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Analysis of Unsteady Squeezing Fluid Flow through Porous Medium Channel with Slip and No-Slip Boundaries Using DTM

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Abstract*:The aim of this paper is to analyze an unsteady axisymmetric flow of no conducting, Newtonian fluid squeezed flow between two circular plates passing through the porous medium channel with slip and no-slip boundary conditions. We have obtained a single fourth order nonlinear ordinary differential equation using similarity transformation. This differential equation can solve by using the differential transformation method (DTM) and evaluate the residual errors for approximate solutions for various M(constant containing permeability).*
Keywords : *Nonlinear ordinary differential equation, Porous medium channel, Squeezing fluid flow, The differential transformation method.*

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I. Introduction

There are many investigations of unsteady axisymmetric squeezing fluid flow through porous media channel with slip and no-slip boundaries. There are various applications of squeezing flow in life such that food industries, especially chemical engineering [1] and [2].

Polymer processing, compression and injection molding, and modeling of lubrication systems are practical examples of squeezing flows. Similarity, another important application is the simulation of ground water pollution, mostly happening due to leakage of chemicals from tanks and oil pipelines. The aim is to consider ground water as one medium and polluted water as another, therefore the deploying in the latter medium and its results can be studied.

In the last times, after the introduction of the adjusted Dray Low [3], analysis through porous medium has been significant subject for the research community like that reservoir petroleum, civil, chemical, and environmental and biomedical engineering. Some applications in these fields consist of ground water hydrology, chemical reactors, geothermal reservoirs, drainage, and recovery of crude oil from pores of reservoir rocks from [4-9].

The initial working on squeezing flow has been done Stefan [10]. Jackson [11] has made an iterative solution for squeezing flow between two parallel plates and has determined the relative magnitudes of the inertial and viscous forces. Longlois [12] and Salbu [13] analyzed isothermal compressible squeeze films, but they neglected inertia effect. Verma [14] has created numerical solutions of squeezing flow between parallel plates.

Qayyum present in [15] model and analysis the unsteady axisymmetric flow of no conducting Newtonian fluid squeezed between two circular plates passing through the porous medium channel with slip boundary condition using HPM. Hemada and Eladdad in [16] had used the Picard and New iterative methods for solving the fractional form of unsteady squeezing fluid flow between two circular plates with slip and no slip boundaries.

The most scientific applications are formalized by nonlinear partial or ordinary differential equations. In later times, the ideas of Homotopy and perturbation have combined together Liao [17] and He [18] and [19] have done the primary work in this regard.

In this paper, DTM is used to solve an unsteadying squeezing fluid flow through porous media channel with slip and no-slip boundaries. We have obtained residual error of approximate solution for various M.

In section (II) include the mathematical formulation of the problem. Sections (III) and (IV) present the basic theory of DTM and its application in case of a slip and no-slip boundaries. Section (V) includes error analysis and numerical results while conclusions are mentioned in section (VI).

II. The Mathematical Formulation Of The Problem

In this section, the unsteady axisymmetric squeezing flow of incompressible Newtonian fluid with density$\boldsymbol{\rho}$, viscosity$\boldsymbol{\mu}$and Kinematic viscosity$\boldsymbol{\nu}$, squeezed between two circular plates having speed$\mathbf{F_{\omega}(\tau)}$ and passing through the porous medium channel is considered. At any time $\boldsymbol{\tau}$it is assumed that the distance between the two circular plates is$\mathbf{2\upsilon(\tau)}$. Too, it is assumed that the r-axis is the central axis of the channel while z-axis was taking normal to it.

Plates move symmetrically with respect to the central axis $\mathbf{z = 0}$ while the flow is axisymmetric about being$\mathbf{r = 0}$. The longitudinal and normal velocity components in radial and axial directions are$\mathbf{E_r(r, z, \tau)}$ and$\mathbf{E_z(r, z, \tau)}$successively, for more details, see [20-22]. The equations of motion are:

$$\frac{\partial E_r}{\partial r} + \frac{E_r}{r} + \frac{\partial E_z}{\partial z} = 0, \tag{1}$$

$$\frac{\partial G}{\partial r} + \rho\left(\frac{\partial E_r}{\partial r} - E_z\Omega\right) = -\mu\left(\frac{\partial \Omega}{\partial r} + \frac{E_r}{k}\right), \tag{2}$$

$$\frac{\partial G}{\partial z} + \rho\left(\frac{\partial E_r}{\partial \tau} + E_r\Omega\right) = \mu\left(\frac{1}{r}\frac{\partial r\Omega}{\partial r} - \frac{E_\omega}{k}\right), \tag{3}$$

