IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 6 Ver. IV (Nov. - Dec. 2017), PP 32-40
www.iosrjournals.org

Minimal Total Dominating Functions of Corona Product Graph of A Cycle With A Star

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Abstract : *Graph Theory is the fast growing area of research in Mathematics. Recently, dominating functions in domination theory have received much attention. In this paper we discuss some results on minimal signed dominating functions and minimal total signed dominating functions of corona product graph of a cycle with a star.*

Keywords : *Corona Product, Signed Dominating Function, Total Dominating Function*

Date of Submission: 05-12-2017 Date of acceptance: 11-01-2018

I. Introduction

The concept of domination number of a graph was first introduced by Berge [4] in his book in Graph Theory. The concepts of total dominating functions and Minimal dominating functions are introduced by Cockayne et al. [3], Jeelani Begum. S [8] has studied some Total Dominating functions of Quadratic Residue Cayley Graphs.

Frutch and Harary [5] introduced a new product on two graphs G_1 and G_2 , called corona product denoted by $G_1 \odot G_2$. The object is to construct a new and simple operation on two graphs G_1 and G_2 called their corona, with the property that the group of the new graph is in general isomorphic with the wreath product of the groups of G_1 and G_2 . In this paper we study the concept of Total dominating functions of corona product graph of a cycle with a star and some results on minimal dominating functions are obtained.

II. CORONA PRODUCT OF $C_n \odot K_{1,m}$

The corona product of a cycle C_n with a star graph $K_{1,m}$ for m ≥ 2, is a graph obtained by taking one copy of a n-vertex graph C_n and n copies of $K_{1,m}$ and then joining the ith vertex of C_n to all vertices of ith copy of $K_{1,m}$ and it is denoted by $C_n \odot K_{1,m}$.

The degree of a vertex v_i in $G = C_n \odot K_{1,m}$ is given by

$$d(v_i) = \begin{cases} m+3, & if \;\; v_i \in C_n \,, \\ m+1, & if \;\; v_i \in K_{1,m} \; and \;\; v \; is \; in \; first \; partition, \\ 2, & if \;\; v_i \in K_{1,m} \; and \;\; v \; is \; in \; second \; partition. \end{cases}$$

Total Dominating Sets And Total Dominating Functions

The concept of Total Dominating functions and Minimal total dominating functions are introduced by Cockayne et al [4]. In this section we prove some results related to total domination functions of the graph $G = C_n \odot K_{1,m}$. First recall some definitions.

Definition: Let $G(V,E)$ be a graph without isolated vertices. A subset T of V is called a total dominating set (TDS) if every vertex in V is adjacent to at least one vertex in T. If no proper subset of T is a total dominating set, then T is called a minimal total dominating set (MTDS) of G.

Definition: The minimum cardinality of a MTDS of G is called a total domination number of G and is denoted by $\gamma_t(G)$.

III. Results And Figures

Theorem 3.1: The total domination number of $G = C_n \odot K_{1,m}$ is n .

Proof: Let T denote a total dominating set of G. Let T contain the vertices of the cycle C_n. By the definition of the graph G, every vertex in C_n is adjacent to all vertices of associated copy of $K_{1,m}$. That is the vertices in C_n dominate the vertices in all copies of $K_{1,m}$ respectively. Further these vertices being in C_n, they dominate among themselves. Thus T becomes a TDS of G. Obviously this set is minimum. Therefore $\gamma_t(G) = n$. ■

Theorem 3.2: Let T be a MTDS of $G = C_n \odot K_{1,m}$. Then a function $f : V \to [\,0,1\,]$ defined by

$$f(v) = \begin{cases} 1, & \text{if } v \in T, \\ 0, & \text{otherwise.} \end{cases}$$

becomes a MTDF of $G = C_n \odot K_{1,m}$.

Proof: Consider the graph $G = C_n \odot K_{1,m}$ with vertex set V.

Let T be a MTDS of G such that it contain all the vertices of C_n . The summation value taken over $N(v)$ of $v \in V$ is as follows:

Case 1: Let $v \in C_n$ be such that $d(v) = m + 3$ in G.

