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General Solution and a Fixed Point Approach to the UlamHyers Stability of Viginti Duo Functional Equation in Multi-Banach Spaces

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Abstract. *In this current work, we compute the general solution and determine the Hyers-Ulam stability for a new form of Viginti duo functional equation in Multi-Banach Spaces by using fixed point technique.*

Key words and phrases: *Hyers-Ulam stability, Multi-Banach Spaces, Vigintic duo FunctionalEquations, Fixed Point Method.*
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I. Introduction

The issue of stability of functional equations has appeared in connection with a question that Ulam[17] posed in 1940. Hyers [7], by using direct method, brilliantly gave a partial answer for the case of the additive Cauchy functional equation for mappings between Banach Spaces. This result was then improved by Aoki [1]and Rassias [15], who weakened the condition for the bound of the norm of the Cauchy difference. The stability phenomena proved in [7] and [15] were named Hyers-Ulam and Hyers-Ulam-Rassias stability duo to the great influence of Hyers and Rassias on this area of research.Some results regarding to the stability of various forms of the quadratic [16], cubic [12], quartic [13],quintic [19],sextic [19], septic and octic [14], decic [2],undecic[10] and quattuordecic [11] functional equations have been investigated by a number of authors with more general domains and co-domains.In this present work, we establish the general solution and Hyers-Ulam stability fora new form of Viginti duo functional equation

$$\mathcal{R}g(a, b) = g(a + 11b) - 22g(a + 10b) + 231g(a + 9b) - 1540g(a + 8b) + 7315g(a + 7b) - 26334g(a + 6b)$$
$$+ 74613g(a + 5b) - 170544g(a + 4b) + 319770g(a + 3b) - 497420g(a + 2b)$$
$$+ 646646g(a + b) - 705432g(a) + 646646g(a - b) + 497420g(a - 2b) +$$

319770g(a – 3b)
–170544g(a – 4b) + 74613g(a – 5b) –26334g(a – 6b) + 7315g(a – 7b) –
1540g(a – 8b)
+ 231g(a – 9b) – 22g(a – 10b) + g(a – 11b) – $1.124000728 \times 10^{21}$g(b) (1.1)

in Multi-Banach Spaces.

II. General Solution OfVigintiDuo Mapping In (1.1)

Theorem 2.1. Let *A* and *B* be the vector spaces. If g: $A \to B$ is a function (1.1) for all a; b $\in A$; then g is Viginti Duo Mapping.

Proof. Substituting a = 0 and b = 0 in (1.1), we obtain that g(0) = 0. Substituting (a, b) with (a, a) and (a,–a) in (1.1), respectively, and subtracting two resulting equations, we can obtain g(–a) = g(a); that is to say, g is an even function.

Changing (a, b) by (11a, a) and (0, 2a), respectively, and subtracting the two resulting equations, we arrive at

22g(21a)– 253g(20a) + 1540g(19a) –7084g(18a) + 26334g(17a)– 76153g(16a) +
170544g(15a)
+ 312455g(14a) + 497420g(13a)– 672980g(12a) + 705432g(11a)–572033g(10a) +
497420g(9a)
– 490314g(8a) + 170544g(7a) – 245157g(6a)+ 26334g(5a) – 504735g(4a) + 1540g(3a)
–(22!/2)g(2a) + 22!g(a) = 0(2.1)

$\forall$a $\in A$. Considering a = 10a and b = a in (1.1), one gets

g(21a) – 22g(20a) + 231g(19a) – 1540g(18a) + 7315g(17a) – 26334g(16a)
+ 74613g(15a) – 170544g(14a) + 319770g(13a) – 497420g(12a) + 646646g(11a)
– 705432g(10a) + 646646g(9a) – 497420g(8a) + 319770g(7a) – 170544g(6a)
+ 74613g(5a) – 26334g(4a) + 7315g(3a) – 1540g(2a) –22!g(a) = 0
(2.2)

$\forall$a $\in A$. Multiplying (2.2) by 22, and then subtracting (2.1) from the resulting equation, we arrive at

231g(20a) – 3542g(19a) + 26796g(18a) – 134596g(17a) + 503195g(16a)– 1470942g(15a)
+ 3439513g(14a) –6537520g(13a) + 10270260g(12a)– 13520780g(11a) +
14947471g(10a)
– 13728792g(9a) + 10452926g(8a)– 6864396g(7a) + 3997125g(6a)–
1615152g(5a)
+ 74613g(4a) – 159390g(3a) – (22!/2)g(2a) + 22!(23)g(a) = 0 (2.3)

$\forall$a $\in A$. Letting a = 9a and b = a in (1.1), one obtains g(20a) – 22g(19a) + 231g(18a) – 1540g(17a) + 7315g(16a)–26334g(15a) + 74613g(14a) –170544g(13a) + 319770g(12a)– 497420g(11a) + 646646g(10a) –705432g(9a) + 646646g(8a) –497420g(7a) + 319770g(6a)– 170544g(5a) + 74613g(4a) – 26334g(3a) + 7316g(2a) –22!g(a) = 0 (2.4)