Where $\Omega(r, z, \tau)$ is the function of velocity and $G(r, z, \tau)$ is the function of generalized pressure and k is the permeability constant. The boundary conditions on $E_r(r, z, \tau)$ and $E_z(r, z, \tau)$ are

$$\begin{aligned} &\text{At } z = v:\ \ E_r(r, z, \tau) = \beta\frac{\partial}{\partial z}E_r(r, z, \tau) \text{ and } E_z(r, z, \tau) = F_\omega(\tau) \\ &\text{At } z = 0: \frac{\partial}{\partial r}E_r(r, z, \tau) = 0 \text{ and } E_z(r, z, \tau) = 0 \end{aligned} \tag{4}$$

Where $F_\omega(\tau) = \dfrac{dv}{d\tau}$ is the velocity of the plates.

The boundary values in (4) are due to slipping at the upper plate when $z = h$ and symmetry at$z = 0$.

By similarity transforms, Verma [14], M. Qayyum [15] and A. A. Hemada [22], the above system can be transformed to a single nonlinear fourth order ordinary differential equation in the form

$$\frac{d^4u}{dt^4} + R\left[(t - u)\frac{d^3u}{dt^3} + 3\frac{d^2u}{dt^2}\right] - M\frac{d^2u}{dt^2} = 0 \tag{5}$$

With the initial conditions are

$$\begin{gathered} u(0) = u''(0) = 0, \\ u'(0) = a \text{ and } u'''(0) = b \end{gathered} \tag{6}$$

Where a and b are constants.

In case of no slip boundary: $a = 1.5$ and $b = -3$.

In case of Slip boundary: $a = 0.75$ and $b = 1.5$.

III. Fundamental Theory of Differential Transform Method (DTM)

Differential transforms method is an approximate method based on Taylor expansion. The concept of DTM was first introduced by Zhou [23]. The differential transformation method of function f(x) is defined as follows:

$$\mathbf{F(k)} = \frac{\mathbf{1}}{\mathbf{k!}}\left[\frac{\mathbf{d^k}}{\mathbf{dx^k}}\mathbf{f(x)}\right]_{\mathbf{x=0}} \tag{7}$$

Where f(x) is the original function, F(k) is transformed function.

The differential inverse transformation of F(k) is defined as:

$$\mathbf{f(x)} = \sum_{\mathbf{k=0}}^{\infty}\mathbf{F(k)x^k} \tag{8}$$

From (7) and (8), we obtain.

$$\mathbf{f(x)} = \sum_{\mathbf{k=0}}^{\infty} \frac{\mathbf{1}}{\mathbf{k!}} \left[\frac{\mathbf{d^k}}{\mathbf{dx^k}} \mathbf{f(x)} \right]_{\mathbf{x=0}} \mathbf{x^k} \tag{9}$$

With the aid of (8), the theorems of DTM we have obtained. The followingTABLE 1contains some important theorems of DTM [24] and [25].

Table 1. Important theorems of the DTM

f(x)	F(k)	f(x)	F(k)
$w(x) \pm g(x)$	$W(k) \pm G(k)$	$cw(x)$	$cW(k)$
x^n	$\delta(k-n) = \begin{cases} 1, k = n \\ 0, k \neq n \end{cases}$	$w(x)g(x)$	$\sum_{r=0}^{K} W(r)G(k-r)$
$\frac{d^m w(x)}{dx^m}$	$\frac{(k+m)!}{k!} W(k+m)$	$\frac{dw(x)}{dx}$	$(k+1)W(k+1)$
		$\frac{d^2 w(x)}{dx^2}$	$(k+1)(k+2)W(k+m)$
$x \frac{d^m w(x)}{dx^m}$	$\frac{(k+m-1)!}{(k-1)!} W(k+m-1)$	$x \frac{dw(x)}{dx}$	$kW(k)$
		$x \frac{d^2 w(x)}{dx^2}$	$(K+1)kW(k+1)$
$x^n \frac{d^m w(x)}{dx^m}$	$\frac{(k+m-n)!}{(k-n)!} W(k+m-n)$	$w(x) \frac{d^2 g(x)}{dx^2}$	$\sum_{r=0}^{K} (k-r+1)(k-r+2)W(k)G(k-r+2)$

