IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 5 Ver. I1 (Sep. - Oct. 2017), PP 90-97
www.iosrjournals.org

Approximate Solution of an Integro-Differential Equation Arising in Oscillating Magnetic Fields Using the Differential Transformation Method

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***Abstract**: In this article, we have discussed an integro-differential equation which describes the charged particle motion for certain configurations of oscillating magnetic fields. The differential transformation method (DTM) is used for solving this equation. Finally, the error analysis of the results of applying this procedure to the integro-differential equation with time-periodic coefficients illustrates the high accuracy, efficiency, simplicity and applicability of this method.*

***Keywords:** Charged particle motion, Differential transformation method, nonlinear integro-differential equation, oscillating magnetic fields.*

Date of Submission: 22-09-2017 Date of acceptance: 07-10-2017

I. Introduction

Mathematical modeling of engineering and physics problems usually results in integro-differential equations, these equations arise in electromagnetics, electrodynamics, fluid dynamics, optics, biological models and chemical kinetics statistical physics, inverse scattering problems and many other practical applications [1-3]. The integro-differential equations are commonly complicated to solve analytically so it is required to get an effective approximate solution. Integro-differential equations of Volterra type arise in many mathematical formulations are naturally nonlinear. Except some number of them, most nonlinear problems do not have analytical solution. Therefore, these nonlinear equations should be solved using different methods such as a numerical or approximate method. This work concerned with the solution of the Volterra integro–differential equation which describes the charged particle motion for certain configurations of oscillating magnetic fields [4] and [5]:

$$\frac{d^2y}{dt^2} = g(t) - a(t)y(t) + b(t)\int_0^t \cos(\omega_p s)\, y(s)ds \tag{1}$$

Where a(t), b(t) and g(t) are given periodic functions of time may be easily found in the charged particle dynamics for some field configurations. Taking for example the three mutually orthogonal magnetic field elements:

Bx = B1sin (ωpt), By = 0, Bz = B0.

The non-relativistic equations of motion of a particle of mass m and charge q in this field configuration are

$$m\frac{d^2x}{dt^2} = q\left(B_0\frac{dx}{dt}\right), \tag{2}$$

$$m\frac{d^2y}{dt^2} = q(B_1 \mathrm{Sin}(\omega_p t)\frac{dz}{dt} - B_0\frac{dx}{dt}), \tag{3}$$

$$m\frac{d^2z}{dt^2} = q\left(-B_1 \mathrm{Sin}(\omega_p t)\frac{dy}{dt}\right), \tag{4}$$

By integration of (2) and (4) and replacement of the time first derivatives of z and x in (3) one has (1) with

$$a(t) = \omega_c^2 + \omega_f^2 + \sin^2(\omega_p t), \tag{5}$$

$$b(t) = \omega_f^2\, \omega_p\, \mathrm{Sin}(\omega_p t), \tag{6}$$

$$g(t) = \omega_f\, \mathrm{Sin}(\omega_p t)\, z'(0) + \omega_c^2\, y(0) + \omega_c\, x'(0), \tag{7}$$

Where $\omega_c = q\frac{B_0}{m}, \omega_f = q\frac{B_1}{m}$.

Making the additional simplification that $x'(0) = 0$ and $y(0) = 0$, equation (1) is finally written as

$$\frac{d^2y}{dt^2} = \omega_f \operatorname{Sin}(\omega_p t)\, z'(0) - \left(\omega_c^2 + \omega_f^2 + \sin^2(\omega_p t)\right) y(t) + \omega_f^2\, \omega_p \operatorname{Sin}(\omega_p t) \int_0^t \cos(\omega_p s)\, y(s) ds \quad (8)$$

In this study, we consider the equation (1) with the following initial conditions:

$$y(0) = \alpha, \quad y'(0) = \beta. \quad (9)$$

There are methods to solve this equation, such as, He's Homotopy perturbation method [5], Chebyshev wavelet [6], Legendre multi-wavelets [7], Local polynomial regression [8], Shannon wavelets [9], Variational iteration method [10] and Homotopy analysis method [11]. The solution of this equation is presented by means of differential transform method.

