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On Supra Bitopological Spaces

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Abstract: *The aim of this paper is to introduce the concept of supra bitopological spaces and discuss the fundamental properties of separation axioms in supra bitopological spaces.*
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Keywords*: supra topological spaces, supra bitopological spaces, supra pairwise T_0-space, supra pairwise T_1-space, supra pairwise T_2 -space.*

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I. Introduction

The concept of bitopological spaces was introduced by J.C.Kelly[7]. A set equipped with two topologies is called a bitopological spaces. In 1980, R.C. Jain[6] was introduced separation axioms in bitopological spaces.The supra topological spaces have been introduced by A.S. Mashhour at[9] in 1983. In topological space the arbitrary union condition is enough to have a supra topological space. Here every topological space is a supra topologicl space but the converse is not always true. Then the authors S.P.Aray and T.M. Nour[1] was discussed some higher separation axioms in bitopological spaces. In this paper we introduce and study the concept of supra bitoplogical spaces and investigate some new separation axioms called supra pairwise T_0, supra pairwise T_1 and supra pairwise T_2 spaces. Also we study some of their basic properties in supra bitopological spaces.

II. Preliminaries

Definition 2.1 *[9] (X, τ) is said to be a supra topological space if it is satisfying these conditions:*
(1) $X, \emptyset \in \tau$
(2) The union of any number of sets in τ belongs to τ.

Definition 2.2 *[9] Each element $A \in \tau$ is called a supra open set in (X, τ), and its compliment is called a supra closed set in (X, τ).*

Definition 2.3 *[9] If (X, τ) is a supra topological spaces, $A \subseteq X$, $A \neq \emptyset$, τ_A is the class of all intersection of A with each element in τ, then (A, τ_A) is called a supra topological subspace of (X, τ).*

Definition 2.4 *[9] The supra closure of the set A is denoted by supra cl(A) and is defined as supra-cl(A)= $\cap \{B : B \text{ is a supra closed and } A \subseteq B\}$.*

Definition 2.5 *[9] The supra interior of the set A is denoted by supra int(A) and is defined as supra-int(A)= $\cup \{B : B \text{ is a supra closed and } B \subseteq A\}$*

III. Supra Bitopological Spaces

Definition 3.1 *If τ_1 and τ_2 are two supra topologies on a non-empty set X, then the triplet (X, τ_1, τ_2) is said to be a supra bitopological space.*

Definition 3.2 *Each element of τ_i is called a supra τ_i-open sets in (X, τ_1, τ_2) for i=1,2. Then the complement of supra τ_i-open sets are called a supra τ_i-closed sets.*

Definition 3.3 *If* (X, τ_1,τ_2) *is a supra bitopological space,* $Y \subseteq X$, $Y \neq \emptyset$ *then* (Y, τ_1^*,τ_2^*) *is a supra bitopological subspace of* (X,τ_1,τ_2) *if*
$\tau_1^* = \{U \cap Y\ ; U \text{ is a supra } \tau_1 - \text{open in } X\}$ *and*
$\tau_2^* = \{V \cap Y\ ; V \text{ is a supra } \tau_2 - \text{open in } X\}$.

Definition 3.4 *The supra* τ_i*-closure of the set A is denoted by supra* τ_i*-cl(A) and is defined as supra* τ_i*-cl(A)=*$\cap \{B : B \text{ is a supra } \tau_i\text{\_closed and } A \subseteq B \text{ for } i = 1,2\}$.

Definition 3.5 *The supra* τ_i *-interior of the set A is denoted by supra* τ_i *–int(A) and is defined as supra* τ_i *-int(A)=* $\cup \{B : B \text{ is a supra } \tau_i\text{\_open and } B \subseteq A \text{ for } i = 1,2\}$.

IV. A New Classes Of Supra Pairwise Separation Axioms

Definition 4.1 *A supra bitopological space* (X,τ_1,τ_2) *is called a supra pairwise* T_0 *if for each pair of distinct points* $x, y \in X$, $x \neq y$, *there is a supra* τ_1 *-open set U and a supra* τ_2 *-open set V such that* $x \in U$, $y \notin V$ *or* $y \in V$, $x \notin V$.

