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Magnetized Anisotropic Bianchi Type-V Cosmological Model for Two Fluids in General Relativity

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Abstract: *In this paper, we have studied Anisotropic Bianchi type-V two fluid cosmological model i.e. with the matter and radiating source in presence of magnetic flux. In this model one of the fluid represents the matter contents of the universe and another fluid is the cosmic microwave background (CMB) radiation. The magnetic field is due to an electric current produced along x-axis, F_{23} is the only non-vanishing component of electromagnetic field tensor F_{ij}. Here we discuss the physical properties of model and behavior of kinematical parameters like expansion, shear are discussed in details with some special cases.*

Keywords*: Bianchi Type-V space-time, Cosmic microwave background (CMB) radiation, Electromagnetic field.*

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I. Introduction

Cosmology is the study of origin, current state and the future of the Universe. The Bianchi cosmologies which are spatially homogeneous and anisotropic play an important role in the theoretical cosmology and have been studied since 1960s. The simplest models of the expanding universe are those which are spatially homogeneous and isotropic at each instant of time. Bianchi type-V cosmological models have been studied by Farnsworth [1], Maartens et al. [2], Wainwright et al. [3], Collins [4], Coley et al. [5].

The occurrence of magnetic fields on galactic scale is well established fact today and it's important for a variety of astrophysical phenomenon is generally acknowledged by several authors, Harrison [6], Asseo [7] and Kim et al. [8] have pointed out the importance of magnetic field in the different context. As natural consequences, we should include magnetic fields in the energy momentum tensor of early universe. Kandalkar et al. [9] have investigated Bianchi type-V magnetized bulk viscous string cosmological model.

The cosmic microwave background (CMB) is one of the cornerstones of the homogeneous, isotropic model. Anisotropies in the CMB are related to small perturbation, superimposed on the perfectly smooth background, which are believed to seed formation of galaxies and large scale structure in the universe. Coley et al. [10] have studied Bianchi type-VI_0 model with two fluid sources. Pant et al. [11] has been investigated two fluid cosmological models using Bianchi type-II space-time. Oli [12] has presented anisotropic, homogeneous two fluid cosmological models in a Bianchi type-I space time with a variable gravitational constant and cosmological constant. Recently, Adhav et al. [13] examined two fluid cosmological models in Bianchi type-V space-time. Also, Pawar et al. [14] has obtained Bianchi type-IX with two fluid cosmological models. Katore et al. [15] has investigated magnetized Bianchi type-I cosmological model in scalar tensor theory of gravitation proposed by Saez and Ballestar in which they used the magnetic field produced due to electric current along x-axis. Recently, Patil et al. [16, 17] has studied non shearing LRS Bianchi type-III and IX string cosmological model in presence of magnetic flux with bulk viscosity. Patil et al. [18, 19] has studied LRS Bianchi type-V cosmological model in presence of perfect fluid and magnetic flux with variable magnetic permeability and Bianchi type-IX cosmological model with two fluids in presence of magnetic flux.

In this paper, we have investigated Bianchi type-V cosmological model in presence of two fluid distributions with electro-magnetic field. By assuming F_{23} is the only non-vanishing component of electromagnetic field tensor F_{ij}. The physical and geometrical aspects of the models are also discussed.

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II. Fundamental Equations And General Solutions

We have considered Bianchi type-V space-time,

$$ds^2 = dt^2 - A^2dx^2 - e^{2ax}(B^2dy^2 + C^2dz^2) \quad (2.1)$$

in which A, B and C are cosmic scale functions of time t.