II. The Differential Transformation Method (DTM)

The basic idea of the differential transformation method was first introduced by Zhou [6] in electrical circuit analysis. The concept of the differential transform is driven from Taylor series expansion. One of the most advantages of DTM is no needing to parameter and transform into a linear form (linearization). Likewise, it does not need to specify the auxiliary function or suitable initial guess against other analytical and mathematical methods. The fundamental definitions, operations and theorems of the differential transformation method are shown in [7-13]. The differential transform of the function $\mathbf{y(t)}$ is the new function $\mathbf{Y(S)}$ that is given by:

$$Y(S) = \frac{1}{S!}\left[\frac{d^S}{dt^S} y(t)\right]_{t=t_0} \quad (10)$$

The differential inverse of Y(S) is defined as:

$$y(t) = \sum_{S=0}^{\infty} Y(S)\, (t - t_0)^S \quad (11)$$

The following TABLE 1 contains some important theorems of the differential transformation method:

Table 1. Some theorems of the DTM

$y(t)$	$Y(S)$
$C\, f(t)$	$C\, F(S)$
$f(t) \pm g(t)$	$F(S) \pm G(S)$
t^n	$\delta(S-n)$
e^{at}	$\frac{a^S}{S!}$
$\sin at$	$\frac{a^S}{S!}\sin(\frac{\pi S}{2})$
$\cos at$	$\frac{a^S}{S!}\cos(\frac{\pi S}{2})$
$f(t).g(t)$	$\sum_{r=0}^{S} F(r)\, G(S-r)$
$\frac{d^n f(t)}{dt^n}$	$\frac{(S+n)!}{S!} F(S+n)$
$f'(t)$	$(S+1)F(S+1)$
$f''(t)$	$(S+1)(S+2)F(S+2)$
$\int_0^t f(t)dt$	$\frac{F(S-1)}{S}$
$\int_0^t f(t).g(t)dt$	$\sum_{R=0}^{S-1} \frac{1}{S} F(R)\, G(S-R-1)$
$f(t).\int_0^t g(t)dt$	$\sum_{R=0}^{S-1} \frac{1}{S-R} F(R)\, G(S-R-1)$
$f(t)\int_0^t g(t).h(t)dt$	$\sum_{R=1}^{S}\sum_{L=1}^{R} \frac{1}{r} G(L-1)\, H(R-L) F(S-R)$

III. Applications

In this section, to illustrate the efficiency of The differential transformation method for solving equation (1), we consider this equation for different values of a(t), b(t) and g(t), which include some test examples with known exact solutions.

Test example 1:

Consider the equation (1) with

$\omega_p = 2, \quad a(t) = \cos t, \quad b(t) = \sin\left(\frac{t}{2}\right),$

$g(t) = \cos t - t\sin t + \cos t(t\sin t + \cos t) - \sin\left(\frac{t}{2}\right)\left[\frac{2}{9}\sin(3t) - \frac{1}{6}t.\cos(3t) + \frac{1}{2}t\cos t\right]$

$\alpha = 1, \quad \beta = 0.$ (12)

Then equation (1) becomes:

$$\frac{d^2y}{dt^2} = \cos t - t\sin t + \cos t(t\sin t + \cos t) - \sin\left(\frac{t}{2}\right)\left[\frac{2}{9}\sin(3t) - \frac{1}{6}t.\cos(3t) + \frac{1}{2}t\cos t\right] - y(t).\cos t + \sin\left(\frac{t}{2}\right)\int_0^t \cos(2s)\, y(s)ds.$$

With initial conditions $y(0) = 1, \quad y'(0) = 0.$ (13)

With the exact solution $y(t) = \cos t + t\sin t.$

To solve the equation (13), we apply the DTM:

$$\begin{aligned}(S+1)(S+2)Y(S+2) &= -\sum_{R=0}^{S}\frac{1}{R!}\cos\left(\frac{R\pi}{2}\right)Y(S-R) \\ &+ \sum_{R=1}^{S}\sum_{L=1}^{R}\frac{2^{L+R-S-1}}{R(S-R)!}\cos\left(\frac{(L-1)\pi}{2}\right)\sin\left(\frac{(S-R)\pi}{2}\right)Y(R-L) \\ &+ \frac{1}{S!}\cos\left(\frac{\pi R}{2}\right)\left(1 + 2^{S-1} + \frac{7^S - 5^S}{9.2^S}\right) \\ &+ \frac{1}{(S-R)!}\sin\left(\frac{(S-1)\pi}{2}\right)\left(-1 + 2^{S-2} + \frac{7^{S-1} - 5^{S-1}}{12.2^{S-1}} + \frac{1 - 3^{R-1}}{4.2^{S-1}}\right)\end{aligned} \quad (14)$$

Since $y(0) = 1$ and $y'(0) = 0.$

In (13) we get $y''(0) = 1.$

Then $Y(0) = 1, Y(1) = 0$ and $Y(2) = 1/2$.