Example 4.2 *Let* $X = \{a, b, c\}$
$\tau_1 = \{\emptyset, X, \{a\}, \{a, c\}, \{a, b\}, \{b, c\}\}$. $\tau_2 = \{\emptyset, X, \{b\}, \{b, c\}, \{a, c\}\}$
Let $a, b \in X$. *Then there is a supra* τ_1*-open set* $U = \{a, c\}$ *and a supra* τ_2*-open set* $V = \{b, c\}$ *such that* $a \in U$, $a \notin V$ *or* $b \in V$, $b \notin U$.
Let $b, c \in X$. *Then there is a supra* τ_1*-open set* $U = \{a, b\}$ *and a supra* τ_2*-open set* $V = \{a, c\}$ *such that* $b \in U$, $b \notin V$ *or* $c \in V$, $c \notin U$.
Let $a, c \in X$. *Then there is a supra* τ_1*-open set* $U = \{a, b\}$ *and a supra* τ_2*-open set* $V = \{b, c\}$ *such that* $a \in U$, $a \notin V$ *or* $c \in V$, $c \notin U$.
Therefore (X,τ_1,τ_2) *is a supra pairwise* T_0*-space.*

Theorem 4.3 *If* (X,τ_1,τ_2) *is a supra pairwise* T_0*-space and* (Y,τ_1^*,τ_2^*) *is a supra bitopological subspace of* (X,τ_1,τ_2) *then* (Y,τ_1^*,τ_2^*) *is also a supra pairwise* T_0*-space.*

Proof: *suppose* $x, y \in Y$, $x \neq y$. *Since* $Y \subseteq X$, $x, y \in X$. *Since* (X,τ_1,τ_2) *is a supra pairwise* T_0*-space, there exist a supra* τ_1*-open set U and a supra* τ_2*-open set V such that* $x \in U$, $y \notin U$ *or* $y \in V$, $x \notin V$. *Then* $U \cap Y$, $V \cap Y$ *are* τ_1^*,τ_2^* *supra open sets in Y respectively such that* $x \in U \cap Y$, $y \notin U \cap Y$ *or* $y \in V \cap Y$, $x \notin V \cap Y$.
Hence (Y,τ_1^*,τ_2^*) *is a supra pairwise* T_0*-space*

Theorem 4.4 *If* (X,τ_1,τ_2), (X',τ_1',τ_2') *are two supra bitopological spaces,* (X,τ_1,τ_2), *is a supra pairwise* T_0*-space and f is a supra open function and bijective then* (X',τ_1',τ_2') *is also a supra pairwise* T_0*-space.*

Proof: *Suppose that* (X,τ_1,τ_2) *is a supra pairwise* T_0*-space. Let* $x', y' \in X'$ $x' \neq y'$, *since f is bijective function then there exist* $x, y \in X$ *such that* $x' = f(x)$, $y' = f(y)$ *and* $x \neq y$. *Since* (X,τ_1,τ_2) *is a supra pairwise* T_0*-space then there exist a supra* τ_1*-open set U and a supra* τ_2*-open set V such that* $x \in U$, $y \notin U$ *or* $y \in V$, $x \notin V$. *Clearly* $f(U) \subseteq X'$ *and* $f(V) \subseteq X'$ *since f is a supra open function, f(U)is a supra* τ_1'*-open set and f(V) is a supra* τ_2'*-open set in*X'. *Also* $f(x) \in f(U)$, $f(y) \notin f(U)$ *or* $f(y) \in f(V)$, $f(x) \notin f(V)$.
Hence (X',τ_1',τ_2') *is a supra pairwise* T_0*-space.*

Theorem 4.5 *If* (X,τ_1,τ_2), (X',τ_1',τ_2') *are two supra bitopological spaces where*(X',τ_1', τ_2') *is a supra pairwise* T_0*-space and* $f : X \rightarrow X'$ *is a bijective continuous function then* (X,τ_1, τ_2) *is a supra pairwise* T_0*-space.*

Proof: *Let* $x, y \in X$, $x \neq y$*.since f is bijective then there exist is a* $x', y' \in X'$ $x' \neq y'$ *such that* $x' = f(x)$, $y' = f(y)$ *Since X ' is a supra pairwise* T_0*-space then there exist a supra* τ_1'*-open set U and a supra* τ_2'*-open set V such that* $f(x) \in U$, $f(y) \notin U$ *or* $f(y) \in V$, $f(x) \notin V$. *Since f is continuous,* $f^{-1}(U)$ *and* $f^{-1}(V)$ *are supra* τ_1*-open set, supra* τ_2*-open set respectively in X. Also* $f(x) \in U$ *implies* $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$ *or* $x \notin f^{-1}(V)$, $y \in f^{-1}(V)$. *Hence* (X,τ_1, τ_2), *is a supra pairwise* T_0*-space.*

Remark 4.6 *Every pairwise* T_0*-space is a supra pairwise* T_0*-space, but the converse is not true as shown in the following example.*