Then $N(v)$ contains $m + 1$ vertices of $K_{1,m}$ and two vertices of C_n in G.

So $\sum_{u \in N(v)} f(u) = 1 + 1 + \underbrace{0 + ... + 0}_{(m+1)-times} = 2.$

Case 2: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Then $N(v)$ contains m vertices of $K_{1,m}$ and one vertex of C_n in G.

So $\sum_{u \in N(v)} f(u) = 1 + \underbrace{0 + ... + 0}_{m-times} = 1.$

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G. Then $N(v)$ contains one vertex of $K_{1,m}$ and one vertex of C_n in G.

So $\sum_{u \in N(v)} f(u) = 1 + 0 = 1.$

Therefore for all possibilities, we get $\sum_{u \in N(v)} f(u) \geq 1, \quad \forall \;\; v \in V.$

This implies that f is a TDF.

Now we check for the minimality of f.

Define $g : V \to [\,0,1\,]$ by

$$g(v) = \begin{cases} r, & \text{for } v = v_k \in T, \\ 1, & \text{for } v \in T - \{v_k\}, \\ 0, & \text{otherwise.} \end{cases} \qquad \text{where } 0 < r < 1.$$

Since strict inequality holds at the vertex $v_k \in T$, it follows that g < f.

The summation value taken over $N(v)$ of $v \in V$ is as follows:

Case (i): Let $v \in C_n$ be such that $d(v) = m + 3$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $\sum_{u \in N(v)} g(u) = r + 1 + \underbrace{0 + ... + 0}_{(m+1)-times} = r + 1 > 1.$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = 1 + 1 + \underbrace{0 + ... + 0}_{(m+1)-times} = 2.$

Case (ii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $\sum_{u \in N(v)} g(u) = r + \underbrace{0 + ... + 0}_{m-times} = r < 1.$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = 1 + \underbrace{0 + ... + 0}_{m-times} = 1.$

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $\sum_{u \in N(v)} g(u) = r + 0 = r < 1.$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = 1 + 0 = 1.$

This implies that $\sum_{u \in N(v)} g(u) < 1$, for some $v \in V$.

So g is not a TDF.

Since g is taken arbitrarily, it follows that there exists no g < f such that g is a TDF.

Therefore f is a MTDF. ■

Theorem 3.3: A function $f: V \to [\,0, 1\,]$ defined by $f(v) = \frac{1}{q}, \ \forall \ v \in V$ is a TDF of $G = C_n \odot K_{1,m}$ if $q \leq 2$. It is a MTDF if q = 2.

Proof: Consider the graph $G = C_n \odot K_{1,m}$ with vertex set V.

Let f be a function defined as in the hypothesis.

Case I: Suppose q < 2.

The summation value taken over $N(v)$ of $v \in V$ is as follows:

Case 1: Let $v \in C_n$ be such that $d(v) = m + 3$ in G.

Then $N(v)$ contains $m + 1$ vertices of $K_{1,m}$ and two vertices of C_n in G.

So $\sum_{u \in N(v)} f(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + ... + \frac{1}{q}}_{(m+3)-times} = \frac{m+3}{q} > 1$, since $q < 2$ and $m \geq 2$.

Case 2: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Then $N(v)$ contains m vertices of $K_{1,m}$ whose degree is 2 and one vertex of C_n in G.

So $\sum_{u \in N(v)} f(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + ... + \frac{1}{q}}_{(m+1)-times} = \frac{m+1}{q} > 1$, since $q < 2$ and $m \geq 2$.

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $N(v)$ contains one vertex of $K_{1,m}$ whose degree is m+1 and one vertex of C_n in G.

So $\sum_{u \in N(v)} f(u) = \frac{1}{q} + \frac{1}{q} = \frac{2}{q} > 1$, since $q < 2$.

Therefore for all possibilities, we get $\sum_{u \in N(v)} f(u) > 1, \ \forall \ v \in V$.

This implies that f is a TDF.

Now we check for the minimality of f.

Define $g : V \to [0,1]$ by

$$g(v) = \begin{cases} r, & \text{for } v = v_k \in V, \\ \frac{1}{q}, & \text{otherwise.} \end{cases}, \qquad \text{where } 0 < r < \frac{1}{q}.$$

Since strict inequality holds at a vertex v_k of V, it follows that $g < f$.