Using the recurrence relation (14):

At $S = 1$: $6Y(3) = -Y(0) \ \rightarrow \therefore \ Y(3) = 0.$

At $S = 2$: $12Y(4) = -2 - \left[Y(2) - \frac{1}{2}Y(0)\right] + \frac{1}{2}Y(0)] \ \rightarrow \therefore \ Y(4) = -\frac{1}{8}.$

At $S = 3$: $20Y(5) = 0 \quad \rightarrow \therefore \ Y(5) = 0$

At $S = 4$: $30Y(6) = \frac{7}{48} - \left[Y(4) - \frac{Y(2)}{2} + \frac{Y(0)}{24}\right] + \left[\frac{-Y(0)}{48} + \frac{Y(2)}{6} - \frac{Y(0)}{3}\right] \ \rightarrow \ \therefore \ Y(6) = \frac{1}{144}$

At $S = 5$: $\therefore \ Y(7) = 0$

At $S = 6$: $56Y(8) = \frac{439}{3840} - \left[Y(6) - \frac{Y(4)}{2} + \frac{Y(2)}{24} - \frac{Y(0)}{720}\right] + \left[\frac{Y(0)}{3840} - \frac{Y(2)}{144} + \frac{Y(0)}{72} + \frac{Y(4)}{10} - \frac{Y(2)}{5} + \frac{Y(0)}{15}\right]$

$\therefore \ Y(8) = -\frac{1}{5760}$

And so on, we get $Y(9) = Y(11) = 0,$

$Y(10) = \frac{1}{403200}$ and $Y(12) = -\frac{1}{43545600}.$

Now, applying the inverse differential transform, we get the approximate solution in the form:

$$y(t) = 1 + \frac{1}{2}t^2 - \frac{1}{8}t^4 + \frac{1}{144}t^6 - \frac{1}{5760}t^8 + \frac{1}{403200}t^{10} - \frac{1}{43545600}t^{12} + \cdots \quad . \quad (15)$$

To illustrate the high accuracy of the differential transformation method (DTM), the absolute errors A.E. $A.E. = |y_{exact}(t) - y_{appr.}(t)|$ have been calculated for different values of t as shown in TABLE 2.

Where $y_{exact}(t) = \cos t + t\sin t$ and $y_{appr.}(t)$ estimated from (15).

Fig. 1: shown the graphs of the approximate solution and exact solution of the test example 1.

Fig. 2: shown the graph of the absolute errors in the DTM solutions for that problem.

It is clear that the solutions obtained by the DTM are in excellent agreement with the exact solution.

Table 2. The absolute errors for different values of t.

t	$y_{appr.}(t)$	$y_{exact}(t)$	Absolute errors $= \lvert y_{exact}(t) - y_{appr.}(t) \rvert$
0	1	1	0
0.1	1.004987507	1.004987507	0
0.2	1.019800444	1.019800444	0
0.3	1.043992551	1.043992551	1.09912E-13
0.4	1.076828331	1.076828331	3.46789E-12
0.5	1.117295331	1.117295331	5.04676E-11
0.6	1.164121098	1.164121099	4.50009E-10
0.7	1.215794565	1.215794568	2.8617E-09
0.8	1.270591568	1.270591582	1.42093E-08
0.9	1.326604129	1.326604187	5.84059E-08
1	1.381773084	1.381773291	2.06826E-07