∀a ∈*A*. Multiplying (2.4) by 231, and then subtracting (2.3) from the resulting equation, we arrive at

1540g(19a) – 26565g(18a) + 221144g(17a) – 1186570g(16a) + 4612212g(15a)–13796090g(14a) + 32858144g(13a) – 63596610g(12a) + 101383240g(11a) –134427755g(10a) + 149226000g(9a) –138922300g(8a) + 108039624g(7a) – 69869745g(6a) + 37780512g(5a) – 17160990g(4a) + 5923764g(3a)– (22!/2)g(2a) + 22!(254)g(a) = 0 (2.5)

∀a ∈*A*. Changing a = 8a and b = a in (1.1), one obtains

g(19a) – 22g(18a) + 231g(17a) – 1540g(16a) + 7315g(15a) – 26334g(14a) + 74613g(13a) – 170544g(12a)+ 319770g(11a) – 497420g(10a) + 646646g(9a) – 70544332g(8a) + 646646g(7a) – 497420g(6a) + 319770g(5a)– 170544g(4a) + 74614g(3a) – 26356g(2a) –22!g(a) = 0 (2.6)

∀a ∈*A*. Multiplying (2.6) by 1540, and then subtracting (2.5) from the resulting equation, we arrive at

7315g(18a) – 134596g(17a) + 1185030g(16a) – 6652888g(15a) + 26758270g(14a) –82045876g(13a) + 199041150g(12a) –391062560g(11a) + 631599045g(10a) –846608840g(9a) + 947442980g(8a) –887795216g(7a) + 696157055g(6a) –454665288g(5a) + 245476770g(4a) – 108981796g(3a) – (22!/2)g(2a) + 22!(1794)g(a) = 0 (2.7)

∀a ∈*A*. Letting a = 7a and b = a in (1.1), one obtains

g(18a) – 22g(17a) + 231g(16a) – 1540g(15a) + 7315g(14a)–26334g(13a) + 74613g(12a) – 170544g(11a) + 319770g(10a) –497420g(9a) + 646646g(8a) –705432g(7a) + 646646g(6a)

$-497420g(5a) + 319771g(4a) - 170566g(3a) + 74844g(2a) - 22!g(a) = 0 \quad (2.8)$

$\forall a \in A$. Multiplying (2.8) by 7315, and then subtracting (2.7) from the resulting equation, we arrive at

$$26334g(17a) - 504735g(16a) + 4612212g(15a) - 26750955g(14a) + 110587334g(13a) -346752945g(12a) + 856466800g(11a) -1707518505g(10a) + 2792018460g(9a) -3782772510g(8a) + 4272439864g(7a) -4034058435g(6a) + 3183962012g(5a) -2093648095g(4a) + 1138708494g(3a) - (22!/2)g(2a) + 22!(9109)g(a) = 0 \quad (2.9)$$

$\forall a \in A$. Letting a = 6a and b = a in (1.1), one gets

$$g(17a) -22g(16a) + 231g(15a) -1540g(14a) + 7315g(13a) -26334g(12a) + 74613g(11a) -170544g(10a) + 319770g(9a) - 497420g(8a) + 646646g(7a) - 705432g(6a) + 646647g(5a) -497442g(4a) + 320001g(3a) -172084g(2a) -22!g(a) = 0 \quad (2.10)$$

$\forall a \in A$. Multiplying (2.10) by 26334, and then subtracting (2.9) from the resulting equation, we arrive at

$$74613g(16a) - 1470942g(15a) + 13803405g(14a) - 82045876g(13a) + 346726611g(12a) -1108391942g(11a) + 2783587191g(10a) - 5628804720g(9a) + 9316285770g(8a) - 12756335900g(7a) + 14542787850g(6a) -13844840090g(5a) + 11005989540g(4a) - 7288197840g(3a) - (22!/2)g(2a) + 22!(35443)g(a) = 0 \quad (2.11)$$

$\forall a \in A$. Changing a = 5a and b = a in (1.1), one gets

$$g(16a) -22g(15a) + 231g(14a) -1540g(13a) + 7315g(12a) - 26334g(11a) + 74613g(10a) - 170544g(9a) + 319770g(8a) -497420g(7a) + 646647g(6a) - 705454g(5a) + 646877g(4a) - 498960g(3a) + 327085g(2a) -22!g(a) = 0 \quad (2.12)$$