The summation value taken over $N(v)$ of $v \in V$ is as follows:

Case (i): Let $v \in C_n$ be such that $d(v) = m + 3$ in G.

Sub case 1: Let $v_k \in N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = r + \underbrace{\frac{1}{q} + \frac{1}{q} + \dots\dots + \frac{1}{q}}_{(m+2)-times}$$

$$< \frac{1}{q} + \frac{m+2}{q} = \frac{m+3}{q} > 1, \text{ since q < 2 and m} \geq 2.$$

Sub case 2: Let $v_k \notin N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots\dots + \frac{1}{q}}_{(m+3)-times} = \frac{m+3}{q} > 1, \text{ since q < 2 and m} \geq 2.$$

Case (ii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Sub case 1: Let $v_k \in N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = r + \underbrace{\frac{1}{q} + \frac{1}{q} + \dots\dots + \frac{1}{q}}_{m-times}$$

$$< \frac{1}{q} + \frac{m}{q} = \frac{m+1}{q} > 1, \text{ since q < 2 and m} \geq 2.$$

Sub case 2: Let $v_k \notin N(v)$.

$$\text{So } \sum_{u \in N(v)} g(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots\dots + \frac{1}{q}}_{(m+1)-times} = \frac{m+1}{q} > 1, \text{ since q < 2 and m} \geq 2.$$

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Sub case 1: Let $v_k \in N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = r + \frac{1}{q} < \frac{2}{q}.$$

Since q < 2, it follows that $\frac{2}{q} > 1$.

Sub case 2: Let $v_k \notin N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = \frac{1}{q} + \frac{1}{q} = \frac{2}{q} > 1, \text{ since q < 2.}$$

Hence, it follows that $\sum_{u \in N(v)} g(u) > 1, \quad \forall \ v \in V.$

Thus g is a TDF.

This implies that f is not a MTDF.

Case II: Suppose q = 2.

Case 4: Let $v \in C_n$ be such that $d(v) = m + 3$ in G.

Then $N(v)$ contains $m + 1$ vertices of $K_{1,m}$ and two vertices of C_n in G.

So $$\sum_{u\in N(v)} f(u) = \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+3)-times} = \frac{m+3}{q} = \frac{m+3}{2} = 1+\frac{m+1}{2} > 1.$$

Case 5: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Then $N(v)$ contains m vertices of $K_{1,m}$ whose degree is 2 and one vertex of C_n in G.

So $$\sum_{u\in N(v)} f(u) = \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+1)-times} = \frac{m+1}{q} = \frac{m+1}{2} > 1, \text{ since } m \geq 2.$$

Case 6: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $N(v)$ contains one vertex of $K_{1,m}$ whose degree is m+1 and one vertex of C_n in G.

So $$\sum_{u\in N(v)} f(u) = \frac{1}{q}+\frac{1}{q} = \frac{2}{q} = \frac{2}{2} = 1.$$

Therefore for all possibilities, we get $\sum_{u\in N(v)} f(u) \geq 1, \ \forall \ v \in V.$

This implies that f is a TDF.

Now we check for the minimality of f.

Define $g : V \to [0,1]$ by

$$g(v) = \begin{cases} r, & \text{for } v = v_k \in V, \\ \frac{1}{q}, & \text{otherwise.} \end{cases}, \text{ where } 0 < r < \frac{1}{q}.$$

Since strict inequality holds at a vertex v_k of V, it follows that $g < f$.