The Einstein's field equations are

$$R_j^i - \frac{1}{2}Rg_j^i = -8\pi T_j^i \, . \quad (2.2)$$

The energy momentum tensor for two fluids source and electromagnetic field tensor is taken in the form,

$$T_j^i = T_j^{i^m} + T_j^{i^r} + E_j^i \quad (2.3)$$

in which,

$$T_j^{i^m} = (p_m + \rho_m)(u^i)^m (u_j)^m - p_m\, g_j^i \quad (2.4)$$

$$T_j^{i^r} = \frac{4}{3}\rho_r (u^i)^r (u_j)^r - \frac{1}{3}\rho_r\, g_j^i \quad (2.5)$$

the commoving coordinates system, for the line element (2.1) is,

$u_i{}^m = (0,0,0,1)$, $u_i{}^r = (0,0,0,1)$, $(u^i)^m = u_i{}^m g^{ij}$ and $(u^i)^r = u_i{}^r g^{ij}$

The electromagnetic field E_j^i is defined as,

$$E_j^i = \frac{1}{4\pi}\left[-F_{jl}F^{il} + \frac{1}{4}g_j^i F_{lm}F^{lm}\right] \quad (2.6)$$

with the Maxwell's equation

$$\frac{\partial}{\partial x^i}(F^{ij}\sqrt{-g}) = 0$$

which leads to, $F_{23} = I$

From equation (2.6), we have

$$\Rightarrow E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{I^2}{8\pi B^2C^2}$$

Using equations (2.4), (2.5), (2.6) in equation (2.2) we have,

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} - \frac{3a^2}{A^2} = 8\pi(\rho_m + \rho_r) + \frac{I^2}{C^2B^2} \quad (2.7)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} - \frac{a^2}{A^2} = -8\pi\left(p_m + \frac{1}{3}\rho_r\right) + \frac{I^2}{C^2B^2} \quad (2.8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} - \frac{a^2}{A^2} = -8\pi\left(p_m + \frac{1}{3}\rho_r\right) - \frac{I^2}{C^2B^2} \quad (2.9)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{a^2}{A^2} = -8\pi\left(p_m + \frac{1}{3}\rho_r\right) - \frac{I^2}{C^2B^2} \quad (2.10)$$

$$\frac{2\dot{A}}{A} - \frac{\dot{C}}{C} - \frac{\dot{B}}{B} = 0 \quad \text{i.e.} \quad \frac{\dot{A}}{A} = \frac{1}{2}\left(\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\right) \quad (2.11)$$

On solving equation (2.11), we obtain,

$$A^2 = (BC) \quad (2.12)$$

Here, we consider constant of integration as zero.

Differentiating equation (2.12) twice, we have

$$\frac{\ddot{A}}{A} = \frac{1}{2}\frac{\ddot{B}}{B} + \frac{1}{2}\frac{\dot{B}\dot{C}}{BC} + \frac{1}{2}\frac{\ddot{C}}{C} - \frac{1}{4}\left(\frac{\dot{C}}{C}\right)^2 - \frac{1}{4}\left(\frac{\dot{B}}{B}\right)^2 \quad (2.13)$$

To obtain more general solution, we assume the metric potential relation,

$$B = C^n \quad (2.14)$$

Differentiating above equation, we have

$$\frac{\dot{B}}{B} = n\frac{\dot{C}}{C} \quad \text{and} \quad \frac{\ddot{B}}{B} = n(n-1)\frac{\dot{C}^2}{C^2} + n\frac{\ddot{C}}{C} \quad (2.15)$$

Subtracting equation (2.10) from equation (2.8) and using equations (2.12), (2.13), (2.14), (2.15) we have,

$$2\ddot{C} + 2\frac{\dot{C}^2}{C}\frac{(3n^2-2n-1)}{2(n-1)} = -\frac{8I^2}{(n-1)C^{2n+1}}$$