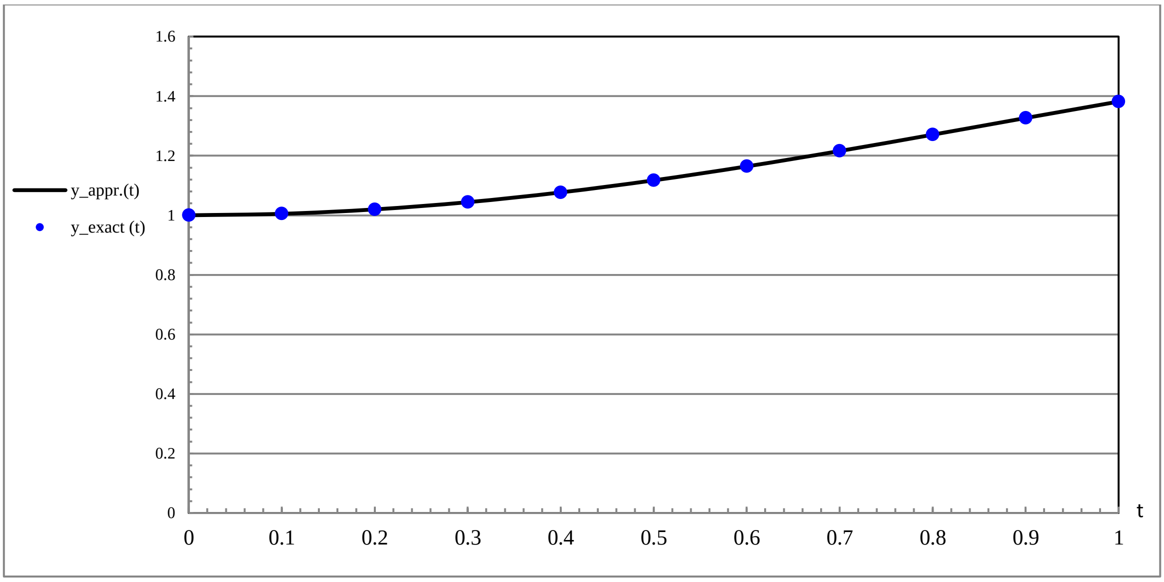

Fig. 1: Shown the graphs of the DTM solutions and the exact solution of the test example 1.

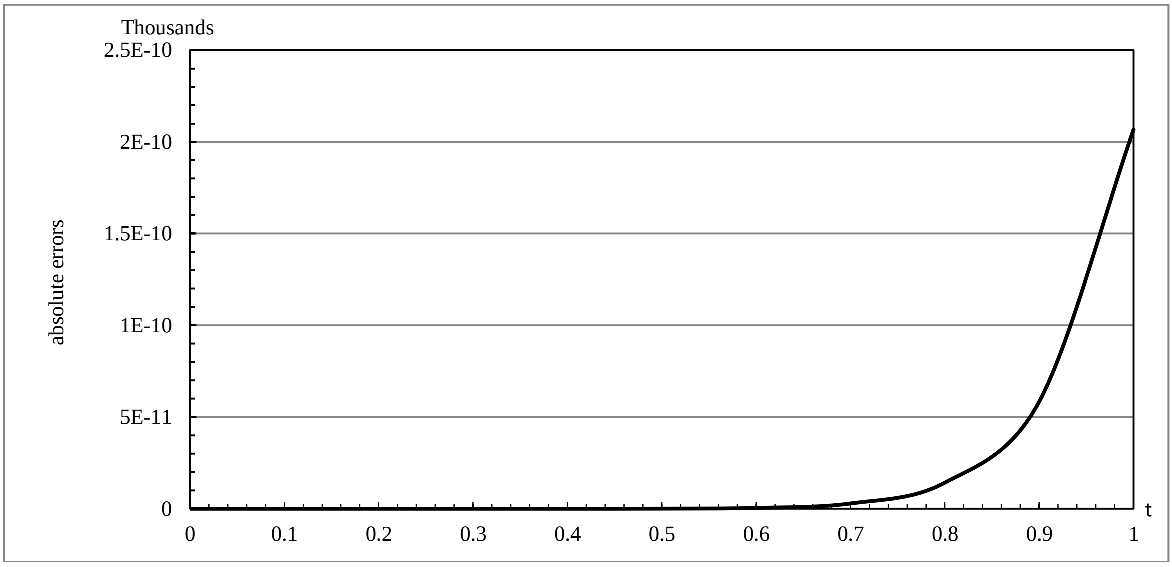

Fig. 2: Shown the graph of the absolute errors in the DTM solutions.

Test example 2:
Consider the equation (1) with
$\omega_p = 1, \quad a(t) = -\sin t, \quad b(t) = \sin t,$

$$g(t) = \frac{1}{9}e^{-\frac{t}{3}} + \sin t\left(e^{-\frac{t}{3}} + t\right) - \sin t\left[-\frac{3}{10}e^{-\frac{t}{3}}\cos t + \frac{8}{9}e^{-\frac{t}{3}}\sin t + \cos t + t\sin t - \frac{7}{10}\right]$$

$\alpha = 1, \quad \beta = 2/3.$ (16)

Then equation (1) becomes:

$$\frac{d^2y}{dt^2} = \frac{1}{9}e^{-\frac{t}{3}} + \sin t\left(e^{-\frac{t}{3}} + t\right) - \sin t\left[-\frac{3}{10}e^{-\frac{t}{3}}\cos t + \frac{8}{9}e^{-\frac{t}{3}}\sin t + \cos t + t\sin t - \frac{7}{10}\right] - y(t)\sin t + \sin t\int_0^t \cos(s)\, y(s)ds.$$

With $y(0) = 1, \quad y'(0) = 2/3$. (17)