Then as in Case (i), for $v \in C_n$ be such that $d(v) = m + 3$, we get

$$\sum_{u\in N(v)} g(u) = r + \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+2)-times} > 1, \text{ if } v_k \in N[v].$$

And $$\sum_{u\in N(v)} g(u) = \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+3)-times} = \frac{m+3}{q} = \frac{m+3}{2} = 1+\frac{m+1}{2} > 1, \text{ if } v_k \notin N[v].$$

Again we can see as in Case (ii) that for $v \in K_{1,m}$ be such that $d(v) = m + 1,$

$$\sum_{u\in N(v)} g(u) = r + \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{m-times} > 1, \text{ if } v_k \in N[v].$$

And $$\sum_{u\in N(v)} g(u) = \underbrace{\frac{1}{q}+\frac{1}{q}+.......+\frac{1}{q}}_{(m+1)-times} = \frac{m+1}{q} = \frac{m+1}{2} > 1, \text{ if } v_k \notin N[v].$$

Similarly we can show as in Case (iii) that $v \in K_{1,m}$ be such that $d(v) = 2,$

$$\sum_{u\in N(v)} g(u) = r + \frac{1}{q} < \frac{2}{q} = \frac{2}{2} = 1, \text{ if } v_k \in N[v],$$

and $\sum_{u\in N(v)} g(u)=\frac{1}{q}+\frac{1}{q}=\frac{2}{q}=\frac{2}{2}=1$, if $v_k \notin N[v]$.

This implies that $\sum_{u\in N(v)} g(u)<1$, for some $v \in V$.

So g is not a TDF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a TDF.

Thus f is a MTDF. ■

Theorem 3.4: A function $f: V \to [0, 1]$ defined by $f(v)=\frac{p}{q}, \forall v \in V$ where $p = min(m,n)$ and $q = max(m,n)$ is a TDF if $\frac{p}{q} \geq \frac{1}{2}$. Otherwise it is not a TDF. Also it becomes a MTDF if $\frac{p}{q} = \frac{1}{2}$.

Proof: Consider the graph $G = C_n \odot K_{1,m}$ with vertex set V.

Let $f: V \to [0, 1]$ be defined by $f(v)=\frac{p}{q}, \forall v \in V$, where $p = \min(m,n)$ and $q = max(m,n)$.

Clearly $\frac{p}{q} > 0$.

The summation value taken over $N(v)$ of $v \in V$ is as follows:

Case 1: Let $v \in C_n$ be such that $d(v) = m+3$ in G.

Then $N(v)$ contains $m+1$ vertices of $K_{1,m}$ and two vertices of C_n in G.

So $\sum_{u\in N(v)} f(u)=\underbrace{\frac{p}{q}+\frac{p}{q}+\dots\dots+\frac{p}{q}}_{(m+3)-times}=(m+3)\frac{p}{q}$.

Case 2: Let $v \in K_{1,m}$ be such that $d(v) = m+1$ in G.

Then $N(v)$ contains m vertices of $K_{1,m}$ whose degree is 2 and one vertex of C_n in G.

So $\sum_{u\in N(v)} f(u)=\underbrace{\frac{p}{q}+\frac{p}{q}+\dots\dots+\frac{p}{q}}_{(m+1)-times}=(m+1)\frac{p}{q}$.

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Then $N(v)$ contains one vertex of $K_{1,m}$ whose degree is $m+1$ and one vertex of C_n in G.

So $\sum_{u\in N(v)} f(u)=\frac{p}{q}+\frac{p}{q}=2\left(\frac{p}{q}\right)$.

From the above three cases, we observe that f is a TDF if $\frac{p}{q} \geq \frac{1}{2}$.

Otherwise f is not a TDF.

Case 4: Suppose $\frac{p}{q} > \frac{1}{2}$.

Clearly f is a TDF.

Now we check for the minimality of f.

Define $g: V \to [0,1]$ by

$$g(v) = \begin{cases} r, & \text{if } v = v_k \in V, \\ \frac{p}{q}, & \text{otherwise.} \end{cases}$$, where $0 < r < \frac{p}{q}$.

Since strict inequality holds at a vertex v_k of V, it follows that $g < f$.