IV. Application.

In this section, we are using (DTM) to solve thedifferential equation (5) subject to initial conditions (6).
Apply DTM on (5), we have

$$(k+1)(k+2)(k+3)(k+4)U(k+4) + (Rk + 3R - M)(k+1)(k+2)U(k+2)$$
$$-R \sum_{r=0}^{k} U(r)(k-r+1)(k-r+2)(k-r+3)U(k-r+3) = 0$$

$$U(k+4) = \frac{1}{(k+1)(k+2)(k+3)(k+4)} \Bigg[-(Rk + 3R - M)(k+1)(k+2)U(k+2)$$
$$+ R \sum_{r=0}^{k} U(r)(k-r+1)(k-r+2)(k-r+3)U(k-r+3) \Bigg] \tag{10}$$

From the initial conditions in (6),we get: $u^{(4)}(0) = 0$.

$\because u(0) = 0 \quad \rightarrow \quad \therefore U(0) = 0$

$\because u'(0) = a \quad \rightarrow \quad \therefore U(1) = a$

$\because u''(0) = 0 \quad \rightarrow \quad \therefore U(2) = 0$

$\because u'''(0) = b \quad \rightarrow \quad \therefore U(3) = \frac{1}{6} b$

$\because u^{(4)}(0) = 0 \quad \rightarrow \quad \therefore U(4) = 0$

<u>For k = 1:</u>

$$U(5) = \frac{1}{5!} [R \sum_{r=0}^{1} U(r)(2-r)(3-r)(4-r)U(4-r) - 6(R + 3R - M)U(3)]$$

$$\therefore U(5) = \frac{1}{5!} [R[U(1)3!\, U(3)] - (4R - M)b\,]$$

$$\therefore \; U(5) = \frac{1}{5!} [(a-4)bR + bM]$$

For k = 2:

$$U(6) = \frac{1}{3.4.5.6}\left[R\sum_{r=0}^{2} U(r)(3-r)(4-r)(5-r)U(5-r) - 12(2R+3R-M)U(4)\right]$$

$$U(6) = \frac{2}{6!}[R[30U(0)U(5) + 24U(1)U(4) + 6U(2)U(3)] - 12(5R-M)U(4)]$$

$$\therefore U(6) = 0$$

For k = 3:

$$U(7) = \frac{1}{4.5.6.7}\left[R\sum_{r=0}^{3}(4-r)(5-r)(6-r)U(r)U(6-r) - (3R+3R-M)20U(5)\right]$$

$$U(7) = \frac{6}{7!}[R[120U(0)U(6) + 60U(1)U(5) + 24U(2)U(4) + 6U(3)U(3)] - 20(6R-M)U(5)]$$

$$U(7) = \frac{1}{7!}[6R[60U(1)U(5) + 24U(2)U(4) + 6U(3)U(3)] - 120(6R-M)U(5)]$$

$$\therefore U(7) = \frac{1}{7!}[3a^2bR^2 - 18abR^2 + 4abMR + bM^2 + b^2R + 24bR^2 - 10bMR]$$

For k = 4:

$$U(8) = \frac{1}{5.6.7.8}\left[R\sum_{r=0}^{4} U(r)(5-r)(6-r)(7-r)U(7-r) - 30(4R+3R-M)U(6)\right]$$

$$U(8) = \frac{24}{8!}[R[210U(0)U(7) + 120U(1)U(6) + 60U(2)U(5) + 24U(3)U(4) + 6U(4)U(3)] - 30(7R-M)U(6)]$$

$$\therefore\ U(8) = 0$$

Therefore, according to (9), we have obtained the following first little components of the differential transform solution:

$$\mathbf{u(t)} = \frac{\mathbf{1}}{\mathbf{1!}}\mathbf{a\,t} + \frac{\mathbf{1}}{\mathbf{3!}}\mathbf{b\,t^3} + \frac{\mathbf{1}}{\mathbf{5!}}\big((\mathbf{a-4})\mathbf{bR} + \mathbf{bM}\big)\mathbf{t^5} + \frac{\mathbf{1}}{\mathbf{7!}}(\mathbf{3a^2bR^2 - 18abR^2 + 4abMR + bM^2 + b^2R + 24bR^2 - 10bMR})\mathbf{t^7} + \cdots \tag{11}$$

In case of slip boundary:

Where $a = 0.75$ and $b = 1.5$
Assume that $R = M = 0.5$ and by computing more other terms of the solution obtained by the DTM in eq. (11), we get:

$$u(t) = 0.75t + 0.25t^3 - 0.0140625t^5 + 0.00068359t^7 - 5.37933\times10^{-5}t^9 + 3.8193\times10^{-6}t^{11} - 2.57439\times10^{-7}t^{13} + 1.27696\times10^{-8}t^{15} - 4.75209\times10^{-10}t^{17} + 1.56632\times10^{-11}t^{19} - 4.75822\times10^{-13}t^{21} + 1.1664\times10^{-14}t^{23} - 2.03097\times10^{-16}t^{25} + 2.3\times10^{-18}t^{27}. \tag{12}$$

In case no-slip boundary:

Where $a = 1.5$ and $b = -3$.
Assume that $R = M = 0.5$ and by computing more other terms of the solution obtained by the DTM in eq. (11), we get:

$$u(t) = 1.5t - 0.5t^3 + 0.01875t^5 + 0.00078125t^7 - 9.9593\times10^{-5}t^9 - 4.67144\times10^{-6}t^{11} + 8.14242\times10^{-7}t^{13} + 2.319\times10^{-8}t^{15} - 2.27551\times10^{-9}t^{17} + 7.48163\times10^{-12}t^{19} + 3.60882\times10^{-12}t^{21} - 1.75116\times10^{-15}t^{23} - 4.14156\times10^{-15}t^{25} - 1.83285\times10^{-17}t^{27}. \tag{13}$$

V. Error Analysis and Numerical Results

Most of the nonlinear problems do not have exact solution. Therefore, these nonlinear equations should be solved using different methods such as, numerical or approximate method. Thus to estimate the error in the approximate and numerical solution, we used the residual error. The residual error is an effective tool to estimate the error of the resulting solution in these problems.

In this work, the discussed problem (7) is a nonlinear differential equation. Therefore, we compute the residual error in the approximate solution obtained from the differential transformation method in both cases of the problem (7) with slip and no slip.The residual error $\mathrm{Re}(t)$ estimated as follows:

$$\mathrm{Re}(t) = \frac{d^4u^*}{dt^4} + R\left[(t - u^*)\frac{d^3u^*}{dt^3} + 3\frac{d^2u^*}{dt^2}\right] - M\frac{d^2u^*}{dt^2} \tag{14}$$

Where Re (t) is residual error, u^* is thefour terms approximate solutionsobtained in (12) and (13).

It is clear that if $\mathrm{Re}(t) = 0$, then the approximate solutionu^*will be the exact solution anyway, butthis usually does not happen in the nonlinear problems.

TABLE 2: show the absolute residual errors |Re(t)|in the DTM for various values of t and M at R=0.3 in the case of slip boundary.

TABLE 3: show the absolute residual errors |Re(t)| in the DTM for various values of t and M at R=0.3 in the no slip boundary case.

Fig. 1: indicate the absolute residual errors in the DTM for different values of t and M at R=0.3 in the case of slip boundary.

Fig. 2: indicate the absolute residual errors in the DTM for different values of t and M at R=0.3 in the case of no slip boundary.

Table 2. The absoluteresidual errors |Re(t)| in the solution of DTM for different values of t and M at R=0.3 in case of slip boundary.

t	|Re(t)| for {R=0.3,M=0.5}	|Re(t)| for {R=0.3,M=0.7}
0	0	0
0.1	2.85428E-08	8.30938E-09
0.2	8.92738E-07	2.57276E-07
0.3	6.52302E-06	1.84736E-06
0.4	2.60194E-05	7.18218E-06
0.5	7.38637E-05	1.96804E-05
0.6	0.000167767	4.26341E-05
0.7	0.000324121	7.73532E-05
0.8	0.000551583	0.00012104
0.9	0.000843922	0.000165144
1	0.00117383	0.000195203

Table 3.The absoluteresidual errors |Re(t)| in the solution of DTM for different values of different t and M at R=0.3 in case of no slip boundary.

t	|Re(t)| for {R=0.3,M=0.5}	|Re(t)| for {R=0.3,M=0.7}
0	0	0
0.1	9.87225E-10	1.69437E-09
0.2	2.18121E-08	5.17797E-08
0.3	5.34534E-08	3.60082E-07
0.4	3.30248E-07	1.30642E-06
0.5	2.60346E-06	3.10861E-06
0.6	8.97699E-06	5.02294E-06
0.7	1.80758E-05	4.36538E-06
0.8	1.05177E-05	3.20404E-06
0.9	8.62395E-05	1.82974E-05
1	0.000470915	1.92411E-05