$\forall a \in A$. Multiplying (2.12) by 74613, and then subtracting (2.11) from the resultingequation, we arrive at

$$170544g(15a) - 3432198g(14a) + 32858144g(13a) - 199067484g(12a) + 856466800g(11a) -2783512578g(10a) + 7095994750g(9a) -14542713240g(8a) + 24357662560g(7a) -33705484760g(6a) + 38791199210g(5a) -37259444060g(4a)$$

$$+ 29940704640g(3a) - (22!/2)g(2a) + 22!(110056)g(a) = 0 \quad (2.13)$$

$\forall a \in A$. Changing a = 4a and b = a in (1.1), one gets

$$g(15a) - 22g(14a) + 231g(13a) - 1540g(12a) + 7315g(11a) - 26334g(10a) + 74613g(9a) - 170544g(8a) + 319771g(7a) - 497442g(6a) + 646877g(5a) - 706972g(4a) + 653961g(3a) - 523754g(2a) - 22!g(a) = 0 \quad (2.14)$$

$\forall a \in A$. Multiplying (2.14) by 170544, and then subtracting (2.13) from the resulting equation, we arrive at

$$319770g(14a) - 6537520g(13a) + 63570276g(12a) - 391062560g(11a) + 1707593118g(10a) - 5628804720g(9a) + 14542542700g(8a) - 30177362860g(7a) + 51130263690g(6a) - 71529791870g(5a) + 83310388740g(4a) - 81588420140g(3a) + 5.620003638 \times 10^{20}g(2a) - 22!(280600)g(a) = 0 \quad (2.15)$$

$\forall a \in A$. Letting a = 3a and b = a in (1.1), one gets

$$g(14a) - 22g(13a) + 231g(12a) - 1540g(11a) + 7315g(10a) - 26334g(9a) + 74614g(8a) - 170566g(7a) + 320001g(6a) - 498960g(5a) + 653961g(4a) - 731766g(3a) + 721259g(2a) - 22!g(a) = 0 \quad (2.16)$$

$\forall a \in A$. Multiplying (2.16) by 319770, and then subtracting (2.15) from the resulting equation, we arrive at

$$497420g(13a) - 10296594g(12a) + 101383240g(11a) - 631524432g(10a) + 2792018460g(9a) - 9316776084g(8a) + 24364526960g(7a) - 51196456080g(6a) + 88022647330g(5a) - 125806720300g(4a) + 152408393700g(3a) - 5.620003641\times10^{20}g(2a) + 22!(600370)g(a) = 0 \quad (2.17)$$

$\forall a \in A$. Changing a = 2a and b = a in (1.1), one obtains

$$g(13a) - 22g(12a) + 231g(11a) - 1540g(10a) + 7316g(9a) - 26356g(8a) + 74844g(7a) - 172084g(6a) + 327085g(5a) - 523754g(4a) + 721259g(3a) - 875976g(2a) - 22!g(a) = 0 \quad (2.18)$$

$\forall a \in A$. Multiplying (2.18) by 497420, and then subtracting (2.17) from the resulting equation, we arrive at

$$646646g(12a) - 13520780g(11a) + 134502368g(10a) - 847106260g(9a) + 3793225436g(8a) - 12864375520g(7a) + 34401567200g(6a) - 74675973370g(5a) + 134718994400g(4a) - 206360258100g(3a) - 5.620003636\times10^{20}g(2a) + 22!(1097790)g(a) = 0 \quad (2.19)$$

$\forall a \in A$. Doing a = a and b = a in (1.1),one gets

$$g(12a) - 22g(11a) + 232g(10a) - 1562g(9a) + 7546g(8a) - 27874g(7a) + 81928g(6a) - 196878g(5a) + 394383g(4a) - 667964g(3a) + 966416g(2a) - 22!g(a) = 0 \quad (2.20)$$

$\forall a \in A$. Multiplying (2.20) by 646646, and then subtracting (2.19) from the resulting equation, we arrive at

$$705432g(11a) - 15519504g(10a) + 162954792g(9a) - 1086365280g(8a) + 5160235080g(7a) - 18576846290g(6a) + 52634397820g(5a) - 120307195000g(4a) + 225575990600g(3a) - 5.620003642\times10^{20}g(2a) + 22!(1744436)g(a) =0 \quad (2.21)$$

$\forall a \in A$. Changing a = 0 and b = a in (1.1), we have

$$g(11a) - 22g(10a) + 231g(9a) - 1540g(8a) + 7315g(7a) - 26334g(6a) + 74613g(5a) - 170544g(4a) + 319770g(3a) - 497420g(2a) - (22!/2)\, g(a) = 0 \quad (2.22)$$

$\forall a \in A$. Multiplying (2.22) by 705432, and then subtracting (2.21) from the resulting equation, we arrive at

$$5.620003639\times10^{20}g(2a) + 2.357200374\times10^{27}g(a) = 0 \quad (2.23)$$

