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Estimation of Baby Weights At Birth Using Generalized Inverse Regression Model And Its Application to Prenatal Care Data

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Abstract: *Maternal weight gain and wait size is one of the most important independent predictors of birth weight. This study was conducted to observe the weight gained and waist size increase by pregnant women and the correlation between the weight gain, waist size increase and the birth weight of baby. In this study women who delivered full term single baby at Webuye sub-county hospital were included after their verbal consent. Baby weights after birth, mothers' weights and their waist sizes before birth were recorded and the data obtained was analysed using general least square method to estimate the parameters of the model. The mean weight of the pregnant women at full term before birth, mean waist size and mean baby weight after delivery were used to develop a more feasible model with uncorrelated errors. The model developed showed the positive effect of gestational weight gain and waist size increase on birth weight. The study concluded that gestational weight gain and waist size increase has a positive relationship on birth weight and can be used to predict birth weight.*

Keywords: *Gestational weight gain, Birth weight, Regression*

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I. Introduction

Birth weight is one important determinant of babys well-being and estimation of Birth weight is an important determinant of babys well-being as low birth weight(below 2.5kgs)is known to increase neonatal morbidity and other adult onset diseases like type 2 diabetes and ischemic heart diseases while macrosomia (above 4kgs) is known to increase birth injuries and other problems like shoulder dystocia, birth trauma and fractures of the clavicle or limbs (kinira,2005).The challenge by medical practitioners is estimation of birth weight at birth(see for instance, Robert 2010).The (WHO,2013) has recommended that the total weight gain of mothers should be according to their pre-pregnancy body mass indices since pre pregnant body mass index varies considerably among women and variance appears to the due to many maternal characteristics therefore maternal characteristics such as pre-pregnant weight and waist size increase if well analysed can also be important predictors of birth weight. Several studies have been carried out in different parts of the world using other maternal and baby characteristics but in Kenya its lacking. This study mainly focuses on mothers' weights, waist sizes and their baby weights after birth. A generalized regression model is then used to construct the inverse regression model that can estimate baby's birth weight at birth with minimal errors.

II. Methodology

The study was conducted over a period of 6 months and Webuye sub-county hospital. The study was undertaken with approval from Webuye sub-county hospital administration. The mothers' weights and waist sizes were taken from different expectant mothers after seeking consent from them. The weight, waist size and baby weight after birth were taken at full term for normal, live singleton deliveries. Pregnancies resulting from still birth and twins and other complications were excluded. The data collected were analysed using R-software and the results used to construct generalized inverse model that can estimate birth weight.

III. The Model

The general regression equation

$$Y = X\beta + \varepsilon_i \tag{1}$$

Where Y is the dependent variable,X is independent variable β is the vector parameters, and ε is the white noise with the assumption that $Var(\varepsilon_i) = \sum$. In OLS theory its assumed that $Var \sum = \sigma^2 I$ implying that white noise is independent as $\text{cov}(\varepsilon_i, \varepsilon_j) = 0 \;\; i \neq j.$ In practice this might not be the case thus

$\operatorname{cov}(\varepsilon_i, \varepsilon_j) \neq 0$ and $i \neq j$. In this study this would be the ultimate assumption and $\sum$ is unknown. Therefore with the assumption that $Cov(\varepsilon_i, \varepsilon_j) \neq 0$ equation (1) cannot be solved by OLS and requires some modification as follows; with $Y = X\beta + \varepsilon_i$ and $\varepsilon \neq \sigma^2$ thus we standardise this equation by multiplying by P vector of eigenvalues of $\sum$ such that $PP' = I$ and (1) becomes

$$P'Y = P'X\beta + P'\varepsilon \tag{2}$$

Simply written as

$$Y^* = \beta X^* + \varepsilon^* \tag{3}$$

$$\arg\min_{\beta} \hat{\varepsilon}\hat{\varepsilon} = \arg\min_{\beta} \left\| Y - X'\hat{\beta} \right\| \tag{4}$$

Which can be simplified to

$$\arg\min_{\beta} \hat{\varepsilon}\hat{\varepsilon} = \arg\min_{\beta} (y'y - yX\hat{\beta} - X'\hat{\beta}'Y + X'\hat{\beta}'X\hat{\beta}) \tag{5}$$

Therefore

$$0 - 2X'W + 2X'X\hat{\beta} = 0 \tag{6}$$

The solution is

$$X'X\hat{\beta} = X'Y \tag{7}$$

Thus,

$$\hat{\beta} = (X'X)^{-1} X'Y \tag{8}$$

Therefore

$$\hat{\beta}^*_{GLS} = (X^{'*} X^*)^{-1} X^{'*} Y^* \tag{9}$$

Notice that (9) has been derived based on $\hat{\Sigma}$ but we don't know $\hat{\Sigma}$ we estimate based on data as follows;

$$X = \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ \vdots & \vdots & \vdots \\ X_{p1} & X_{p2} & X_{p3} \end{pmatrix} \tag{10}$$

