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Analytical Study of Instability Phenomenon by Using Fourier Transform

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Abstract: *The aim of the study is to analyse the instability phenomenon by using Fourier Transform. Here we adopted this study with two phase immiscible flow through homogenous porous media. The concern equation is solved to obtain the saturation of water by using Fourier Transform. The graphical representation suggest that injected fluid is increasing the Saturation of water decreasing while time T is increasing Saturation of water increasing.*

Keywords: *Cauchy – Residue Theorem,Fourier Transform, Instability Phenomenon, Porous Medium*

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I. Introduction

A well-known phenomenon of fingering (Instabilities has) been discussed which occurs in two immiscible phase flow through homogeneous porous media. The flow of two immiscible fluids in a large medium can be investigated fairly simply if it is unidirectional, in other words if the different values such as pressures, saturations, fluid speeds, etc. vary only in a single space direction corresponding to the movement direction.

If the porous medium is thicker, the vertical component of the velocities cannot be ignored, and the analysis of the forces acting in the porous medium shows that the interfaces are “ fronts ” (front means the zone of the medium where saturation of injecting phase rises sharply) are generally distorted (encroachment) . Theses encroachment occurs on the scale of the front, called the **tonguephenomenon**, but also a smaller scale, called **fingering** [1]. These encroachments are governed by conditions of stability of instability. Many experiment shows that these instabilities (tongue, fingering) depend in particular on the mobility ratio. More precisely, instabilities are more likely to appear if the mobility ratio (M) is higher than 1. In other words, injected fluids that are more mobile than native fluid can cause harmful instabilities. The difference in viscosities of flowing fluids causes the occurrence of the phenomenon of fingering. Most of the earlier authors have completely neglected the capillary pressure. [2] and [3] have included capillary pressure in the analysis of fingers. [4] has given an up-to-date review on the topic. It is one of the most practically important problem for oil production in the oil reservoir engineering. Therefore, the fingers should be stabilized at time in oil recovery processes.

Here, the authors have considered this phenomenon with capillary pressure. It is assumed that the individual pressure of the two flowing phases may be replaced by their common mean pressure and the behaviour of the fingers is determined by a statistical treatment. The governing law of Darcy, governing equation of continuity and certain basic assumptions yields a nonlinear partial differential equation for motion of saturation of injecting fluid. The solution is obtained using Fourier transform and Complex Residue theorem.

[5] investigated the study of Instabilities in Displacement Process through Homogeneous Porous Media. With the reference of [5] the aim of the study is to discuss the analytical solution of Instability Phenomenon by using Fourier Transform which was not found in previous literature.

II. StatementoftheProblem:

When a fluid contained in porous medium is displaced by another fluid of lesser viscosity, instead of regular displacement of the whole front, protuberance may occur which shoot through the porous medium at relatively great speed. Those protuberances are called fingers [2] and the phenomenon is called fingering.

We consider that a finite cylindrical piece of homogeneous porous medium of length L, fully saturated with oil, which is displaced by injecting water which give rise to fingers (protuberance). Since the entire oil at the initial boundary $x = 0$ (x being measured in the direction of displacement), is displaced through a small distance due to water injection, therefore, it is assumed that complete saturation exists at the initial boundary.

Here, an analytical expression for the cross- sectional area occupied by fingers has been obtained. For the mathematical formulation, we consider the governing law which is Darcy's law, here, as valid for the investigated flow system and assumed further that the macroscopic behaviour of fingers is governed by statistical treatment.

In the statistical treatment of fingers only the average behaviour of the two fluids involved is taken into consideration. It was shown by [6] that this treatment of motion with the introduction of the concept of fictitious relative permeability become formally identical to the Buckley-Leverett description of two immiscible of injected fluid flow through porous media. The saturation of injected fluid (Si) is then defined in average cross-sectional area occupied by the injected fluid level$x$at time t, i.e.$S_i(x,t)$. Thus the saturation of injected fluid in porous medium represents the average cross-sectional area occupied by fingers [7].

III. Mathematical Formulation of the Problem:

3.1 Fundamental Equation:

Assuming validity of Darcy's law, the seepage velocity of water (V_w) and oil (V_o) may be written as:

$$V_w = -\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \text{------} (1)$$

$$V_o = -\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x} \text{------} (2)$$

Where$k$ is the permeability of the homogeneous medium,$k_w$and $k_o$are relative permeability of water and oil respectively, $P_w and P_o$ are the pressures of water and oil respectively and $\mu_w and \mu_o$are viscosities of water and oil respectively. Again $k_w$and $k_o$are assumed to be functions of water saturation S_w and oil saturation $S_o$respectivly.

