In this section, the arrival rate was controlled by giving appointments to patientswithin 30 minutes of time intervals. The following table illustrates the comparisonbetween current and suggested number of arrivals.

Table 6: The current and suggested model on average number of arrivals

Phase	Wednesday			Thursday			Friday		
	Time	C	S	Time	C	S	Time	C	S
Medical Officer	1.15-1.45	69	36	12.45-1.15	62	18	8.45-9.15	67	12
	1.45-2.15	40	36	1.15-1.45	16	18	9.15-9.45	7	12
	2.15-2.45	0	36	1.45-2.15	8	18	9.45-10.15	0	12
				2.15-2.45	5	18	10.15-10.45	0	12
				2.45-3.15	0	18	10.45-11.15	0	12
							11.15-11.45	0	12
	Total	109	108	Total	91	90	Total	74	72
Surgeon	1.00-1.30	13	4	Not available			7.45-8.15	9	4
	1.30-2.00	6	4				8.15-8.45	1	7
	2.00-2.30	2	4				8.45-9.15	1	2
	2.30-3.00	0	4				9.15-9.45	1	4
	3.00-3.30	0	4				9.45-10.15	1	1
							10.15-10.45	1	1
	Total	21	20				Total	24	24
Consultant	Not available			12.45-1.15	5	2	7.45-8.15	25	6
				1.15-1.45	4	2	8.15-8.45	9	6
				1.45-2.15	3	2	8.45-9.15	6	6
				2.15-2.45	1	1	9.15-9.45	2	1
				2.45-3.15	0	2	9.45-10.15	0	6
				3.15-3.45	0	2	10.15-10.45	0	6
				3.45-4.15	0	2	10.45-11.15	0	6
				Total	15	14	Total	43	42
Pharmacy	1.15-1.45	31	19	Crowd is manageable			9.00-9.30	50	22
	1.45-2.15	52	19				9.30-10.00	27	22
	2.15-2.45	35	19				10.00-10.30	19	22
	2.45-3.15	12	19				10.30-11.00	1	1
	3.15-3.45	2	19				11.00-11.30	2	22
	3.45-4.15	0	19						
	4.15-4.45	0	19						
	Total	132	133				Total	111	110

C- Current average number of arrivals, S- Suggested average number of arrivals

It can be concluded that a constant arrival rate is possible to manage in the clinic tooptimize the waiting time of patients.

According to the current system, most of the patients try to come early as possible toobtain a token to see a doctor soon. This may cause wastage in time of the patients.But according to the suggested system, they can come at a particular time given forthem at their previous clinic which they have come. And the appointments can begiven in 30 minutes of time intervals according to the suggested system. There canbe some patients who comes to the clinic for the first time. Appointments cannot begiven for them as in the suggested system. Since that number is very small, it will notsignificantly affect the suggested system. Therefore they can be treated according tothe prevailing system.

3.3 Comparison between Part I and Part II

In this research, two queuing models are suggested to optimize the waiting time ofpatients at Neuro-Trauma Center of National hospital. A comparison between them isneeded to identify the best model which is efficient.
By comparing the two proposed scenarios of Part 1 and Part 2, it can be concluded thatin most of the cases the suggested number of channels in Part 2 is less than that of inPart 1. Therefore, Part 2 scenario will be more cost effective than Part 1 model. And acost analysis of the two scenarios should be conducted in order to determine the bestone.

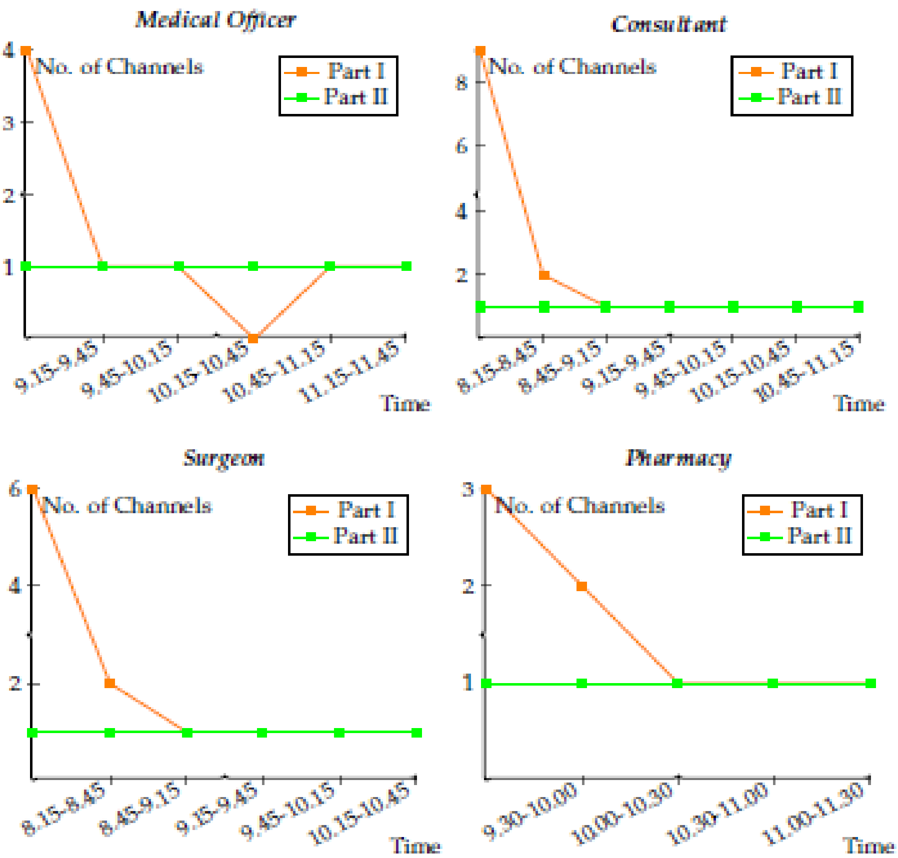

Figure 2: Comparison between part I and part II on Friday

III. Limitations

This research is heavily depended on the observed data collected at each clinic. Thereforethere may be human errors in collecting data which may cause incorrect output ofthe mathematical model. Another restraint is there may be slight deviations from thetaken values due to many reasons such as bus strikes, public holidays etc. The resultsfor Part 1(i.e. without controlling the arrival rate) are somewhat impractical in the realworld since a large number of channels must be allocated to optimize the system insome cases. Therefore to implement the suggested systems successfully and all thestakeholders must be well informed about it.

IV. Conclusion

According to the results of this research, it is clear that the queuing model can be appliedto analyze the queuing system in Neuro-Trauma Centre at the National Hospitalof Sri Lanka. By analyzing the observed data obtained from different clinics, optimalsystem models were developed to achieve the least waiting time and length of patientsto receive their required service at each clinic. Optimal system can be determined bydoing a cost analysis of the systems which have been developed in this research.

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