The exact solution: $y(t) = t + e^{-\frac{t}{3}}$.
To solve the equation (17), we apply the DTM:

$$\begin{aligned}(S+1)(S+2)Y(S+2) &= -\sum_{R=0}^{S}\frac{1}{R!}\sin\left(\frac{\pi R}{2}\right)Y(S-R) \\ &+ \sum_{R=1}^{S}\sum_{L=1}^{R}\frac{1}{R(L-1)!\,(S-R)!}\cos\left(\frac{(L-1)\pi}{2}\right)\sin\left(\frac{(S-R)\pi}{2}\right)Y(R-L) \\ &+ \frac{-1}{2}\delta(S-1) + \frac{61}{180}\frac{(-1)^{S+1}}{3^S.S!} + \frac{0.7-2^{S-1}}{S!}\sin(\frac{\pi S}{2}) \\ &+ \frac{1}{(S-1)!}\left[2^{S-2}\cos\left(\frac{(S-1)\pi}{2}\right) + \sin\left(\frac{(S-1)\pi}{2}\right)\right] \\ &+ \sum_{R=0}^{S}\frac{(-1)^R}{20(3)^R(R!)((S-R)!)}\left(9(2)^{S-R}\cos\left(\frac{(S-R)\pi}{2}\right)\right. \\ &\left. + \{3(2)^{S-R} + 20\}\sin\left(\frac{(S-R)\pi}{2}\right)\right)\end{aligned} \quad (18)$$

Since $y(0) = 1$ and $y'(0) = \frac{2}{3}$.

In (17) we get $y''(0) = \frac{1}{9}$.

Then $Y(0) = 1, Y(1) = \frac{2}{3}$ and $Y(2) = \frac{1}{18}$.

Using the recurrence relation (18):

At $S = 1$: $6Y(3) = \frac{26}{27} - Y(0) \;\rightarrow\; \therefore\; Y(3) = -\frac{1}{162}$

At $S = 2$: $12Y(4) = -\frac{53}{162} - Y(1) + Y(0) \;\rightarrow\; \therefore\; Y(4) = \frac{1}{1944}$

At $S = 3$: $20Y(5) = -\frac{649}{1458} - \left[Y(2) - \frac{1}{6}Y(0)\right] + \frac{1}{2}Y(1) \;\rightarrow\; \therefore\; Y(5) = -\frac{1}{29160}$

And so on, we get

At $S = 4$: $\rightarrow \therefore Y(6) = \frac{1}{524880}$

At $S = 5$: $\rightarrow \therefore\; Y(7) = -\frac{1}{11022480}$

Now, applying the inverse differential transform, we get the approximate solution in the form:

$$y(t) = 1 + \frac{2}{3}t + \frac{1}{18}t^2 - \frac{1}{162}t^3 + \frac{1}{1944}t^4 - \frac{1}{29160}t^5 + \frac{1}{524880}t^6 - \frac{1}{11022480}t^7 + \cdots. \quad (19)$$

As in Test example 1, we calculate the absolute errors= $\left|y_{exact}(t) - y_{appr.}(t)\right|$ in the results for different values of t as shown in TABLE 3. Where $y_{exact}(t) = \cos t + t\sin t$ and $y_{appr.}(t)$ estimated from (19).
Fig. 3: shown the graphs of the approximate solution and exact solution of the test example 2.
Fig. 4: shown the graph of the absolute errors in the DTM solutions for that problem.
It is clear that the solutions obtained by the DTM are in excellent agreement with the exact solution.

Table 3. The absolute errors for different values of t.

t	$y_{appr.}(t)$	$y_{exact}(t)$	Absolute errors $= \lvert y_{exact}(t) - y_{appr.}(t) \rvert$
0	1	1	0
0.1	1.0672161	1.0672161	2.22045E-16
0.2	1.135506985	1.135506985	9.76996E-15
0.3	1.204837418	1.204837418	2.45581E-13
0.4	1.275173319	1.275173319	2.44116E-12
0.5	1.346481725	1.346481725	1.44973E-11
0.6	1.418730753	1.418730753	6.21088E-11
0.7	1.491889566	1.491889566	2.12398E-10
0.8	1.565928338	1.565928338	6.15903E-10
0.9	1.640818219	1.640818221	1.57457E-09
1	1.716531307	1.716531311	3.64468E-09