The equation of continuity (phase densities and regarded constants) are:

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \text{------} (3)$$

$$\varphi \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \text{------} (4)$$

Where $\varphi$is the porosity of the medium,

The porous medium considered to be fully saturated, from the definition of phase saturation, it is evident that

$$S_w + S_o = 1 \text{------} (5)$$

The capillary pressure P_c, defined as the pressure discontinuity of the flowing phases across their common interface, maybe written as:

$$P_c = P_o - P_w \text{------} (6)$$

For definiteness of the mathematical analysis, we assume standard form of an analytical expression for the relationship between the relative perm abilities, phase saturation and capillary pressure as (for instabilities)

$$k_w = S_w \ , k_o = S_o = 1 - S_w \text{------} (7)$$

$$P_c = -\beta S_w \text{------} (8)$$

(For physical significance, we consider negative sign which shows the direction of saturation of water is opposite to capillary pressure). For definiteness, we consider β to be small parameter.

The value of pressure of oil P_o can be written as [8]

$$P_o = \frac{1}{2}(P_o + P_w) + \frac{1}{2}(P_o - P_w)$$

$$P_o = \underline{P} + \frac{P_c}{2} \ , P_o = \frac{1}{2}(P_o + P_w) \text{------} (9)$$

Where $\underline{P}$ is the mean pressure which is constant.

3.2. Equation for Motion for Saturation

The equation for motion for saturation can be obtained by substituting the values of $V_w and V_o$ from (1) and (2) in to (3) and (4)

$$\varphi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left[-\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x}\right] \text{------} (10)$$

$$\varphi \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x}\left[-\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x}\right] \text{------} (11)$$

Eliminating $\frac{\partial P_w}{\partial x}$ from equations (10) and (6), we get

$\varphi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}[-\frac{k_w}{\mu_w} k\{\frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x}\}]$ -- (12)

Combining (11) and (12) by using (5), we obtain

$\frac{\partial}{\partial x}[\{\frac{k_o}{\mu_o} + \frac{k_w}{\mu_w}\}k \frac{\partial P_o}{\partial x} - \frac{k_w}{\mu_w} k \frac{\partial P_c}{\partial x}] = 0$ -- (13)

Integrating (13) with respect to x , we get

$[\{\frac{k_o}{\mu_o} + \frac{k_w}{\mu_w}\}k \frac{\partial P_o}{\partial x} - \frac{k_w}{\mu_w} k \frac{\partial P_c}{\partial x}] = -A$ -- (14)

Where A is the constant of integration (negative sign on right hand side is considered for our convenience). Simplifying equation 14, we get

$\frac{\partial P_o}{\partial x} = -\frac{A}{k\{\frac{k_o}{\mu_o}+\frac{k_w}{\mu_w}\}} + \frac{k_w}{\mu_w\{\frac{k_o}{\mu_o}+\frac{k_w}{\mu_w}\}}\frac{\partial P_c}{\partial x}$ -- (15)

Now substituting the value of $\frac{\partial P_o}{\partial x}$ from (15) into (12), we have

$$\varphi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}[\frac{k_w}{\mu_w} k\{-\frac{A}{k\{\frac{k_o}{\mu_o} + \frac{k_w}{\mu_w}\}} + \frac{k_w}{\mu_w\{\frac{k_o}{\mu_o} + \frac{k_w}{\mu_w}\}}\frac{\partial P_c}{\partial x} - \frac{\partial P_c}{\partial x}\}]$$

$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}[\frac{k_o}{\mu_o} k \frac{\partial P_c}{\partial x}\frac{1}{\{1+\frac{k_o \mu_w}{k_w \mu_o}\}} + \frac{A}{\{1+\frac{k_o \mu_w}{k_w \mu_o}\}}] = 0$ -------------------------------- (16)

Now from (9), $\frac{\partial P_o}{\partial x} = \frac{1}{2}\frac{\partial P_c}{\partial x}$
Substituting this value in (14), we have

$A = \{\frac{k_w}{\mu_w} - \frac{k_o}{\mu_o}\}\frac{k}{2}\frac{\partial P_c}{\partial x}$ -- (17)

Substituting the value of A from (17) into (16), we get

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}[k \frac{k_w}{2\mu_w}\frac{\partial P_c}{\partial x}] = 0$$

$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}[k \frac{k_w}{2\mu_w}\frac{dP_c}{dx}\frac{\partial S_w}{\partial x}] = 0$ -- (18)

From (7), (8) and (18), we have

$\varphi \frac{\partial S_w}{\partial t} - \frac{\beta k k_w}{2\mu_w}\frac{\partial^2 S_w}{\partial x^2} = 0$ --- (19)

Let $k = C_0 \tau \frac{\varphi^3}{M_s(1-\varphi)^2}$, $\tau = (\frac{L}{L_e})^2$ -- (20)

Where φ is porosity

M_s is specific surface area
C_0 Kozeny constant
L_e effective length of the path of the fluid.