$\forall a \in A$. On Simplification, we arrive at

$$41944304^{-1}g(2a) - g(a) = 0 \quad (2.24)$$

III. Stability of the functional equation (1.1) in Multi-Banach Spaces

Theorem 3.1. Let P be an linear space and let $((Q^k,\|.\|_k)$: K $\in\mathbb{N})$ be a multi-Banach space. Suppose that δ is a non-negativereal number and g: $P\rightarrow Q$ be a function fulfills

$$\sup_{k\in\mathbb{N}} \|(\mathcal{R}g(a_1, b_1), \ldots, \mathcal{R}g(a_k, b_k))\|_k \leq \delta \quad (3.1)$$

$\forall a_1, \ldots, a_k, b_1, \ldots, b_k \in P$. Then there exists a unique Viginti duo function $D_{22}: P \to Q$ such that

$$\sup_{k\in\mathbb{N}} \|(g(a_1) - D_{22}(a_1), \ldots, g(a_k) - D_{22}(a_k))\|_k \leq 4194305\delta/(22!\times 4194303) \quad (3.2)$$

$\forall a_i \in P$, where i = 1, 2, …, k

Proof. Taking $a_i = 0$ and changing $b_i$by $2a_i$ in (3.1), and also dividing by 2 in the resulting equation, we arrive at

$$\begin{aligned}&\sup_{k\in\mathbb{N}}\|(g(22a_1) - 22g(20a_1) + 231g(18a_1) - 1540g(16a_1) + 7315g(14a_1) - 6334g(12a_1)\\&+ 74613g(10a_1) - 170544g(8a_1) + 319770g(6a_1) - 497420g(4a_1) - 5.620003639\times 10^{20}g(2a_1),\\&\ldots,\\&g(22a_k) - 22g(20a_k) + 231g(18a_k) - 1540g(16a_k) + 7315g(14a_k) - 26334g(12a_k) +\\&74613g(10a_k)\\&- 170544g(8a^k) + 319770g(6a^k) - 497420g(4a^k) - 5.620003639\times 10^{20}g(2a^k))\| \leq \delta/2 \quad (3.3)\end{aligned}$$

$\forall a_1, \ldots, a_k \in P$. Plugging $a_1, \ldots, a_k$ into $11a_1, \ldots, 11a_k$ and replacing $b_1, b_2, \ldots, b_k$ by $a_1, \ldots, a_k$ in (3.1), we then have

$$\begin{aligned}&\sup_{k\in\mathbb{N}} \|(g(22a_1) - 22g(21a_1) + 231g(20a_1) - 1540g(19a_1) + 7315g(18a_1) - 26334g(17a_1)\\&+74613g(16a_1) - 170544g(15a_1) + 319770g(14a_1) - 497420g(13a_1) + 646646g(12a_1)\\&- 705432g(11a_1) + 646646g(10a_1) - 497420g(9a_1) + 319770g(8a_1) - 170544g(7a_1)\\&+74613g(6a_1) - 26334g(5a_1) + 7315g(4a_1) - 1540g(3a_1) + 231g(2a_1) - 22!g(a_1), \ldots,\\&g(22a_k) - 22g(21a_k) + 231g(20a_k) - 1540g(19a_k) + 7315g(18a_k) - 26334g(17a_k)\\&+74613g(16a_k) - 170544g(15a_k) + 319770g(14a_k) - 497420g(13a_k) + 646646g(12a_k)\\&- 705432g(11a_k) + 646646g(10a_k) - 497420g(9a_k) + 319770g(8a_k) - 170544g(7a_k)\\&+74613g(6a_k) - 26334g(5a_k) + 7315g(4a_k) - 1540g(3a_k) + 231g(2a_k) - 22!g(a_k))\| \leq \delta \quad (3.4)\end{aligned}$$

forall $a_1, \ldots, a_k \in P$. Combining from (3.3) and (3.4), we have

$$\begin{aligned}&\sup_{k\in\mathbb{N}} \|(22g(21a_1) - 253g(20a_1) + 1540g(19a_1) - 7084g(18a_1) + 26334g(17a_1)\\&- 76153g(16a_1)\\&+ 170544g(15a_1) - 312455g(14a_1) + 497420g(13a_1) - 672980g(12a_1) + 705432g(11a_1)\\&- 572033g(10a_1) + 497420g(9a_1) - 490314g(8a_1) + 170544g(7a_1) - 245157g(6a_1) +\\&26334g(5a_1)\\&- 504735g(4a_1) + 1540g(3a_1), \ldots, 22g(21a_k) - 253g(20a_k) + 1540g(19a_k) - 7084g(18a_k)\\&+ 26334g(17a_k) - 76153g(16a_k) + 170544g(15a_k) - 312455g(14a_k) + 497420g(13a_k)\\&- 672980g(12a_k) + 705432g(11a_k) - 572033g(10a_k) + 497420g(9a_k) - 490314g(8a_k)\\&+ 170544g(7a_k) - 245157g(6a_k) + 26334g(5a_k) - 504735g(4a_k) + 1540g(3a_k))\| \leq 3\delta/2\end{aligned}$$