Let $\dot{C} = f$ and $\ddot{C} = \frac{d\dot{C}}{dt} = \frac{df}{dt}$

$\ddot{C} = \frac{df}{dC}\frac{dC}{dt} = f'f$, Let $f' = \frac{df}{dC}$

Using these values in above equation and further solving, it implies

$$\frac{dC}{dt} = \frac{I\,2\sqrt{2}\,C^{-n}}{\sqrt{(1-n^2)}} \quad (2.16)$$

$$\Rightarrow dt = \frac{\sqrt{(1-n^2)}}{I\,2\sqrt{2}\,C^{-n}}\,dC$$

$$\Rightarrow dt^2 = \frac{(1-n^2)}{I^2\,8\,C^{-2n}}\,dC^2 \quad (2.17)$$

Using equation (2.16), we have

$$\frac{\dot{C}}{C} = \frac{I\,2\sqrt{2}\,C^{-n-1}}{\sqrt{(1-n^2)}} \quad (2.18)$$

Using equation (2.12), (2.14), (2.17) in equation (2.1), we have

$$ds^2 = \frac{(1-n^2)dC^2}{8I^2C^{-2n}} - C^{n+1}dx^2 - e^{2ax}(C^{2n}dy^2 + C^2dz^2) \quad (2.19)$$

Transforms, $C = T,\ x = X,\ y = Y,\ z = Z$ in above equation, we have

$$ds^2 = \frac{(1-n^2)dT^2}{8T^{-2n}I^2} - T^{n+1}dX^2 - e^{2aX}(T^{2n}dY^2 + T^2dZ^2) \quad (2.20)$$

III. Physical And Geometrical Properties

Using equations (2.12), (2.14), (2.15) in equation (2.7), and $\rho = \rho_m + \rho_r$.

$$\Rightarrow \rho = \frac{1}{8\pi}\left(4\frac{I^2N}{(1-n^2)C^{2+2n}} - \frac{3a^2}{C^{n+1}} - \frac{I^2}{C^{2n+2}}\right) \quad (3.1)$$

where $(n^2 + 4n + 1) = N$.

We have the equation of state in the form $p_m = (\gamma - 1)\rho_m$ (3.2)

Total energy density is, $\rho = \rho_m + \rho_r$ (3.3)

From equation (3.2) and (3.3) we obtain the relation,

$$\left(p_m + \frac{1}{3}\rho_r\right) = (\gamma - 1)\rho + \rho_r\left(\frac{4}{3} - \gamma\right) \quad (3.4)$$

Using equation (2.12), (2.13), (2.14), (2.15), (3.3) and (3.4) in equation (2.10), we obtain

$$\rho_r = \frac{1}{-8\pi\left(\frac{4}{3}-\gamma\right)}\left(\frac{4I^2(2\beta+(\gamma-1)N)}{(1-n^2)C^{2n+2}} - \frac{2I^2(n+3)}{(n-1)C^{2n+2}} - \frac{a^2(1+3(\gamma-1))}{C^{n+1}} - \frac{I^2(2-\gamma)}{C^{2n+2}}\right) \quad (3.5)$$

where, $\beta = \frac{(-2n^3-6n^2+6n+2)}{4(n-1)}$

Since, $\rho_m = \rho - \rho_r$

$$\rho_m = \frac{1}{8\pi}\left[\frac{4I^2N}{(1-n^2)C^{2+2n}} - \frac{6a^2}{(4-3\gamma)C^{n+1}} + \frac{2I^2}{(4-3\gamma)C^{2n+2}} + \frac{12I^2(2\beta+(\gamma-1)N)}{(4-3\gamma)(1-n^2)C^{2n+2}} - \frac{6I^2(n+3)}{(4-3\gamma)(n-1)C^{2n+2}}\right] \quad (3.6)$$

From equation (3.2), we have

$$p_m = (\gamma - 1)\rho_m$$

$$p_m = \frac{(\gamma-1)}{8\pi}\left[\frac{4I^2N}{(1-n^2)C^{2+2n}} - \frac{6a^2}{(4-3\gamma)C^{n+1}} + \frac{2I^2}{(4-3\gamma)C^{2n+2}} + \frac{12I^2(2\beta+(\gamma-1)N)}{(4-3\gamma)(1-n^2)C^{2n+2}} - \frac{6I^2(n+3)}{(4-3\gamma)(n-1)C^{2n+2}}\right] \quad (3.7)$$