From (19) and (20), we have

$\frac{\partial S_w}{\partial t} - \frac{\beta C_0 \tau k_w \varphi^2}{2\mu_w M_s(1-\varphi)^2}\frac{\partial^2 S_w}{\partial x^2} = 0$ -- (21)

This is a non-linear partial differential equation of motion for the saturation of the injected fluid through the homogeneous porous media.Here, the capillary pressure co-efficient, from our assumption, it is small enough to consider. It is a perturbation parameter. Again β is multiplied to the highest derivative in (21), therefore, the problem (21) is a singular perturbation problem. Such problem together with appropriate conditions has been solved analytically or numerically.

A set of conditions are written as

$\frac{\partial}{\partial x} S_w(1,t) = 0, 0 \leq x \leq 1$ --- (22)

$S_w(1,t) = \frac{e^{-x}}{2}$ -- (23)

Also we assume that there is no flow across the face $x = L$ (because the face at $x = L$ is assumed to be impermeable), that is,

Let $X = \frac{x}{L}, T = \frac{C_0 \tau k_w \varphi^2}{2\mu_w M_s (1-\varphi)^2 L^2} t$

From 21, we have

$\frac{\partial S_w}{\partial T} - \beta \frac{\partial^2 S_w}{\partial X^2} = 0$ -- (24)

$\frac{\partial}{\partial X} S_w(1,T) = 0, 0 \leq X \leq 1$ --- (25)

$S_w(X,0) = \frac{e^{-X}}{2}$ --- (26)

IV. MathematicalSolution:

We have to solve (24)

$$\frac{\partial S_w}{\partial T} - \beta \frac{\partial^2 S_w}{\partial X^2} = 0$$

$$\frac{\partial S_w}{\partial T} = \beta \frac{\partial^2 S_w}{\partial X^2}$$

Now using Fourier transform on X,

$$\frac{\partial \underline{S_w}}{\partial T} = F_C\left(\beta \frac{\partial^2 S_w}{\partial X^2}\right)$$

$= -S^2 \beta F_C(S_w)(S,T) - \frac{\partial S_w}{\partial x}(1,t)$

$= -S^2 \beta F_C(S_w)(S,T)$

Now S is fixed,

$\frac{d\underline{S_w}}{dT} + S^2 \beta F_C(S_w) = 0$

$\frac{d\underline{S_w}}{dT} + S^2 \beta \underline{S_w} = 0$-- (27)

Solving (27) by first order ordinary differential equation method (variable separable method), so solution is,

$\underline{S_w} = A(S) e^{-\beta S^2 T}$-- (28)

Now using the condition $S_w(x,0) = \frac{e^{-x}}{2}$

$F_C(S_w)(S,0) = \int_0^\infty (S_w(s,0) cossx)\, dx$

$$= \int_0^\infty \left(\frac{e^{-x}}{2} cossx\right) dx$$

$$= \frac{1}{2}\left[\frac{e^{-x}}{1+S^2}(-cossx + ssinsx)\right]_0^\infty$$

$$F_C(S_w)(S,0) = \frac{1}{2}\left(\frac{1}{1+S^2}\right) = A(s)$$

$$\underline{S_w} = \frac{1}{2}\left(\frac{1}{1+S^2}\right) e^{-\beta S^2 T}$$

Now applying Inverse Fourier transform

$$S_w(X,T) = \frac{2}{\pi}\int_0^\infty \frac{1}{2}\left(\frac{1}{1+S^2}\right) e^{-\beta S^2 T} cossx\, ds \text{ ---------------------------- (29)}$$

Solving equation 29, by Cauchy residue theorem,
Here we have function $F(s) = \frac{1}{2}(\frac{1}{1+s^2})e^{-\beta s^2 T} cossx$ which is not analytic at $= s = \pm i$ and we are consider this equation as   so we use only real axis limit for this. Therefore $s = i$ is in real axis part. And the final solution of the given integral is $S_w(X,T) = e^{\beta T - X}$

Table 1: Values of S_w

	T=0.1	T=0.2	T=0.3	T=0.4
X=	S_w			
0	1.01005	1.020201	1.030455	1.040811
0.1	0.913931	0.923116	0.932394	0.941765
0.2	0.826959	0.83527	0.843665	0.852144
0.3	0.748264	0.755784	0.763379	0.771052
0.4	0.677057	0.683861	0.690734	0.697676
0.5	0.612626	0.618783	0.625002	0.631284
0.6	0.554327	0.559898	0.565525	0.571209
0.7	0.501576	0.506617	0.511709	0.516851
0.8	0.453845	0.458406	0.463013	0.467666
0.9	0.410656	0.414783	0.418952	0.423162
1	0.371577	0.375311	0.379083	0.382893

