IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 6 Ver. I (Nov. - Dec. 2017), PP 56-63
www.iosrjournals.org

Zero Average Method to Finding an Optimal Solution of Fuzzy Transportation Problems

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Abstract: *In this paper, a proposed method, namely, Zero Average Method (ZAM) is used for solving fuzzy transportation problems by assuming that a decision maker is uncertain about the precise values of the transportation cost only but there is no uncertainty about the demand and supply of the product. In this proposed method transportation cost are represented by generalized fuzzy numbers. To illustrate the proposed method a numerical example is solved and the obtained results are comparing with the results of existing methods. The proposed method is very easy to understand and apply on real life transportation problems for the decision makers.*
Mathematics Subject Classification: 90C08, 90C90, 90C70, 90B06, 90C29.
Keywords: *Fuzzy Transportation Problem (FTP); Generalized Trapezoidal Fuzzy Number (GTrFN); Ranking function; Zero Average Method (ZAM).*

Date of Submission: 07-11-2017 Date of acceptance: 21-11-2017

I. Introduction

Transportation problem is an important network structured in linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in the problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling, and many others. In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way i.e., in crisp environment. However, in many cases the decision makers has no crisp information about the coefficients belonging to the transportation problem. In these cases, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and the fuzzy transportation problem (FTP) appears in natural way.

The basic transportation problem was originally developed by Hitchcock [18].The transportation problems can be modeled as a standard linear programming problem, which can then be solved by the simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to the transportation problem can be made quite efficient in terms of how to evaluate the necessary simplex method information (Variable to enter the basis, variable to leave the basis and optimality conditions). Charnes and Cooper [4] developed a stepping stone method which provides an alternative way of determining the simplex method information. Dantzig and Thapa [8] used simplex method to the transportation problem as the primal simplex transportation method. An Initial Basic Feasible Solution (IBFS) for the transportation problem can be obtained by using the North- West Corner rule, Row Minima Method, Column Minima Method, Matrix Minima Method or Vogel's Approximation Method (VAM) [27]. The Modified Distribution Method (MODI) [5] is useful for finding the optimal solution of the transportation problem. In general, the transportation problems are solved with the assumptions that the coefficients or cost parameters are specified in a precise way i.e., in crisp environment.

In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy number [30] may represent the data. Hence fuzzy decision making method is used here.

Zimmermann [31] showed that solutions obtained by fuzzy linear programming method and are always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Chanas et al. [2] presented a fuzzy linear programming model for solving transportation problems with crisp cost coefficient, fuzzy supply and demand values. Chanas and Kuchta [3] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution.

Saad and Abbas [28] discussed the solution algorithm for solving the transportation problem in fuzzy environment.

Liu and Kao [22] described a method for solving fuzzy transportation problem based on extension principle. Gani and Razak [17] presented a two stage cost minimizing fuzzy transportation problem (FTP) in which supplies and demands are trapezoidal fuzzy numbers. A parametric approach is used to obtain a fuzzy solution and the aim is to minimize the sum of the transportation costs in two stages. Lin [20] introduced a genetic algorithm to solve transportation problem with fuzzy objective functions. Dinagar and Palanivel [9] investigated FTP, with the aid of trapezoidal fuzzy numbers. Fuzzy modified distribution method is proposed to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [26] proposed a new algorithm namely, fuzzy zero point method for finding a fuzzy optimal solution for a FTP, where the transportation cost, demand and supply are represented by trapezoidal fuzzy numbers. Ganesan [25] proposed a new approach for solving FTP, where the transportation cost, demand and supply are represented by trapezoidal fuzzy numbers. Vimala [29] proposed a new algorithm namely MAM'S for solving FTP, where the transportation cost, demand and supply are represented by trapezoidal fuzzy numbers. Maliniand [24] solving fuzzy transportation problems in which transportation cost, demand and supply quantities are trapezoidal fuzzy number.

Edward Samuel [13] proposed algorithmic approach to unbalanced fuzzy transportation problems, where the transportation cost, demand and supply are represented by triangular fuzzy number. Edward Samuel [14-16] showed the unbalanced fuzzy transportation problems without converting into balanced one getting an optimal solution, where the transportation cost, demand and supply are represented by triangular fuzzy number. Edward Samuel [10-12] a solving generalized trapezoidal fuzzy transportation problems, where precise values of the transportation costs only, but there is no uncertain about the demand and supply.

In this paper, a proposed method, namely, Zero Average Method (ZAM) is used for solving a special type of fuzzy transportation problem by assuming that a decision maker is uncertain about the precise values of transportation costs only . In the proposed method transportation costs are represented by generalized trapezoidal fuzzy numbers. To illustrate the proposed method ZAM a numerical example is solved. The proposed method ZAM is easy to understand and to apply in real life transportation problems for the decision makers.