forall $a_1, \ldots, a_k \in P$. Switching $a_1, \ldots, a_k$ into $10a_1, \ldots, 10a_k$ and replacing $b_1, \ldots, b_k$ by $a_1, \ldots, a_k$

in (3.1) and taking into consideration the fact that g; is an even function, we then have

$$\sup_{k\in\mathbb{N}}\|(g(21a_1) - 22g(20a_1) + 231g(19a_1) - 1540g(18a_1) + 7315g(17a_1) - 26334g(16a_1) +74613g(15a_1) - 170544g(14a_1) + 319770g(13a_1) - 497420g(12a_1) + 646646g(11a_1) -705432g(10a_1) + 646646g(9a_1) -497420g(8a_1) + 319770g(7a_1) -170544g(6a_1) +74613g(5a_1) - 26334g(4a_1) + 7315g(3a_1) -1540g(2a_1) -22!g(a_1), \ldots, g(21a_k) -22g(20a_k) + 231g(19a_k) -1540g(18a_k) + 7315g(17a_k) -26334g(16a_k) +74613g(15a_k) -170544g(14a_k) + 319770g(13a_k) -497420g(12a_k) + 646646g(11a_k)-705432g(10a_k) + 646646g(9a_k) - 497420g(8a_k) + 319770g(7a_k) - 170544g(6a_k) +74613g(5a_k) - 26334g(4a_k) + 7315g(3a_k) - 1540g(2a_k) -22!g(a_k))\| \leq \delta \quad (3.6)$$

for all $a_1, \ldots, a_k \in P$. Multiplying by 22 on both sides of (3.6),then it follows from (3.5) and the resulting equation one gets

$$\sup_{k\in\mathbb{N}} \|(231g(20a_1) - 3542g(19a_1) + 26796g(18a_1) - 134596g(17a_1)+503195g(16a_1) - 1470942g(15a_1) + 3439513g(14a_1) - 6537520g(13a_1)+10270260g(12a_1) - 13520780g(11a_1) + 14947471g(10a_1) - 13728792g(9a_1)+10452926g(8a_1) - 6864396g(7a_1) + 3997125g(6a_1) - 1615152g(5a_1) + 74613g(4a_1) - 159390g(3a_1) -(22!/2)g(2a_1) + 22!23g(a_1), \ldots, (231g(20a_k) - 3542g(19a_k) + 26796g(18a_k) - 134596g(17a_k) +503195g(16a_k) - 1470942g(15a_k) + 3439513g(14a_k) - 6537520g(13a_k) +10270260g(12a_k) - 13520780g(11a_k) + 14947471g(10a_k) - 13728792g(9a_k) +10452926g(8a_k) - 6864396g(7a_k) + 3997125g(6a_k) - 1615152g(5a_k) + 74613g(4a_k) - 159390g(3a_k) -(22!/2)g(2a_k) + 22!23g(a_k))\| \leq 47\delta/2 \quad (3.7)$$

for all $a_1, \ldots, a_k \in P$. Plugging $a_1, \ldots, a_k$ into $9a_1, \ldots, 9a_k$ and replacing $b_1, \ldots, b_k$ by $a_1, \ldots, a_k$ in (3.1) and again taking into consideration the fact that g is an even function, we arrive at

$$\sup_{k\in\mathbb{N}} \|(g(20a_1) -22g(19a_1) + 231g(18a_1) -1540g(17a_1) + 7315g(16a_1) -26334g(15a_1) + 74613g(14a_1) -170544g(13a_1) + 319770g(12a_1)- 497420g(11a_1)+646646g(10a_1) -705432g(9a_1) + 646646g(8a_1) -497420g(7a_1) + 319770g(6a_1)- 170544g(5a_1) + 74613g(4a_1)- 26334g(3a_1) + 7316g(2a_1) -22!g(a_1), \ldots, g(20a_k) - 22g(19a_k) + 231g(18a_k) - 1540g(17a_k) + 7315g(16a_k)-26334g(15a_k) + 74613g(14a_k)$$

$-170544g(13a_k) + 319770g(12a_k) - 497420g(11a_k) + 646646g(10a_k) - 705432g(9a_k) + 646646g(8a_k)$
$-497420g(7a_k) + 319770g(6a_k) - 170544g(5a_k) + 74613g(4a_k) - 26334g(3a_k) + 7316g(2a_k) - 22!g(a_k))\| \leq \delta$ (3.8)

forall $a_1, \ldots, a_k \in P$. Multiplying by 231 on both sides of (3.8),then it follows from (3.7) and the resulting equation, we reach at