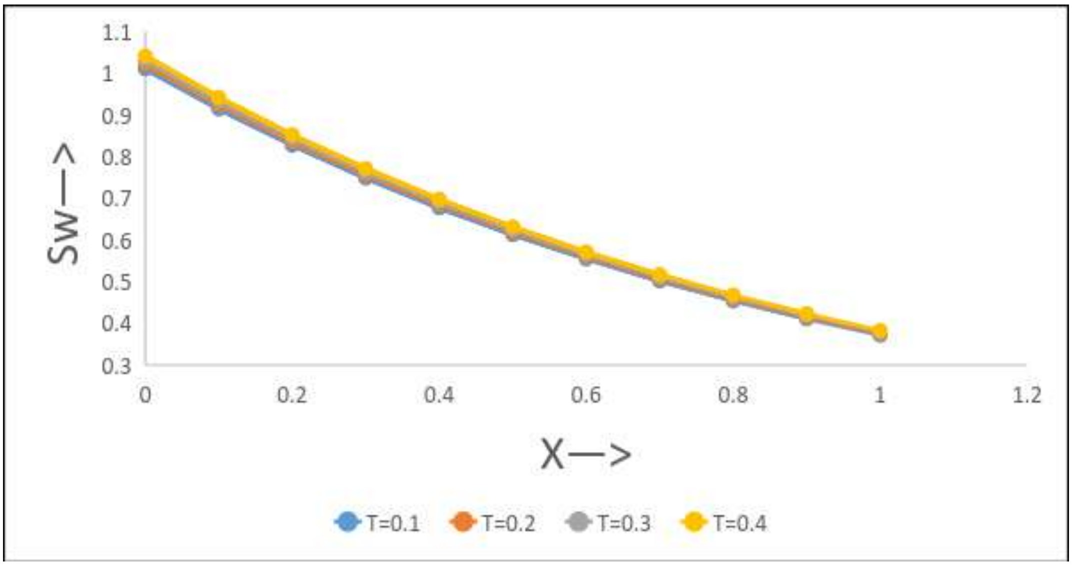

Fig 1: Injected fluid —> Water Saturation

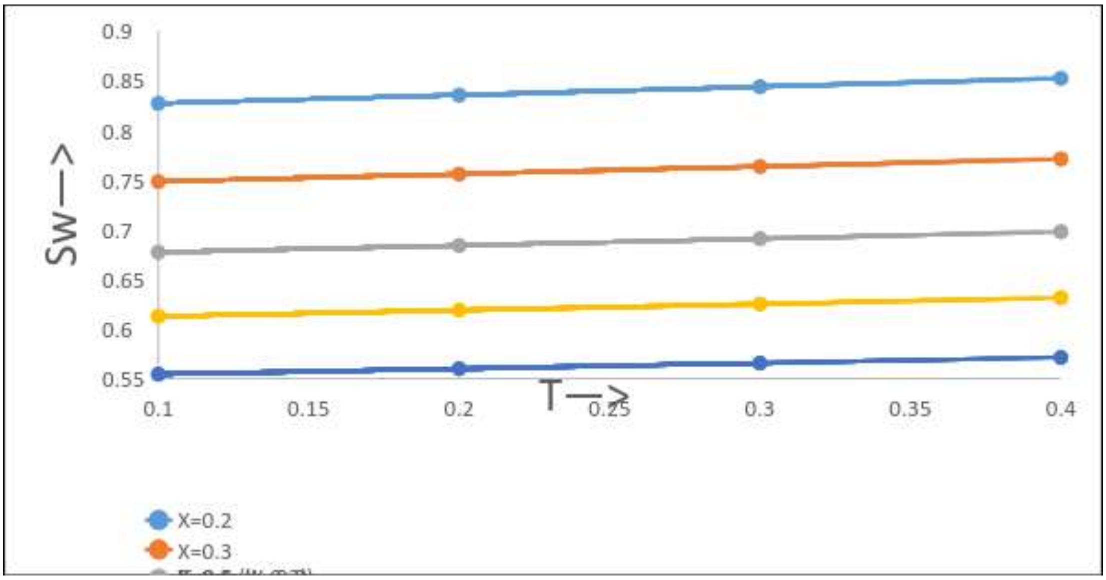

Fig 2: Injected fluid —> Water Saturation

V. Conclusion

From Fig. 1, we can say that as injected fluid (X) increases the saturation (S_w) decreases. Keeping T constant as X increases saturation decreases exponentially. From Fig. 2, it is clear that as time T increases saturation (S_w) increases. Keeping X constant, for different values of T the saturation increases linearly as time T increases.

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