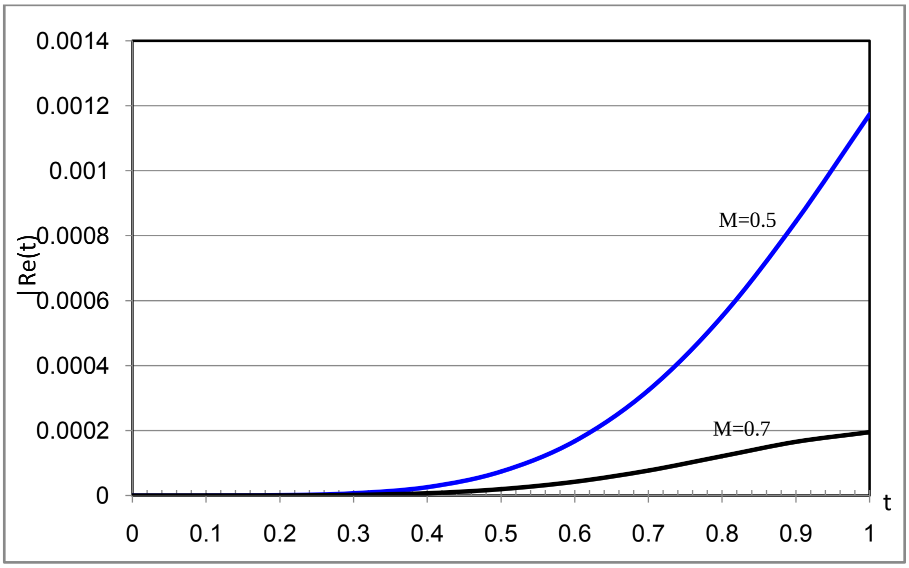

Fig1. Indicate the absolute residual errors in theDTM for different values of t and M at R=0.3in the case ofslip boundary.

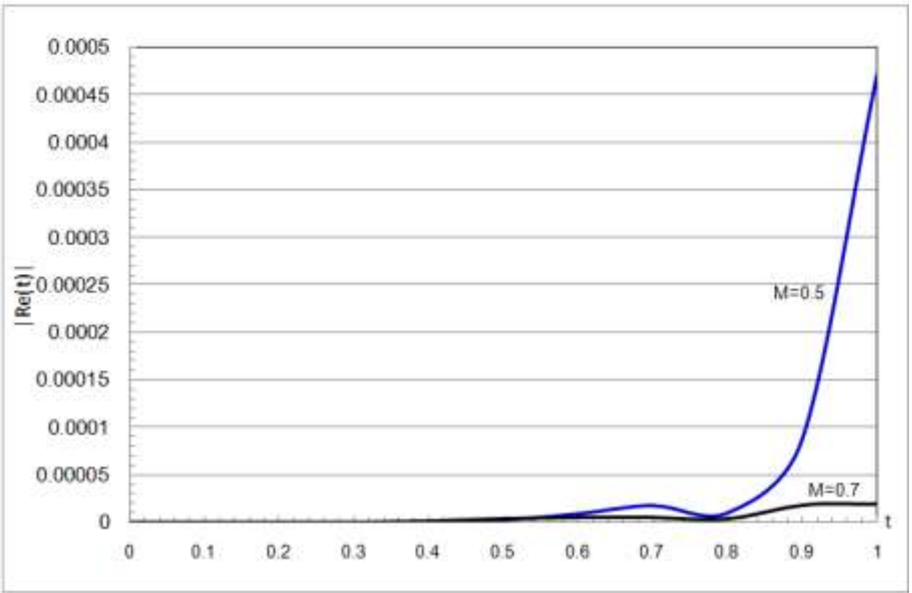

Fig. 2indicates the absolute residual errors in theDTM for different values of t and M at R=0.3in the case ofno slip boundary.

VI. Conclusion

In this article, we used the differential transformation method DTM to find the approximate solution of fourth order nonlinear ordinary differential equation that represents the unsteady axisymmetric flow of no conducting, Newtonian fluid squeezed flow between two circular plates passing through the porous medium channel with slip and no-slip boundary conditions. Convergence of the DTM is confirmed by absolute residual errors for various values of M. Therefore, we deduced that the DTM can be effectively used in different fields of engineering and science just as it gives best results in term accuracy.

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