II. Preliminaries

In this section, some basic definitions, arithmetic operations and an existing method for comparing generalized fuzzy numbers are presented.

2.1. Definition [19]

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers $\Re$ is said to be fuzzy number if it is membership function has the following characteristics:

(i) $\mu_{\tilde{A}}(x) : \Re \rightarrow [0,1]$ is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.

(iii) $\mu_{\tilde{A}}(x)$ Strictly increasing on [a, b] and strictly decreasing on [c, d]

(iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c], where\ \ a < b < c < d.$

2.2. Definition [19]

A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)}, & a \le x < b, \\ 1, & b \le x \le c, \\ \dfrac{(x-d)}{(c-d)}, & c < x \le d, \\ o, & otherwise. \end{cases}$$

2.3. Definition [6]

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers $\Re$, is said to be generalized fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}(x) : \Re \rightarrow [0, \omega]$ is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.

(iii) $\mu_{\tilde{A}}(x)$ Strictly increasing on [a, b] and strictly decreasing on [c, d]

(iv) $\mu_{\tilde{A}}(x) = \omega$, for all $x \in [b, c]$, *where* $0 < \omega \leq 1$.

2.4. Definition [6]

A fuzzy number $\tilde{A} = (a, b, c, d; \omega)$ is said to be generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \omega \dfrac{(x-a)}{(b-a)}, & a \leq x < b, \\ \omega \quad , & b \leq x \leq c, \\ \omega \dfrac{(x-d)}{(c-d)}, & c < x \leq d, \\ o \quad , & otherwise. \end{cases}$$

2.5. Arithmetic operations:

In this section, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on the universal set of real numbers $\Re$, are presented [1,6,7].

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$ are two generalized trapezoidal fuzzy numbers, then the following is obtained.

(i) $\tilde{A}_1 + \tilde{A}_2 = (a_1 + a_2,\ b_1 + b_2,\ c_1 + c_2,\ d_1 + d_2;\ \min(\omega_1, \omega_2)),$

(ii) $\tilde{A}_1 - \tilde{A}_2 = (a_1 - d_2,\ b_1 - c_2,\ c_1 - b_2,\ d_1 - a_2;\ \min(\omega_1, \omega_2)),$

(iii)
$$\begin{aligned} &\tilde{A}_1 \otimes \tilde{A}_2 \cong (a, b, c, d; \min(\omega_1, \omega_2)), \\ &where \\ &a = \min(a_1a_2, a_1d_2, a_2d_1, d_1d_2),\ \ b = \min(b_1b_2, b_1c_2, c_1b_2, c_1c_2), \\ &c = \max(b_1b_2, b_1c_2, c_1b_2, c_1c_2), \quad d = \max(a_1a_2, a_1d_2, a_2d_1, d_1d_2). \end{aligned}$$

(iv)
$$\lambda \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1;\ \omega_1),\ \ \lambda > 0, \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1;\ \omega_1),\ \ \lambda < 0. \end{cases}$$

2.6. Ranking function

An efficient approach for comparing the fuzzy numbers is by the use of ranking function[7,21,23], $\Re : F(\Re) \rightarrow \Re$, where F($\Re$) is a set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line, where a natural order exists, i.e.,

(i) $\tilde{A} >_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) > \Re(\tilde{B})$

(ii) $\tilde{A} <_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) < \Re(\tilde{B})$

(iii) $\tilde{A} =_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ *and* $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$ *be two generalized trapezoidal fuzzy numbers and* $\omega = \min(\omega_1, \omega_2)$*. Then* $\Re(\tilde{A}_1) = \frac{\omega(a_1 + b_1 + c_1 + d_1)}{4}$ *and* $\Re(\tilde{A}_2) = \frac{\omega(a_2 + b_2 + c_2 + d_2)}{4}$.

III. Proposed Method

In this section, a proposed method namely, Zero Average Method (ZAM) for finding an optimal solution in which transportation costs as generalized trapezoidal fuzzy numbers.