And, the scalar of expansion, shear scalar are obtained as,

$$\theta = u^i_{;i} = u^i_{,i} + u^k\,\Gamma^i_{ki}$$

$$\theta = \left[\frac{3(n+1)I\sqrt{2}}{C^{n+1}\sqrt{(1-n^2)}}\right] \quad (3.8)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$$

where $\sigma_{ij} = \frac{1}{2}(u_{i;j} - u_{j;i}) + \frac{1}{2}(u_{i,j}u^k u_j - u_i u_{j;k}u^k)$

$$\sigma^2 = \frac{3(n+1)I^2}{(1-n)C^{2n+2}} \quad (3.9)$$

IV. Some Special Cases

Case I : For dust model, $\gamma = 1, n \neq 1$.

From equations (3.5) and (3.6) yields,

$$\rho_r = \frac{-3}{8\pi}\left(\frac{8I^2\beta}{(1-n^2)C^{2n+2}} - \frac{2I^2(n+3)}{(n-1)C^{2n+2}} - \frac{a^2}{C^{n+1}} + \frac{I^2}{C^{2n+2}}\right)$$

$$\rho_m = \frac{1}{8\pi}\left(\frac{4I^2N}{(1-n^2)C^{2+2n}} - \frac{6a^2}{C^{n+1}} + \frac{2I^2}{C^{2n+2}} + \frac{24\beta\, I^2}{(1-n^2)C^{2n+2}} - \frac{6I^2(n+3)}{(n-1)C^{2n+2}}\right)$$

$$\Omega_r = \frac{\frac{-3}{8\pi}\left(\frac{8I^2\beta}{(1-n^2)C^{2n+2}} - \frac{2I^2(n+3)}{(n-1)C^{2n+2}} - \frac{a^2}{C^{n+1}} + \frac{I^2}{C^{2n+2}}\right)}{\frac{1}{3}\left[\frac{18(n+1)I^2}{C^{2n+2}(1-n)}\right]}$$

$$\Omega_m = \frac{\frac{1}{8\pi}\left(\frac{4I^2N}{(1-n^2)C^{2+2n}} - \frac{6a^2}{C^{n+1}} + \frac{2I^2}{C^{2n+2}} + \frac{24\beta\, I^2}{(1-n^2)C^{2n+2}} - \frac{6I^2(n+3)}{(n-1)C^{2n+2}}\right)}{\frac{1}{3}\left[\frac{18(n+1)I^2}{C^{2n+2}(1-n)}\right]}$$