$\sup_{k\in\mathbb{N}} \|(1540g(19a_1) - 26565g(18a_1) + 221144g(17a_1) - 1186570g(16a_1) + 4612212g(15a_1)$
$-13796090g(14a_1) + 32858144g(13a_1) - 63596610g(12a_1) + 101383240g(11a_1)$
$-134427755g(10a_1) + 149226000g(9a_1) - 138922300g(8a_1) + 108039624g(7a_1)$
$- 69869745g(6a_1) + 37780512g(5a_1) - 17160990g(4a_1) + 5923764g(3a_1) - (22!/2)g(2a_1)$
$+ 22!(254)g(a_1), \ldots, 1540g(19a_k) - 26565g(18a_k) + 221144g(17a_k) - 1186570g(16a_k)$
$+ 4612212g(15a_k) - 13796090g(14a_k) + 32858144g(13a_k) - 63596610g(12a_k)$
$+ 101383240g(11a_k) - 134427755g(10a_k) + 149226000g(9a_k) - 138922300g(8a_k)$
$+ 108039624g(7a_k) - 69869745g(6a_k) + 37780512g(5a_k) - 17160990g(4a_k)$
$+5923764g(3a_k) - (22!/2)g(2a_k) + 22!(254)g(a_k))\| \leq 509\delta/2$
(3.9)

forall $a_1, \ldots, a_k \in P$. Switching $a_1, \ldots, a_k$ into $8a_1, \ldots, 8a_k$ and changing $b_1, \ldots, b_k$ by $a_1, \ldots, a_k$ in (3.1) and using the fact that g is an even function, we arrive at

$\sup_{k\in\mathbb{N}}\|(g(19a_1) - 22g(18a_1) + 231g(17a_1) - 1540g(16a_1) + 7315g(15a_1) - 26334g(14a_1) + 74613g(13a_1)$
$- 170544g(12a_1) + 319770g(11a_1) - 497420g(10a_1) + 646646g(9a_1) - 705432g(8a_1) + 646646g(7a_1)$
$-497420g(6a_1) + 319770g(5a_1) - 170544g(4a_1) + 74614g(3a_1) - 26356g(2a_1) - 22!g(a_1), \ldots,$
$g(19a_k) - 22g(18a_k) + 231g(17a_k) - 1540g(16a_k) + 7315g(15a_k) - 26334g(14a_k)$
$+74613g(13a_k)$
$-170544g(12a_k) + 319770g(11a_k) - 497420g(10a_k) + 646646g(9a_k) - 705432g(8a_k) + 646646g(7a_k)$
$-497420g(6a_k) + 319770g(5a_k) - 170544g(4a_k) + 74614g(3a_k) - 26356g(2a_k) - 22!g(a_k))\|$
$\leq \delta$ (3.10)

forall $a_1, \ldots, a_k \in P$. Multiplying by 1540 on both sides of (3.10),then it follows from (3.9) and the resulting equation, one gets

$\sup_{k\in\mathbb{N}} \|(7315g(18a_1) - 134596g(17a_1) + 1185030g(16a_1) - 6652888g(15a_1)$
$+26758270g(14a_1)$
$-82045876g(13a_1) + 199041150g(12a_1) - 391062560g(11a_1) + 631599045g(10a_1)$

$-846608840g(9a_1) + 947442980g(8a_1) - 887795216g(7a_1) + 696157055g(6a_1)$
$-454665288g(5a_1)$
$+ 245476770g(4a_1) - 108981796g(3a_1) - (22!/2)g(2a_1) + 22!(1794)g(a_1), \ldots,$
$7315g(18a_k)$
$-134596g(17a_k) + 1185030g(16a_k) - 6652888g(15a_k) + 26758270g(14a_k)$
$-82045876g(13a_k)$
$+ 199041150g(12a_k) - 391062560g(11a_k) + 631599045g(10a_k) - 846608840g(9a_k)$
$+947442980g(8a_k) - 887795216g(7a_k) + 696157055g(6a_k) - 454665288g(5a_k)$
$+245476770g(4a_k) - 108981796g(3a_k) - (22!/2)g(2a_k) + 22!(1794)g(a_k))\| \leq 3589\delta/2$
(3.11)

On simplification, we get

$$\sup_{k\in\mathbb{N}} \|(g(2a_1)/4194304 - g(a_1), \ldots, g(2a_k)/4194304 - g(a_k))\| \leq 4194305\delta/(22!\times 4194304) \quad (3.12)$$

forall $a_1, \ldots, a_k \in P$. Let Ψ $\{\ell = P\to Q \mid \ell(0) = 0\}$ and introduce the generalized metric d defined on $\Psi$by

$$d(\ell,m) = \inf\{\Psi\in[0,\infty] \mid \sup_{k\in\mathbb{N}} \|\ell(a_1) - m(a_1), \ldots, \ell(a_k) - m(a_k)\|_k \leq \Psi, \forall\, a_1, \ldots, a_k \in P\}$$