Example 4.7 *Let* $X = \{a, b, c\}$
$\tau_1 = \{\emptyset, X, \{b\}, \{a, c\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{a, c\}, \{b, c\}\}$
Therefore (X, τ_1, τ_2) *is a supra pairwise* T_0*-space but not a pairwise* T_0*-space*

Definition 4.8 *A supra bitopological space* (X, τ_1, τ_2) *is called a supra pairwise* T_1 *if for each pair of distinct points* $x, y \in X$, $x \neq y$ *there is a supra* τ_1*-open set U and a supra* τ_2*-open set V such that* $y \in U$, $y \notin V$ *and* $y \in V$, $x \notin V$.

Example 4.9 *Let* $X = \{a, b, c\}$
$\tau_1 = \{\emptyset, X, \{b\}, \{a, c\}, \{a, b\}\}$. $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}, \{a, c\}\}$
Let $a, b \in X$. *Then there is a supra* τ_1*-open set* $U = \{a, c\}$ *and a supra* τ_2*-open set* $V = \{b, c\}$ *such that* $a \in U$, $b \notin U$ *and* $b \in V$, $a \notin V$.
Let $a, c \in X$. *Then there is a supra* τ_1*-open set* $U = \{a, b\}$ *and a supra* τ_2*-open set* $V = \{b, c\}$ *such that* $a \in U$, $c \notin U$ *and* $c \in V$, $a \notin V$.
Let $b, c \in X$. *Then there is a supra* τ_1*-open set* $U = \{a, b\}$ *and a supra* τ_2*-open set* $V = \{a, c\}$ *such that* $b \in U$, $c \notin U$ *and* $c \in V$, $b \notin V$.
Therefore (X, τ_1, τ_2) *is a supra pairwise* T_1*-space.*

Theorem 4.10 *If* (X, τ_1, τ_2) *is a supra pairwise* T_1*-space and* (Y, τ_1^*, τ_2^*) *is a supra bitopological subspace of* (X, τ_1, τ_2) *then* (Y, τ_1^*, τ_2^*) *is also a supra pairwise* T_1*-space.*

Proof: *suppose* $x, y \in Y$, $x \neq y$. *Since* $Y \subseteq X$, $x, y \in X$. *Since* (X, τ_1, τ_2) *is a supra pairwise* T_1*-space, there exist a supra* τ_1*-open set U and a supra* τ_2*-open set V such that* $x \in U$, $y \notin U$, $y \in V$, $x \notin V$. *Then* $U \cap Y$, $V \cap Y$ *are* τ_1^*, τ_2^* *supra open sets in Y respectively such that* $x \in U \cap Y$, $y \notin U \cap Y$ *and* $y \in V \cap Y$, $x \notin V \cap Y$. *Hence* (Y, τ_1^*, τ_2^*) *is a supra pairwise* T_1*-space*

Theorem 4.11 *If* (X, τ_1, τ_2), (X', τ_1', τ_2') *are two supra bitopological spaces,* (X, τ_1, τ_2) *is a supra pairwise* T_1*-space and f is a supra open function and bijective then* (X', τ_1', τ_2') *is a supra pairwise* T_1*-space.*

Proof: *Suppose that* (X, τ_1, τ_2) *is a supra pairwise* T_1*-space. Let* $x', y' \in X'$, $x' \neq y'$, *since f is bijective there exist* $x, y \in X$ *such that* $f(x) = x'$, $f(y) = y'$ *and* $x \neq y$.
Since (X, τ_1, τ_2) *is a supra pairwise* T_1*-space, there exist a supra* τ_1*-open set U and a supra* τ_2*-open set V such that* $x \in U$, $y \notin U$ *and* $y \in V$, $x \notin V$. *Clearly* $f(U) \subseteq X'$ *and* $f(V) \subseteq X'$ *since f is a supra open function,* $f(U)$ *is a supra* τ_1'*-open set and* $f(V)$ *is a supra* τ_2'*-open set in* X'. *Also* $f(x) \in U$, $f(y) \notin U$ *and* $f(y) \in V)$, $f(x) \notin V)$.
Hence (X', τ_1', τ_2') *is a supra pairwise* T_1*-space.*

Remark 4.12 *Every supra* T_1*-space is a supra* T_0*-space but the converse is not true as shown in the following example.*

Example 4.13 *Let* $X = \{a, b, c\}$
$\tau_1 = \{\emptyset, X, \{b\}, \{a, c\}, \{a, b\}\}$. $\tau_2 = \{\emptyset, X, \{a\}, \{a, c\}, \{b, c\}\}$
Here (X, τ_1, τ_2) *is a supra pairwise* T_0*-space but not a supra pairwise* T_1*-space*

Definition 4.14 *A supra bitopological space* (X, τ_1, τ_2) *is called a supra pairwise* T_2 *(or supra pairwise Hausdorff space) if for each pair of distinct points* $x, y \in X$, $x \neq y$, *there is a supra* τ_1*-open set U and a supra* τ_2*-open set V such that* $x \in U$ *and* $y \in V$, $U \cap V = \emptyset$.