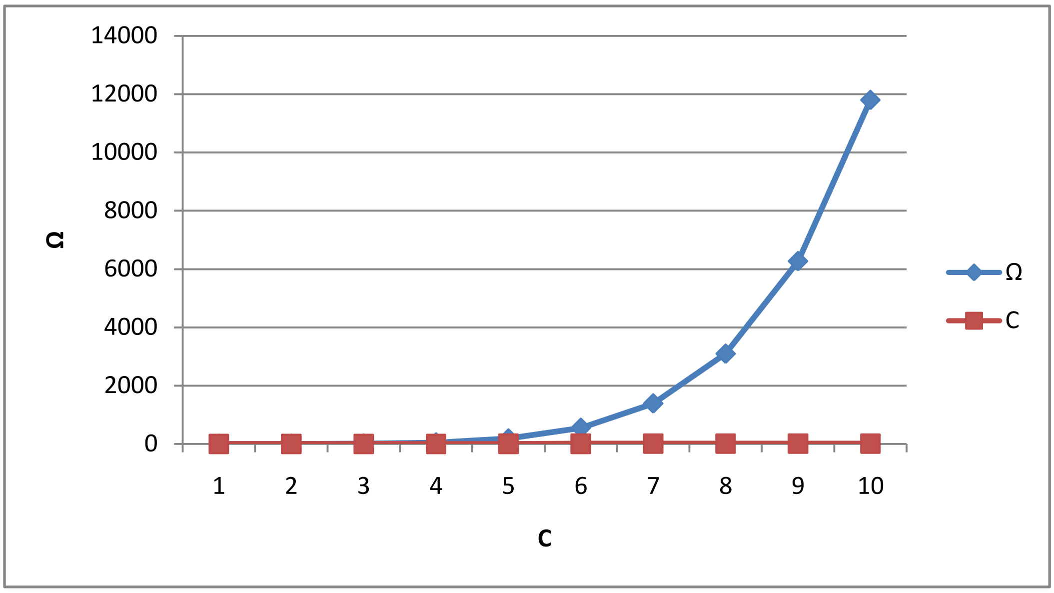
$$\Omega = \Omega_m + \Omega_r$$

$$\Omega = \frac{C^{2n+2}(1-n)}{48\pi(n+1)I^2}\left(\frac{4I^2N}{(1-n^2)C^{2+2n}} - \frac{3a^2}{C^{n+1}} - \frac{I^2}{C^{2n+2}}\right)$$

Graphical representation:

$$\Omega = \frac{C^{2n+2}(1-n)}{48\pi(n+1)I^2}\left(\frac{4I^2N}{(1-n^2)C^{2+2n}} - \frac{3a^2}{C^{n+1}} - \frac{I^2}{C^{2n+2}}\right)$$

For, $n = 0.5\,, N = 1, a = 1, I = 1$



Case II: For radiating universe, $\gamma = \frac{4}{3}$, $n \neq 1$.

From equations (3.5) and (3.6) yields,

$\rho_m = \rho_r = \infty\,, \Omega_m = \Omega_r = \infty\,, \Rightarrow \Omega = \infty$.

Case III: For hard universe, $\gamma \in \left(\frac{4}{3}\,,\ 2\right)$, i.e. $\gamma = \frac{5}{3}$, $n \neq 1$.

From equations (3.5) and (3.6) we have obtain,

$$\rho_r = \frac{3}{8\pi}\left(\frac{4I^2\left(2\beta+\frac{2}{3}N\right)}{(1-n^2)C^{2n+2}} - \frac{2I^2(n+3)}{(n-1)C^{2n+2}} - \frac{3a^2}{C^{n+1}} - \frac{I^2}{3C^{2n+2}}\right)$$

$$\rho_m = \frac{1}{8\pi}\left[\frac{4I^2N}{(1-n^2)C^{2+2n}} + \frac{6a^2}{C^{n+1}} - \frac{2I^2}{C^{2n+2}} - \frac{12I^2\left(2\beta+\frac{2}{3}N\right)}{(1-n^2)C^{2n+2}} + \frac{6I^2(n+3)}{(n-1)C^{2n+2}}\right]$$

$$\Omega_r = \frac{\frac{3}{8\pi}\left(\frac{4I^2\left(2\beta+\frac{2}{3}N\right)}{(1-n^2)C^{2n+2}} - \frac{2I^2(n+3)}{(n-1)C^{2n+2}} - \frac{3a^2}{C^{n+1}} - \frac{I^2}{3C^{2n+2}}\right)}{\frac{1}{3}\left[\frac{18(n+1)I^2}{C^{2n+2}(1-n)}\right]}$$

$$\Omega_m = \frac{\frac{1}{8\pi}\left(\frac{4I^2N}{(1-n^2)C^{2+2n}} + \frac{6a^2}{C^{n+1}} - \frac{2I^2}{C^{2n+2}} - \frac{12I^2\left(2\beta+\frac{2}{3}N\right)}{(1-n^2)C^{2n+2}} + \frac{6I^2(n+3)}{(n-1)C^{2n+2}}\right)}{\frac{1}{3}\left[\frac{18(n+1)I^2}{C^{2n+2}(1-n)}\right]}$$