Then itis easy to show that (Ψ,d) is a generalized complete metric space (see [8]).
We define an operator $\mathcal{J}: \Psi\to\Psi$ by

$$\mathcal{J}\ell(a) = \ell(2a)/2^{22}, \quad a \in P$$

We assert that $\mathcal{J}$ is a strictly contractive operator. Givenℓ, m $\in\Psi$, let $\Psi\in [0, \infty]$ be an arbitrary constant with $d(\ell, m)$. From the definition if follows that

$$\sup_{k\in\mathbb{N}} \|\ell(a_1) - m(a_1), \ldots, \ell(a_k) - m(a_k)\|_k \leq \Psi, \forall\, a_1, \ldots, a_k \in P.$$

Therefore,

$$\sup_{k\in\mathbb{N}} \|\mathcal{J}\ell(a_1) - \mathcal{J}m(a_1), \ldots, \mathcal{J}\ell(a_k) - \mathcal{J}m(a_k)\|_k \leq \Psi/2^{22}, \forall\, a_1, \ldots, a_k \in P$$

for all $a_1, \ldots, a_k \in P$. Hence, it holds that

$$d(\mathcal{J}\ell,\mathcal{J}m) \leq d(\ell,m)/2^{22}$$

$\forall \ell$, m $\in\Psi$.
This means that $\mathcal{J}$ is strictly contractive operator on Ψ with the Lipschitz constant$L = 1/2^{22}$.
By (3.12), we have

$$d(\mathcal{J}g, g) \leq 4194305\delta/(22!\times 4194304).$$

Applying the Theorem 2.2 in [9]we deduce the existence of a fixed point of $\mathcal{J}$ that is the existence of mappingD_{22}: $P \to Q$ such that

$$D_{22}(2a) = 2^{22} D_{22}(a), \quad \forall a \in P.$$

Moreover, we have $d(\mathcal{J}^n g, D_{22}) \to 0$; which implies

$$D_{22}(a) = \lim_{n\to\infty} \mathcal{J}^n g(a) = \lim_{n\to\infty} g(2^n a)/2^{22n}$$

for all $a \in P$.
Also,

$$d(g, D_{22}) \leq d(\mathcal{J} g, g)/(1 - L)$$

implies the inequality

$$d(g, D_{22}) \leq 4194305\delta/(22! \times 4194303).$$

Doing $a_1 = \ldots = a_k = 2^n a$ and $b_1 = \ldots = b_k = 2^n b$ in (3.1) and dividing by 2^{22n}, we have

$$\|\mathcal{R} D_{22}(a, b)\| = \lim_{n\to\infty} \|\mathcal{R} g(2^n a, 2^n b)\| \leq \lim_{n\to\infty} 2^{-22n} = 0$$

for all $a, b \in P$.
Hence the proof is complete.

Corollary 3.2. Let P be a linear space, and let $(Q^n, \|.\|_n)$ be a multi-Banach space. Let $\theta > 0$; $0 < p < 22$ and g: $P \to Q$ be a mapping satisfying $g(0) = 0$

$$\sup_{k\in\mathbb{N}} \|\mathcal{R}\, g(a_1, b_1), \ldots, \mathcal{R}\, g(a_k, b_k)\|_k \leq \theta(\|a_1\|^p + \|b_1\|^p, \ldots, \|a_k\|^p + \|b_k\|^p) \quad (3.13)$$

for all $a_1, \ldots, a_k, b_1, \ldots, b_k \in P$. Then there exists a unique mapping D_{22}: $P \to Q$ such that

$$\sup_{k\in\mathbb{N}} \|g(a_1) - D_{22}(a_1), \ldots, g(a_k) - D_{22}(a_k)\|_k \leq \Delta_A(\|a_1\|^p, \ldots, \|a_k\|^p)/(2^{22} - 2^p) \quad (3.14)$$

where

$$\Delta_A = (2/22!)\, \theta\, (2^{p-1} + (11^p + 1) + 22(10^p + 1) + 231(9^p + 1)$$
$$+ 1540(8^p + 1) + 7315(7^p + 1) + 26334(6^p + 1) + 74613(5^p + 1)$$
$$+ 170544(4^p + 1) + 319770(3^p + 1) + 497420(2^p + 1) + 1646008).$$

Proof. Proof is similar to that of Theorem 3.1 replacing δ by $\theta(\|a_1\|^p + \|b_1\|^p, \ldots, \|a_k\|^p + \|b_k\|^p)$, we arrive the result.