The summation value taken over $N(v)$ of $v \in V$ is as follows:

Case (i): Let $v \in C_n$ be such that $d(v) = m + 3$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $\sum_{u \in N(v)} g(u) = r + \underbrace{\frac{p}{q} + \frac{p}{q} + ... + \frac{p}{q}}_{(m+2)-times}$

$$< \frac{p}{q} + (m+2)\frac{p}{q} = (m+3)\frac{p}{q} > 1, \text{ since } \frac{p}{q} > \frac{1}{2} \ and\ m \geq 2.$$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + ... + \frac{p}{q}}_{(m+3)-times} = (m+3)\frac{p}{q} > 1$, since $\frac{p}{q} > \frac{1}{2}\ and\ m \geq 2.$

Case (ii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $\sum_{u \in N(v)} g(u) = r + \underbrace{\frac{p}{q} + \frac{p}{q} + ... + \frac{p}{q}}_{m-times}$

$$< \frac{p}{q} + m\left(\frac{p}{q}\right) = (m+1)\frac{p}{q} > 1, \text{ since } \frac{p}{q} > \frac{1}{2} \ and\ m \geq 2.$$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + ... + \frac{p}{q}}_{(m+1)-times} = (m+1)\frac{p}{q} > 1$, since $\frac{p}{q} > \frac{1}{2}\ and\ m \geq 2.$

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G.

Sub case 1: Let $v_k \in N(v)$.

Then $\sum_{u \in N(v)} g(u) = r + \frac{p}{q} < 2\left(\frac{p}{q}\right)$.

Since $\frac{p}{q} > \frac{1}{2}\ and\ m \geq 2,\ it\ follows\ that\ 2\left(\frac{p}{q}\right) > 1.$

Sub case 2: Let $v_k \notin N(v)$.

Then $\sum_{u \in N(v)} g(u) = \frac{p}{q} + \frac{p}{q} = 2\left(\frac{p}{q}\right) > 1$, since $\frac{p}{q} > \frac{1}{2}$.

It follows that $\sum_{u \in N(v)} g(u) > 1, \quad \forall \ \ v \in V.$

Thus for all possibilities, we get that g is a TDF.

This implies that f is not a MTDF.

Case 5: Suppose $\frac{p}{q} = \frac{1}{2}$.

As in Case 1 and 2, we have that

$$\sum_{u\in N(v)} f(u) = \underbrace{\frac{p}{q}+\frac{p}{q}+\dots\dots+\frac{p}{q}}_{(m+3)-times} = (m+3)\left(\frac{p}{q}\right) > 1, \text{ since } m \geq 2 \text{ and } \frac{p}{q}=\frac{1}{2}.$$

And $\sum_{u\in N(v)} f(u) = \underbrace{\frac{p}{q}+\frac{p}{q}+\dots\dots+\frac{p}{q}}_{(m+1)-times} = (m+1)\left(\frac{p}{q}\right) > 1,$ since $m \geq 2$ and $\frac{p}{q}=\frac{1}{2}$.

Again as in Case 3 , we have that

And $\sum_{u\in N(v)} f(u) = \frac{p}{q}+\frac{p}{q} = 2\left(\frac{p}{q}\right) = 2\left(\frac{1}{2}\right) = 1.$

Therefore for all possibilities, we get $\sum_{u\in N(v)} f(u) \geq 1, \ \forall \ v \in V.$

This implies that f is a TDF.

Now we check for the minimality of f.

Define $g : V \to [0,1]$ by

$$g(v) = \begin{cases} r, & \text{if } v = v_k \in V, \\ \frac{p}{q}, & \text{otherwise.} \end{cases}$$

, where $0 < r < \frac{p}{q}$.

Since strict inequality holds at a vertex v_k of V, it follows that $g < f$.

Then we can show as in Case (i) and (ii) of Case 4 that

$\sum_{u\in N(v)} g(u) > 1,$ if $v \in C_n$ and $v_k \in N(v)$ or $v_k \notin N(v)$.

Again as in Case (ii) of Case 4, we can show that

$\sum_{u\in N(v)} g(u) > 1,$ if $v \in K_{1,m}$ and $v_k \in N(v)$ or $v_k \notin N(v)$.

As in Case (iii) of Case 4, we can show that

$\sum_{u\in N(v)} g(u) = r + \frac{p}{q} < 2\left(\frac{p}{q}\right) = 2\left(\frac{1}{2}\right) = 1,$ if $v \in K_{1,m}$ and $v_k \in N(v)$.

And $\sum_{u\in N(v)} g(u) = \frac{p}{q}+\frac{p}{q} = 2\left(\frac{p}{q}\right) = 2\left(\frac{1}{2}\right) = 1,$ if $v \in K_{1,m}$ and $v_k \notin N(v)$.