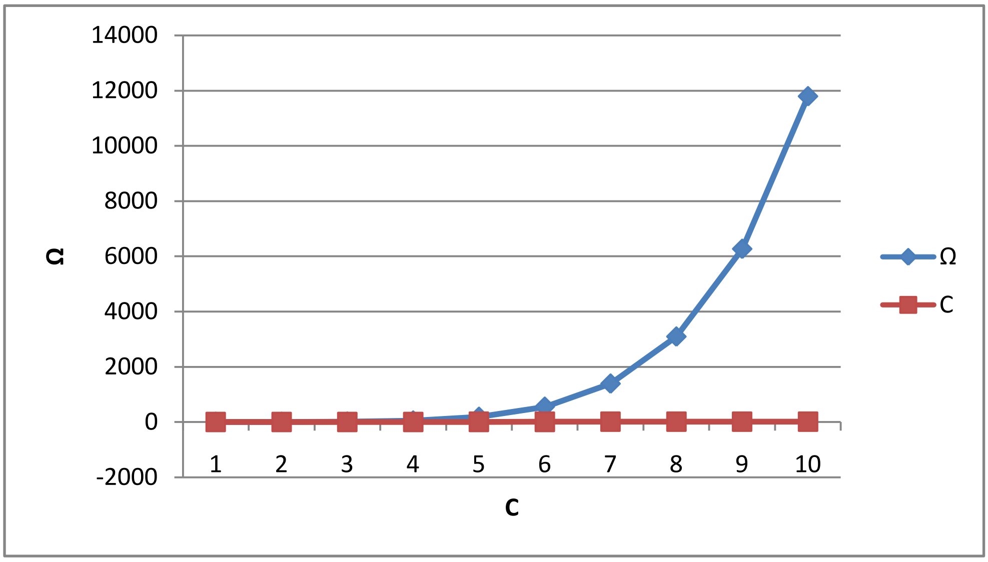
$$\Omega = \Omega_m + \Omega_r$$

$$\Omega = \frac{C^{2n+2}(1-n)}{48\pi(n+1)I^2}\left[\frac{I^2 4N}{(1-n^2)C^{2+2n}} - \frac{3a^2}{C^{n+1}} - \frac{3I^2}{C^{2n+2}}\right]$$

Graphical representation:

$\Omega = \frac{C^{2n+2}(1-n)}{48\pi(n+1)I^2}\left[\frac{I^2 4N}{(1-n^2)C^{2+2n}} - \frac{3a^2}{C^{n+1}} - \frac{3I^2}{C^{2n+2}}\right]$

For, $n = 0.5\,, N = 1, a = 1, I = 1$



Case IV: For Zeldovich universe, $\boldsymbol{\gamma = 2,\ n \neq 1}$.

From equations (3.5) and (3.6) we have obtain,

$\rho_r = \frac{3}{16\pi}\left(\frac{4I^2(2\beta+N)}{(1-n^2)C^{2n+2}} - \frac{2I^2(n+3)}{(n-1)C^{2n+2}} - \frac{4a^2}{C^{n+1}}\right)$

$\rho_m = \frac{1}{8\pi}\left[\frac{4I^2 N}{(1-n^2)C^{2+2n}} + \frac{3a^2}{C^{n+1}} - \frac{I^2}{C^{2n+2}} - \frac{6I^2(2\beta+N)}{(1-n^2)C^{2n+2}} + \frac{3I^2(n+3)}{(n-1)C^{2n+2}}\right]$

$\Omega_r = \frac{\frac{3}{16\pi}\left(\frac{4I^2(2\beta+N)}{(1-n^2)C^{2n+2}} - \frac{2I^2(n+3)}{(n-1)C^{2n+2}} - \frac{4a^2}{C^{n+1}}\right)}{\frac{1}{3}\left[\frac{18(n+1)I^2}{C^{2n+2}(1-n)}\right]}$