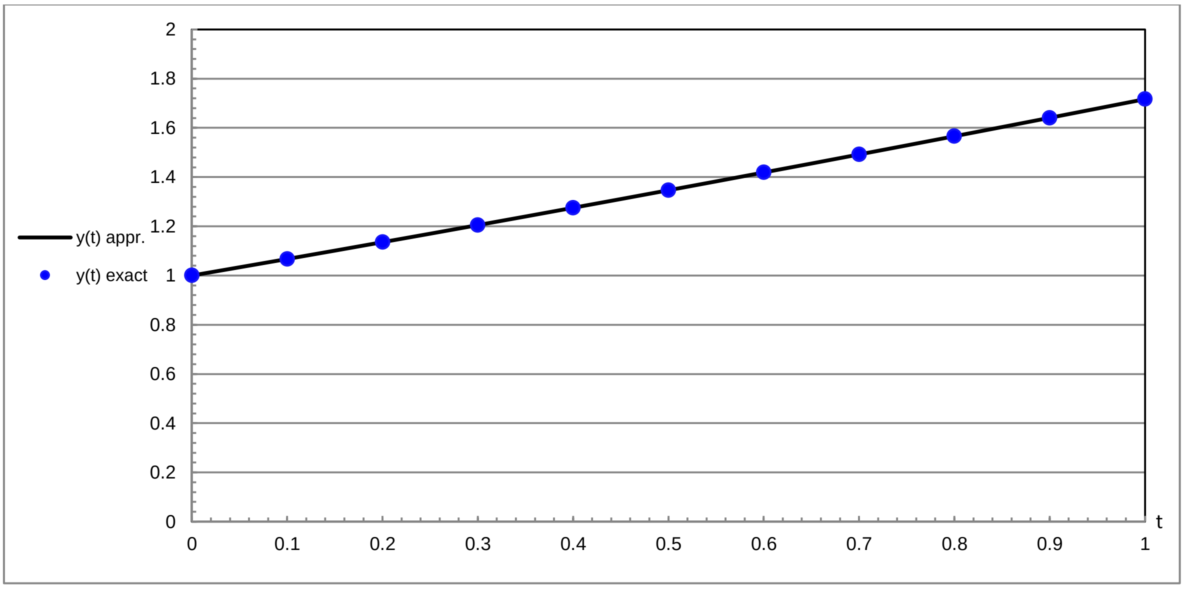

Fig. 3: Shown the graphs of the DTM solutions and exact solutions of test example 2.

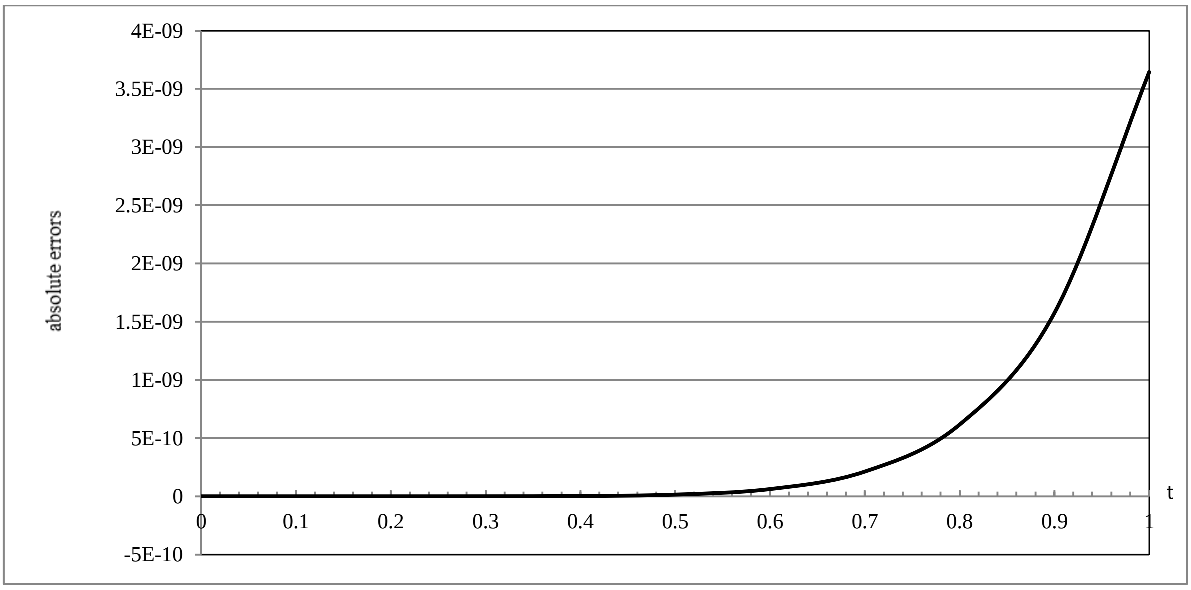

Fig. 4: Shown the graph of the absolute errors in the DTM solutions.