Step 1. (a). Ensure whether the given fuzzy transportation problem (FTP) is balanced or not, if not go to step 2 otherwise go to step 3
Step 2. If the given FTP is unbalanced, then any one of the following two cases may arise.
If the total demand exceeds total supply, then go to case1 if else go to case 2
Case 1.
(a). Locate the smallest fuzzy cost in each row of the given cost table and then subtract that from each fuzzy cost of that row.
(b). Convert in to balanced one and then replace the dummy fuzzy cost of the largest unit fuzzy transportation cost in the reduced matrix obtain from case 1(a).
(c). In the reduced matrix obtained from case 1(b), locate the smallest fuzzy cost in each column and then subtract that from each fuzzy cost of that column and then go to step 4.
Case 2.
(a). Locate the smallest fuzzy cost in each column of the given cost table and then subtract that from each fuzzy cost of that column.
(b). Convert in to balanced one and then replace the dummy fuzzy cost of the largest unit fuzzy transportation cost in the reduced matrix obtain from case 2(a).
(c). In the reduced matrix obtained from case 2(b), locate the smallest fuzzy cost in each row and then subtract that from each fuzzy cost of that row and then go to step 4.
Step 3. (a). Locate the smallest fuzzy cost in each row of the given cost table and then subtract that from each fuzzy cost of that row, and
(b). In the reduced matrix obtained from 3(a), locate the smallest fuzzy cost in each column and then subtract that from each fuzzy cost of that column.
Step 4. Select the smallest fuzzy costs (not equal to zero) in the reduced matrix obtained from step 2 or step 3 and then subtract it by selected smallest fuzzy cost only.
Step 5. (a). Select the fuzzy zero (row wise) and count the number of fuzzy zeros (excluding selected one) in row and column and record as a subscript of selected zero. Repeat the process for all zeros in the matrix.
(b). Now, choose the value of subscript is minimum and allocate maximum possible to that cell. If tie occurs for some fuzzy zero values, then find the average of demand and supply value and then choose one with minimum.
Step 6. Adjust the supply and demand and cross out the satisfied the row or column.
Step 7. Check whether the resultant matrix possesses at least one fuzzy zero in each and column. If not repeat step 3, otherwise go to step 8.
Step 8. Repeat step 3 and step 5 to step 7 until all the demand and supply are exhausted.
Step 9. Compute total fuzzy transportation cost for the feasible allocation from the original fuzzy cost table.

Important Remarks
If there is a tie in the values of average of demand and supply, then calculate their corresponding row and column value and select one with minimum.

IV. Numerical Example.

To illustrate the proposed method, namely, Zero Average Method (ZAM) the following Fuzzy Transportation Problem is solved

4.1. Problem 1

A company manufactures motor cars and it has three factories S_1, S_2 and S_3 whose weekly production capacities are 13, 20 and 5 pieces of cars respectively. The company supplies motor cars to its three showrooms located at D_1, D_2 and D_3 whose weekly demands are 12, 15 and 11 pieces of cars respectively. The transportation costs per piece of motor cars are given in the transportation Table 1. Find out the schedule of shifting of motor cars from factories to showrooms with minimum cost.

Table 1

	D_1	D_2	D_3	Supply (a_i)
S_1	(11,13,14,18;.5)	(20,21,24,27;.7)	(14,15,16,17;.4)	13
S_2	(6,7,8,11;.2)	(9,11,12,13;.2)	(20,21,24,27;.7)	20
S_3	(14,15,17,18;.4)	(15,16,18,19;.5)	(10,11,12,13;.6)	5
Demand (b_j)	12	15	11	

Since, $\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j = 38$, so the chosen problem is a balanced FTP.

Iteration 1. Using step3 and step4, we get

	D_1	D_2	D_3	Supply (a_i)
S_1	**(-14,-2,2,14;.5)**	(-5,2,8,18;.2)	**(-16,-4,4,16;.4)**	13
S_2	**(-12,-2,2,12;.2)**	**(-9,-2,2,9;.2)**	(6,12,18,24;.2)	20
S_3	(-6,2,7,15;.4)	(-5,-1,4,11;.2)	**(-6,-2,2,6;.6)**	5
Demand (b_j)	12	15	11	

Iteration 2. Using step5 and step6, we get

	D_1	D_2	D_4	Supply (a_i)
S_1	**(-14,-2,2,14;.5)**	(-5,2,8,18;.2)	**(-16,-4,4,16;.4)**	13
S_2	**(-12,-2,2,12;.2)**	**(-9,-2,2,9;.2)**	(6,12,18,24;.2)	20
S_3	*	*	**(-6,-2,2,6;.6)** **5**	*
Demand (b_j)	12	15	6	

Iteration 3. Using step7 and step 8, we get

	D_1	D_2	D_4	Supply (a_i)
S_1	(-14,-2,2,14;.5) **7**	(-5,2,8,18;.2)	(-16,-4,4,16;.4) **6**	*
S_2	(-12,-2,2,12;.2) **5**	(-9,-2,2,9;.2) **15**	(6,12,18,24;.2	*
S_3	(-6,2,7,15;.4)	(-5,-1,4,11;.2)	(-6,-2,2,6;.6) **5**	*
Demand (b_j)	*	*	*	