$\Omega_m = \frac{\frac{1}{8\pi}\left(\frac{4I^2 N}{(1-n^2)C^{2+2n}} + \frac{3a^2}{C^{n+1}} - \frac{I^2}{C^{2n+2}} - \frac{6I^2(2\beta+N)}{(1-n^2)C^{2n+2}} + \frac{3I^2(n+3)}{(n-1)C^{2n+2}}\right)}{\frac{1}{3}\left[\frac{18(n+1)I^2}{C^{2n+2}(1-n)}\right]}$

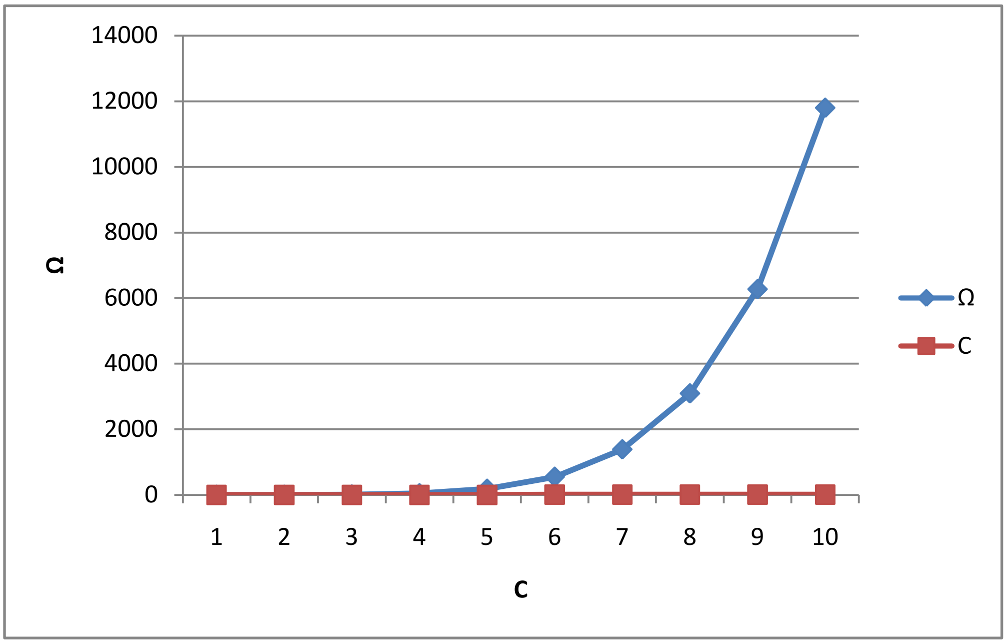
$\Omega = \Omega_m + \Omega_r$

$\Omega = \frac{C^{2n+2}(1-n)}{48\pi(n+1)I^2}\left(\frac{4I^2 N}{(1-n^2)C^{2+2n}} - \frac{3a^2}{C^{n+1}} - \frac{I^2}{C^{2n+2}}\right)$

Graphical representation:

$$\Omega = \frac{C^{2n+2}(1-n)}{48\pi(n+1)I^2}\left(\frac{4I^2N}{(1-n^2)C^{2+2n}} - \frac{3a^2}{C^{n+1}} - \frac{I^2}{C^{2n+2}}\right)$$

For, $n = 0.5\,, N = 1, a = 1, I = 1$



V. Conclusion

From equation (2.20), it is observed that, as T increases the universe expands, and for $T = 0$, it shrinks i.e. in the presence of electromagnetic field, the universe is expanding and absence of electromagnetic field it shrinks. It is clear from cases I, III and IV for $T = 0$, the density parameter Ω tends to infinity. While in case II, for $\gamma = \frac{4}{3}$, all the density parameters ρ_m, ρ_r, Ω_m, Ω_r and Ω are tends to infinity. Also, from graph for $\gamma = 1$, $\frac{5}{3}$, 2 it is observed that, the density parameter gradually increases as time increases. In all above cases, we have the ratio $\left(\sigma/_{\theta}\right)^2 \neq 0$, therefore these universe does not approach to isotropy for large value of T.

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