Test example 3:

Consider the equation (1) with

$$\omega_p = 3,\ \ a(t) = 1,\ \ b(t) = \text{sint} + \text{cost},$$

$$g(t) = -(\text{sint} + \text{cost})\left[-\frac{1}{3}t^3 sin3t + \frac{1}{3}t^2(sin3t - cos3t) - \frac{1}{9}t(13sin3t - 2cos3t) + \frac{1}{27}(16sin3t - 13cos3t) + \frac{13}{27}\right] + (-t^3 + t^2 - 11t + 4).$$

$$\alpha = 2, \quad \beta = -5. \tag{20}$$

Then equation (1) becomes:

$$\frac{d^2y}{dt^2} = -(\text{sint} + \text{cost})\left(-\frac{1}{3}t^3 sin3t + \frac{1}{3}t^2(sin3t - cos3t) - \frac{1}{9}t(13sin3t - 2cos3t) + \frac{1}{27}(16sin3t - 13cos3t) + \frac{13}{27}\right) + (-t^3 + t^2 - 11t + 4) - y(t) + (\text{sint} + \text{cost})\int_0^t \cos(3s)\, y(s)ds \tag{21}$$

With $y(0) = 2, \quad y'(0) = -5$.
The exact solution: $y(t) = -t^3 + t^2 - 5t + 2$.
To solve the equation (21), we apply the DTM:

$$\begin{aligned}(S+1)(S+2)Y(S+2) &= -Y(S) + \sum_{R=1}^{S}\sum_{L=1}^{R}\frac{3^{L-1}}{R(L-1)!\,(S-R)!}\cos\left(\frac{(L-1)\pi}{2}\right)\sin\left(\frac{(S-R)\pi}{2}\right)Y(R-L)\\ &+ \sum_{R=1}^{S}\sum_{L=1}^{R}\frac{3^{L-1}}{R(L-1)!\,(S-R)!}\cos\left(\frac{(L-1)\pi}{2}\right)\cos\left(\frac{(S-R)\pi}{2}\right)Y(R-L) - \delta(S-3)\\ &+ \delta(S-2) - 11\delta(S-1) + 4\delta(S)\\ &- \frac{1}{54(S!)}\left(\sin\left(\frac{\pi S}{2}\right)\{26 + 29(2)^S + 3(4)^S\} + \cos\left(\frac{\pi S}{2}\right)\{26 + 3(2)^S - 29(4)^S\}\right)\\ &+ \frac{1}{18(S-1)!}\left(\sin\left(\frac{(S-1)\pi}{2}\right)\{15(2)^{S-1} + 11(4)^{S-1}\}\right.\\ &\left.+ \cos\left(\frac{(S-1)\pi}{2}\right)\{11(2)^{S-1} - 15(4)^{S-1}\}\right)\\ &- \frac{(2)^{S-2}}{3((S-2)!)}\left[\sin\left(\frac{(S-2)\pi}{2}\right) - \cos\left(\frac{(S-2)\pi}{2}\right)\right]\\ &+ \frac{1}{6((S-3)!)}\left(\sin\left(\frac{(S-2)\pi}{2}\right)\{(2)^{S-3} + (4)^{S-3}\}\right.\\ &\left.+ \cos\left(\frac{(S-2)\pi}{2}\right)\{(2)^{S-3} - (4)^{S-3}\}\right).\end{aligned} \tag{22}$$

Since $y(0) = 2$ and $y'(0) = -5$.
In (21) we get $y''(0) = 2$.
Then $Y(0) = 2, Y(1) = -5$ and $Y(2) = 1$.
Using the recurrence relation (22):
At $S = 1$: $6Y(3) = -13 - Y(1) + Y(0) = -6 \rightarrow \therefore\ Y(3) = -1$
At $S = 2$: $12Y(4) = \frac{3}{2} - Y(2) + Y(0) + \frac{Y(1)}{2} = 0 \rightarrow \therefore\ Y(4) = 0$
At $S = 3$: $20Y(5) = \frac{31}{6} - Y(3) + \frac{Y(1)}{2} - 2Y(0) + \frac{Y(2)}{3} = 0 \rightarrow \therefore\ Y(5) = 0$
And so on, we get
At $S = 4$: $\therefore Y(6) = 0$
At $S = 5$: $\therefore Y(7) = 0$
At $S = 6$: $\therefore Y(8) = 0$
Then it is clear that $Y(n) = 0$, for $n \geq 4$.
Now, applying the inverse differential transform, we get the approximate solution in the form:

$$y(t) = 2 - 5t + t^2 - t^3, \tag{23}$$

This approximate solution is the exact solution for our test example 3.