Corollary 3.3. Let P be a linear space, and let $(Q^n, \|.\|_n)$ be a multi-Banach space. Let $\theta > 0$, $0 < r + s = p < 24$ and ϕ: $P \to Q$ be a mapping satisfying $g(0) = 0$

$$\sup_{k\in\mathbb{N}} \|Rg(a_1, b_1), \ldots, Rg(a_k, b_k)\|_k \leq \theta(\|a_1\|^r . \|b_1\|^s, \ldots, \|a_k\|^r . \|b_k\|^s) \quad (3.15)$$

for all $a_1, \dots, a_k, b_1, \dots, b_k \in P$. Then there exists a unique mapping $D_{22}: P \to Q$ such that

$$\sup_{k\in\mathbb{N}} \|g(a_1)-D_{22}(a_1), \dots, g(a_k)-D_{22}(a_k)\|_k \le \Delta_{AA}(\|a_1\|^{r+s}, \dots, \|a_k\|^{r+s})/(2^{22}-2^p) \tag{3.16}$$

where

$$\Delta_{AA} = (2/22!)\,\theta(11^r + 22 \times 10^r + 231 \times 9^r + 1540 \times 8^r + 7315 \times 7^r + 26334 \times 6^r + 74613 \times 5^r + 170544 \times 4^r + 319770 \times 3^r + 497420 \times 2^r + 1646008).$$

Proof. Proof is similar to that of Theorem 3.1 replacing δ by $\theta(\|a_1\|^r.\|b_1\|^s, \dots, \|a_k\|^r.\|b_k\|^s)$, we arrive the result.

References

[1] T. Aoki, On the stability of the linear transformation in Banach spaces, *J. Math. Soc. Japan.2 (1950)*, 64-66.

[2] M. Arunkumar, A. Bodaghi, J. M. Rassias and E. Sathiya, The general solution and approximations of a decic type functional equation in various normed spaces, *J. Chungcheong Math. Soc., 29 (2) (2016)*, 287-328.

[3] P. Gavruta,A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings, *J. Math. Anal. Appl. 184, no. 3 (1994)*, 431-436.

[4] J.B. Diaz and B. Margolis, A fixed point theorem of the alternative, for contraction on a generalized complete metric space, *Bulletin of the American Mathematical Society, vol. 74 (1968)*, 305-309.

[5] Dales, H.G and Moslehian, Stability of mappings on multi-normed spaces, *Glasgow Mathematical Journal, 49 (2007)*, 321-332.

[6] K. W. Jun and H. M. Kim, The generalized Hyers-Ulam-Russias stability of a cubic functional equation, *J. Math. Anal. Appl. 274, no. 2 (2002)*, 867-878.

[7] D. H. Hyers, On the stability of the linear functional equation, *Proc. Natl. Acad. Sci. USA, 27 (1941)*, 222-224.

[8] D. Mihet and V. Radu, On the stability of the additive Cauchy functional equation in random normed spaces, *Journal of mathematical Analysis and Applications, 343 (2008)*, 567-572.

[9] V.Radu, The fixed point alternative and the stability of functional equations, *Fixed Point Theory 4 (2003)*, 91-96.

[10] K. Ravi, J.M. Rassias and B.V. Senthil Kumar, Ulam-Hyers stability of undecic functional equation in quasi-beta normed spaces fixed point method, *Tbilisi Mathematical Science 9 (2) (2016)*, 83-103.

[11] K. Ravi, J.M. Rassias, S. Pinelas and S.Suresh, General solution and stability of Quattuordecic functional equation in quasi beta normed spaces, *Advances in pure mathematics 6 (2016)*, 921-941.

[12] J. M. Rassias, Solution of the Ulam stability problem for cubic mappings, *GlasnikMatematic Ser. III 36 (56) (2001)*, 63-72.

[13] J. M. Rassias, Solution of the Ulam stability problem for quartic mappings, *GlasnikMatematicki Series III, 34 (2) (1999)*, 243-252.

[14] J. M. Rassias and M. Eslamian, Fixed points and stability of nonic functional equation inquasi- -normed spaces, *Cont. Anal. Appl. Math.3, No. 2 (2015)*, 293-309.

[15] Th. M. Rassias, On the stability of the linear mapping in Banach spaces, *Proc. Amer. Math. Soc., 72 (2) (1978)*, 297-300.

[16] F. Skof, Proprietlocali e approssimazione di operatori, *Rend. Sem. Mat. Fis. Milano, 53 (1983)*, 113-129.

[17] S.M. Ulam, *A collection of the mathematical problems*, Interscience, New York, (1960).

[18] T. Z. Xu, J. M. Rassias, and W. X. Xu,A generalized mixed quadratic-quartic functional equation, *Bulletin Malay. Math. Sci. Soc. 35, No 3 (2012)*, 633-649.

[19] T. Z. Xu, J. M. Rassias, M. J. Rassias and W. X. Xu, A fixed point approach to the stability of quintic and sextic functional equations in quasi- -normed spaces, *J. Inequal. Appl. (2010), Article ID 423231*, 23 pages doi:10.1155/2010/423231.

[20] Younghong Shen and Wei Chen, On the stability of septic and octic functional equations, *J.Computational Analysis and Applications, 18 (2015)*, 277-290.

R. Murali. "General Solution and a Fixed Point Approach to the Ulam Hyers Stability of Viginti Duo Functional Equation in Multi-Banach Spaces." IOSR Journal of Mathematics (IOSR-JM) , vol. 13, no. 4, 2017, pp. 48–59.