X_{i1} =Baby weight observations

X_{i2} =Mothers weight observations

X_{i3} = Waist size observations

The estimate of $\hat{\Sigma}$ here is denoted by $\hat{\Sigma}$ and

$$\hat{\Sigma} = \frac{1}{(n-1)(k-1)} \Sigma(\bar{X}_{ii} - X_i)(X_{ii} - X_i)$$

$$= \begin{pmatrix} Var(X_{ii}) & Cov(X_i X_i) \\ Cov(X_i X_i) & Var(X_{ii}) \end{pmatrix} \tag{11}$$

Therefore the p used in (2) is now based on $\hat{\Sigma}$ and here is denoted by P such that

$$\hat{\Sigma} = P\Lambda P \tag{12}$$

hence forth we utilize (9) is vital to this study. Hence could be the cornerstone to the application of this model. From (3) we shall get our generalised inverse equation.

$$Y^* = B_0 + B_1 X_{1i} + B_2 X_{2i} \tag{13}$$

In the next section we apply the developed model on the data collected from Webuye sub county hospital from January 2016 to June 2016.

IV. Application

Next we apply (10) on our data in matrix form is given as

$$X = \begin{pmatrix} 2.5 & 58 & 32 \\ 3.0 & 68 & 37 \\ 3.2 & 70 & 38 \\ 3.4 & 74 & 43 \\ 3.6 & 78 & 43 \\ 3.9 & 80 & 49 \\ \vdots & \vdots & \vdots \end{pmatrix} \qquad (14)$$

First, we estimate $\hat{\Sigma}$ from the data and here we require $\bar{X}_{1i}$, $\bar{X}_{2i}$ and $\bar{X}_{3i}$, they are $\bar{X}_{1i}$ = 3.3, $\bar{X}_{2i}$ = 70; and $\bar{X}_{3i}$ = 40 , the means for baby weights, mothers weights and waist sizes respectively. The deviations from their means is given by matrix X as

$$X = X_{1i} - X_i = \begin{pmatrix} -0.8 & -13 & -8 \\ -0.8 & -3 & -3 \\ -0.1 & -1 & -2 \\ -0.1 & 3 & 0 \\ \vdots & \vdots & \vdots \end{pmatrix}$$

(15)

Finally

$$\hat{\Sigma} = \frac{X'X}{n-1} = \begin{pmatrix} 0.2 & 3.2 & 2.3 \\ 3.2 & 53 & 36 \\ 2.3 & 36 & 27 \end{pmatrix}$$

(16)

And $P'P = I$. Then we decompose $\hat{\Sigma}$ to $\hat{\Sigma} = P\Lambda P$. Therefore

$$Det(\Sigma - \lambda I) = \begin{pmatrix} 0.2-\lambda & 3.2 & 2.3 \\ 3.2 & 53-\lambda & 36 \\ 2.3 & 36 & 27-\lambda \end{pmatrix}$$

$$= (0.2-\lambda)(53-\lambda)(27-\lambda) = 0 \qquad (17)$$

With eigenvalues

$$(0.0005,\ 0.0017,\ 0.7704)$$

and corresponding eigenvectors in matrix form given as

$$P = \begin{pmatrix} 0.9984 & -0.0252 & 0.0503 \\ 0.0266 & 0.5755 & 0.08174 \\ 0.0496 & -0.8174 & 0.5739 \end{pmatrix} \qquad (18)$$

from (12) we have

$$\begin{pmatrix} 0.9984 & -0.0252 & 0.0503 \\ -0.0267 & 0.5735 & 0.0817 \\ -0.0095 & 0.8174 & 0.5787 \end{pmatrix} \begin{pmatrix} 0.9984 & 0.0267 & -0.0495 \\ -0.0252 & 0.5755 & 0.08174 \\ 0.0503 & 0.8174 & 0.5739 \end{pmatrix} = \begin{pmatrix} 1.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{pmatrix} \qquad (19)$$

$$P'\Lambda P = \begin{pmatrix} 0.9984 & -0.0252 & 0.0503 \\ -0.0267 & 0.5735 & 0.0817 \\ -0.0095 & 0.8174 & 0.5787 \end{pmatrix} \begin{pmatrix} 0.0005 & 0 & 0 \\ 0 & 0.0017 & 0 \\ 0 & 0 & 0.7704 \end{pmatrix} \tag{20}$$

$$= \begin{pmatrix} 0.984 & -0.0267 & 0.0495 \\ -0.0252 & 0.5755 & -0.2174 \\ 0.503 & 0.8174 & 0.5737 \end{pmatrix} \tag{21}$$