Iteration 4. Using step9, we get

	D_1	D_2	D_3	Supply (a_i)
S_1	(11,13,14,18;.5) **7**	(20,21,24,27;.7)	(14,15,16,17;.4) **6**	*
S_2	(6,7,8,11;.2) **5**	(9,11,12,13;.2) **15**	(20,21,24,27;.7)	*
S_3	(14,15,17,18;.4)	(15,16,18,19;.5)	(10,11,12,13;.6) **5**	*
Demand (b_j)	*	*	*	

The minimum fuzzy transportation cost is
= **7** x (11,13,14,18;.5) + **6** x (14,15,16,17;.4) + **5** x (6,7,8,11;.2) +**15** x (9,11,12,13;.2)
+ **5** x (10,11,12,13;.6)
= **(376,436,474,543; .2)**
The ranking function R(A) = 91.45

4.2. Result with normalization process
If all the values of the parameters used in problem1 are first normalized and then the Problem is solved by using the ZAM, then the fuzzy optimal value is $\tilde{x}_0 = (376, 436, 474, 543; 1)$.

4.3. Result without normalization process
If all the values of the parameters of the same problem1 are not normalized and then the Problem is solved by using the ZAM, then the fuzzy optimal value is $\tilde{x}_0 = (376, 436, 474, 543; .2)$.

4.4. Remark
Results with normalization process represent the overall level of satisfaction of decision maker about the statement that minimum transportation cost will lie between 436 and 474 units as 100% while without normalization process, the overall level of satisfaction of the decision maker for the same range is 20%. Hence, it is better to use generalized fuzzy numbers instead of normal fuzzy numbers, obtained by using normalization process.

4.5. Problem 2

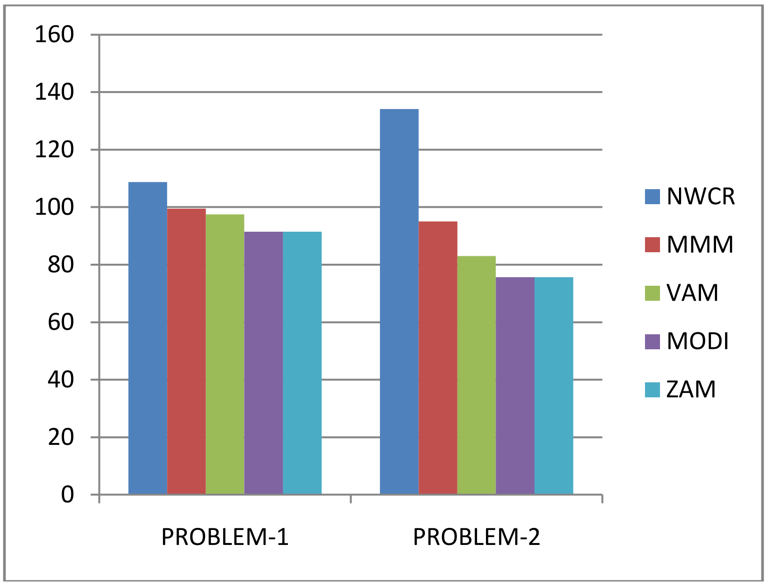
	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(11,12,13,15;.5)	(16,17,19,21;.6)	(28,30,34,35;.7)	(4,5,8,9;.2)	8
S_2	(49,53,55,60;.8)	(18,20,21,23;.4)	(18,22,25,27;.6)	(25,30,35,42;.7)	10
S_3	(28,30,34,35;.7)	(2,4,6,8;.2)	(36,42,48,52;.8)	(6,7,9,11;.3)	11
Demand (b_j)	4	7	6	12	

4.6. Comparative Study and Result Analysis
From the investigations and the results given in Table 2 it clear that ZAM is better than **NWCR**, **MMM** and **VAM[27]** for solving fuzzy transportation problem and also, the solution of the fuzzy transportation problem is given by ZAM is an optimal solution.

Table 2

S.NO	ROW	COLUMN	NWCR	MMM	VAM	MODI	ZAM
1.	3	3	108.80	99.50	97.50	**91.45**	**91.45**
2.	3	4	134.18	95.00	83.00	**75.60**	**75.60**

Table 2 represents the solution obtained by **NWCR**, **MMM, VAM [27], MODI [5]** and **ZAM.** This data speaks the better performance of the proposed method. The graphical representation of solution obtained by varies methods of this performance, displayed in graph 1.



Graph -1

V. Conclusion

In this paper, a proposed method, namely, ZAM is proposed for finding an optimal solution of fuzzy transportation problems in which the transportation cost are represented by generalized fuzzy numbers. The advantage of the proposed method is discussed and a numerical example is solved to illustrate the ZAM. The ZAM is very easy to understand and to apply for solving the fuzzy transportation problems occurring in real life situations.

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