IV. Conclusion

In this work we have successfully applied differential transform method (DTM) on an integro-differential equation with time periodic coefficients which describes the charged particle motion for certain configurations of oscillating magnetic fields. We have considered three test problems to solve and illustrated its efficiency, wider applicability and high accuracy. This study shows that the differential transformation method (DTM) is a useful method for such type of problems because the time periodic coefficients are very complex expressions and other numerical methods add more complexity to it because of their lengthy procedures. The precision of the resulted solution can be improved by obtaining more terms in the solution. In some cases, the approximate solution obtained with the differential transformation method (DTM) can be resulted the exact solution. It can be easily observed from the test problems results that the approximate solution is approaching towards an exact solution in some problems and the absolute error has very small values in other problems.

References

[1]. G. C. Wu, E. W. M. Lee, Fractional variational iteration method and its application. Phys. Lett., A 374, (2010), 2506–2509.
[2]. A.Yildirim, Solution of BVPs for fourth-order integro-differential equations by using homotopy perturbation method. Comput. Math. Appl. 56, (2008), 3175–3180.
[3]. A.M. Wazwaz, A comparison study between the modified decomposition method and the traditional methods, Applied Mathematics and Computation181, (2006) 1703-1712.
[4]. K. Parand and M. Delkhosh, Numerical solution of an integro-differential equation arising in oscillating magnetic fields, J. K SIAM Vol.20, No.3, 261–275, 2016.
[5]. M. Dehghan, F. Shakeri, Solution of an integro-differential equation arising in oscillating magnetic fields using He's homotopy perturbation method. Progress in Electromagnetics Research 78, (2008), 361–376.
[6]. M. Ghasemi, Numerical technique for integro-differential equations arising in oscillating magnetic fields, Iranian J. Sci. Tech., 38A4, (2014) 473–479.
[7]. Y. Khan, M. Ghasemi, S. Vahdati, M. Fardi, Legendre Multi-Wavelets to Solve oscillating magnetic fields integro-differential equations, U.P.B. Sci. Bull. Ser. A, 76(1), (2014) 51–58.
[8]. F. Li, T. Yan, L. Su, Solution of an Integral-Differential Equation Arising in Oscillating Magnetic Fields Using Local Polynomial Regression, Adv. Mech. Eng., (2014) 9 pages.
[9]. K. Maleknejad, M. Hadizadeh, M. Attary, On the Approximate Solution of Integro-Differential Equation Arising in Oscillating Magnetic Fields, App. Math., 58(5), (2013) 595–607.
[10]. M.R Pathak, P. Joshi, High Order Numerical Solution of a Volterra Integro-Differential Equation Arising in Oscillating Magnetic Fields using Variational Iteration Method, Int. J. Adv. Sci. Tech., 69, (2014) 47–56.
[11]. H. Saberi-Nik, S. Effati, R. Buzhabadi, Analytic approximate solution for an integro-differential equation arising in oscillating magnetic fields using homotopy analysis method, Iranian J. Optim., 3, (2010) 518–535.
[12]. J.K. Zhou, Differential Transformation and its Application for Electrical Circuits (Huazhong University Press, Wuhan, 1986).
[13]. H. Seiheii, K. Ghorbanpoor, M. Rezaii, Numerical method for Solving a kind of volterra integro-differential equation using differential transform method, Applied mathematics in Engineering, Management and Technology1 (4), 2013:54-59.
[14]. A. Arikoglu, I. Ozkol, Solution of boundary value problems for integro-differential equations by using differential transform method, Appl. Math. Comput. 168, (2005) 1145-1158.
[15]. J. Biazar, M. Eslami, M. R. Islam, Differential transform method for special systems of integral equations, Journal of King Saud University – Science, (2012) 24, 211-214.
[16]. Fatma Ayaz, Application of differential transform method to differential-algebraic equations, Appl. Math. Comput. 152, (2004) 649-657.
[17]. I.H. Hassan, Comparison differential transformation technique with Adomian decomposition method for linear and nonlinear initial value problems, Chaos Solitons Fractals 36 (1), (2008) 53-65.
[18]. S. H. Behiry and S. I. Mohamed, Solving high-order nonlinear Volterra-Fredholm integro-differential equations by differential transform method, Vol.4, No.8, 581-587 (2012).
[19]. S. H. Behiry, Nonlinear integro-differential equations by differential transform method with Adomian polynomials, Australian Journal of Basic and Applied Sciences, 7(7), 128-139, 2013.

A. Hemada. "Approximate Solution of an Integro-Differential Equation Arising in Oscillating Magnetic Fields Using the Differential Transformation Method." IOSR Journal of Mathematics (IOSR-JM), vol. 13, no. 5, 2017, pp. 90–97.