Therefore

$$P'\Lambda P = \begin{pmatrix} 0.9984 & 0.0267 & -0.0495 \\ -0.0252 & 0.5755 & 0.08174 \\ 0.0503 & 0.8174 & 0.5739 \end{pmatrix} \begin{pmatrix} 0.0005 & 0 & 0 \\ 0 & 0.0017 & 0 \\ 0 & 0 & 0.7704 \end{pmatrix}$$

$$= \begin{pmatrix} 0.47 & 0.83 & 0.87 & \cdots \\ 3.7 & 4.4 & 4.51 & \cdots \\ 17.55 & 21 & 21.6 & \cdots \end{pmatrix} \tag{22}$$

By OLS theory

$$\beta = \begin{pmatrix} 1 & 1 & 1 & \cdots \\ 2.5 & 3.0 & 3.2 & \cdots \\ 32 & 37 & 381 & \cdots \end{pmatrix} \begin{pmatrix} 1 & 2.5 & 32 \\ 1 & 3.0 & 37 \\ 1 & 3.2 & 38 \\ . & . & . \\ \vdots & \vdots & \vdots \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1 & 1 & 1 & \cdots \\ 2.5 & 3.0 & 3.2 & \cdots \\ 32 & 37 & 381 & \cdots \end{pmatrix} \begin{pmatrix} 58 \\ 68 \\ 70 \\ \vdots \end{pmatrix}$$

$$= (48.64 \quad 1.604 \quad 1.3015) \tag{23}$$

Therefore

$$Y^* = (48.64 \ \ 1.604 \ \ 1.3015) \begin{pmatrix} 0.47 & 0.83 & 0.87 & \cdots \\ 3.7 & 4.4 & 4.51 & \cdots \\ 17.55 & 21 & 21.6 & \cdots \end{pmatrix}$$

$$= (51.6 \ \ 74.7 \ \ 76.8 \ldots) \tag{24}$$

$$\beta = \begin{pmatrix} 48.64 \\ 1.6040 \\ 1.3015 \end{pmatrix} \tag{25}$$

Thus from (9) we have

$$\beta^* = \begin{pmatrix} 21.04 \\ 1.29 \\ 1.249 \end{pmatrix} \tag{26}$$

Then the generalised model is

$$Y^* = 21.04 + 1.29\ X_{1i} + 1.249\ X_{2i}$$
(27)

The coe_cient of baby weight is 1.29 so every unit increase in baby weight a 1.29 unit increase in weight of the mother is predicted holding other values constant. The coefficient of mothers waist size is 1.249 so every unit increase in mothers waist size a 1.249 unit increase in mothers waist size is predicted holding other values constant.

4.1 Prediction of the baby's birth weight by the model

The main objective of the study was to estimate the baby's weight before birth so that interventions can be put in place before delivery of the baby. Hence, calling for prediction of baby's weight which is the subject of the section. From the above model, baby's weight can be predicted using the following generalised inverse model.

$$Baby's\ weight(X_{1i}) = \frac{Y_{1i} - 21.04 - 1.249 * X_{2i}}{1.29}$$
(28)

where

Y_{1i} = mother's weight.

X_{1i} = Baby's weight

X_{2i} =Waist size.

Example

A prenatal information from one of the mother's records in the data collected had the following values;
Waist size=36.0cm
Mother's weight=70Kg

Fitting the information into the model gives baby weight=(70-21.04-1.249*36)/1.29=3.10kgs from data given, the calculated vaue is 3.10 which shows a slight variation from true value(3.00).This shows that in a given situation where the mothers pre-natal information (weight and waist size) are known the weight of the baby can be predicted with or determined with small error.

V. Conclusion

In this study, generalised inverse method was applied on a data set from a Webuye sub county hospital of Bungoma County (Western Kenya) for a period of 6 months. The study investigated the relation between mothers' weight, waist size and birth weight. The model developed enabled us to predict birth weight which occurred at full term before delivery. The study results could be used in prediction and estimation of birth weights before birth so that neccessary interventions can be put in place by medical practitioners to avoid complications associated with low and high birth weights.

VI. Recommendation

The study concentrated on singleton birth at full term and it could be interesting for further investigations to be carried out on birth resulting in twins triplets or more then results compared for efficiency. We also recommend a suitable model that includes other maternal characteristics.

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