Example 4.15 *Let* $X = \{a, b, c\}$
$\tau_1 = \{\emptyset, X, \{b\}, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. $\tau_2 = \{\emptyset, X, \{c\}, \{a, b\}, \{b, c\}\}$.
Let $a, b \in X$. *Then there is a supra* τ_1*-open set* $U = \{a\}$ *and a supra* τ_2*-open set* $V = \{b, c\}$ *such that* $a \in U$ *and* $b \in V$, $U \cap V = \emptyset$.
Let $b, c \in X$. *Then there is a supra* τ_1*-open set* $U = \{b\}$ *and a supra* τ_2*-open set* $V = \{c\}$ *such that* $b \in U$ *and* $c \in V$, $U \cap V = \emptyset$.

Let a, c ∈ X. Then there is a supra τ_1-open set U = $\{a\}$ and a supra τ_2-open set V =$\{c\}$ such that a ∈ U and c ∈ V , U ∩ V =∅.
Therefore (X,τ_1, τ_2) is a supra pairwise T_2-space.

Theorem 4.16 *If (X,τ_1, τ_2) is a supra pairwise T_2-space and (Y,τ_1^*, τ_2^*) is a supra bitopological subspace of (X,τ_1, τ_2) then (Y,τ_1^*, τ_2^*) is also a supra pairwise T_2-spac*

Proof: *suppose x, y ∈ Y , x ≠ y. Since Y ⊆ X, x, y ∈ X. Since (X,τ_1, τ_2) is a supra pairwise T_2-space, there exist a supra τ_1-open set U and a supra τ_2-open set V such that x ∈ U, y ∈ V ,U ∩ V=∅. Then U ∩ Y ,V ∩ Y are τ_1^*, τ_2^* supra open sets in Y respectively such that $x \in U \cap Y$, $y \notin U \cap Y$ and $y \in V \cap Y$, $x \notin V \cap Y$*

$U \cap V = (U \cap Y) \cap (V \cap Y)$
$= (U \cap Y) \cap Y$
$= \emptyset \cap Y$
$= \emptyset$. *Hence(Y,τ_1^*, τ_2^*) is a supra pairwise T_2-space.*

Theorem 4.17 *If (X,τ_1, τ_2), (X ',τ_1', τ_2') are two supra bitopological spaces, (X,τ_1, τ_2) is a supra pairwise T_2-space and f is a supra open function and bijective then (X ',τ_1', τ_2') is a supra pairwise T_2-space.*

Proof: *Suppose that (X,τ_1, τ_2) is a supra pairwise T_2-space. Let $x', y' \in X'$, $x' \neq y'$, since f is bijective there exist x, y ∈ X such that $f(x) = x'$, $f(y) = y'$ and x ≠y.*
Since (X,τ_1, τ_2) is a supra pairwise T_1-space, there exist a supra τ_1 - open set U and a supra τ_2- open set V such that x ∈ U and y ∈ V, U ∩ V =∅. Clearly f(U) ⊆ X ' and f (V) ⊆ X', since f is a supra open function, f (U) is a supra τ_1'- open set and f (V) is a supra τ_2'-open set in X'. Also f(x) ∈ f(U) and f(y) ∈ $f(V)$, f(U) ∩ f(V) = ∅
Hence (X ',τ_1', τ_2') is a supra pairwise T_2-space.

Remark 4.18 *Every supra pairwise T_2-space is a supra pairwise T_1-space but the converse is not true such as shown in the following example.*

Example 4.19 *Let X =* $\{a, b, c\}$
$\tau_1 = \{\emptyset, X, \{a, b\}, \{a, c\}, \{b, c\}\}$. $\tau_2 = \{\emptyset, X, \{a\}, \{a, c\}, \{b, c\}\}$.
Here (X,τ_1, τ_2) is a supra pairwise T_1-space but not a supra pairwise T_2-space.

V. Conclusion

In this paper, basic concepts of supra bitopological spaces are introduced and also separation axioms of supra bitopological spaces are analysed.

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