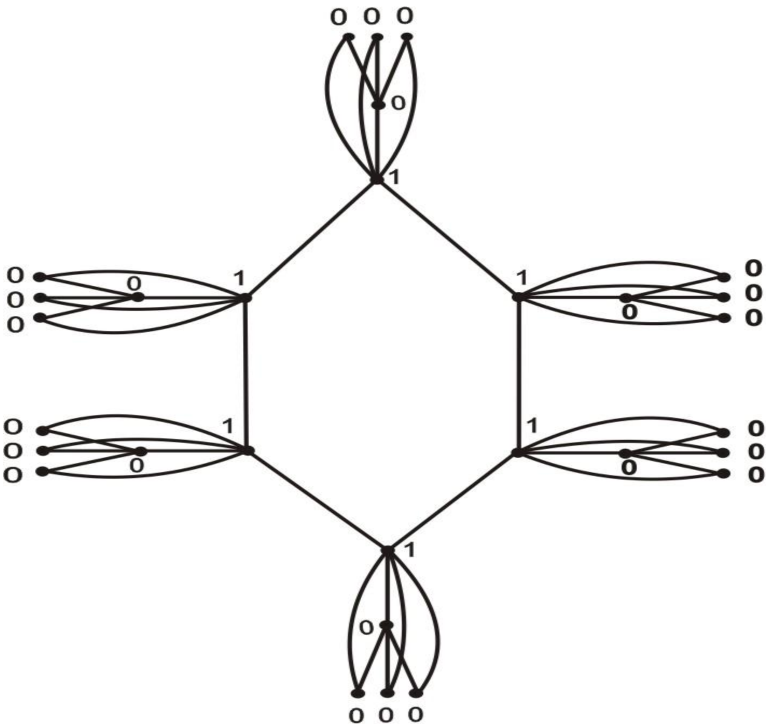
This implies that $\sum_{u\in N(v)} g(u) < 1$, for some $v \in V$.

So g is not a TDF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a TDF.

Thus f is a MTDF. ■

Figure 1: $G = C_6 \odot K_{1,3}$



The function f takes the value 1 for the vertices of C_6 and the value 0 for the vertices in each copy of $K_{1,3}$.

References

Books:

[1]. Allan R.B and Laskar, R.C – On domination, independent domination numbers of a graph, Discrete Math, 23(1978), 73-76.

[2]. Cockayne E.J, and Hedetniemi, S.T – Towards theory on domination in graphs, Networks, 7 (1977), 247-261.

[3]. Cockayne C.J, Dawes, R.M amd Hedentiemi, S.T – Total domination in graphs, Networks, 10(1980), 211-219.

[4]. Cockayne, E.J, Mynhardt, C.M and Yu, B-Total dominating functions in trees: Minimality and convexity, Journal of Graph Thoery, 19(1995), 83-92.

[5]. Frutch R and Harary F – on the corona of two graphs, Aequationes Mathematicae,Volume 4, Issue 3 (1970), 322-325.

[6]. [6]. Haynes, T.W, Hedetniemi, S.T and Slater P.J – Domination in Graphs: Advanced topics, Marcel Dekker, Inc, New York(1998).

[7]. Haynes, T.W, Hedentiemi, S.T and Slater P.J – Fundamentals of domination in graphs, Marcel Dekker, Inc, New York(1998).

Thesis:

[8]. Jeelani Begum.S – Some Studies on dominating functions of Quadratic Residue Cayley graphs, Ph.D Thesis, Sri Padmavathi Mahila Viswa Vidyalayam, Tirupati, Andhra Pradesh, India (2011).

[9]. Siva Parvati.M, Some Studies on dominating functions of corona product graphs, Ph.D Thesis, Sri Padmavathi Mahila Viswa Vidyalayam, Tirupati, Andhra Pradesh, India (2013).

M.S.Vimala Devi . "Minimal Total Dominating Functions of Corona Product Graph of A Cycle With A Star." IOSR Journal of Mathematics (IOSR-JM) , vol. 13, no. 6, 2017, pp. 32-40.