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# Classical Statistical Distribution Involving Multivariable H-Function

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***Abstract:*** *In this paper an attempt has been made to present a unified theory of the classical statistical distributions associated with generalized beta and gamma distributions of one variable. The probability density function is taken in terms of the Multivariable H-function. In particular, the characteristic function and the distribution function are investigated.*

## I. Introduction

In probability theory, several authors have studied a large number of statistical distributions from time to time. For example, Mathai and Saxena [5] introduced a general hypergeometric distribution, whose probability density function involves a hypergeometric functions ${}_2F_1$. Again, Shrivastava and Singhal [6] studied another general class of distributions, whose probability density function involves the H-function. It may be readily seen that the distributions, considered by Mathai and Saxena [5] and all other well-known classical statistical distributions, such as the generalized beta and gamma distributions, the exponential distribution, the generalized F-distribution, students t-distribution, the normal distribution, etc. can be derived as specialized or confluent cases of the class of distributions, considered by Shrivastava and Singhal [6]. More recently, Exton [4] considered the family of distributions which have the probability density function in terms of the product of several generalized hypergeometric functions ${}_pF_q$.

In an attempt to present a further generalization of the probability distributions studied by Shrivastava and Singhal [6], Exton [4] etc. here we introduce and study a general family of statistical probability distributions involving the Multivariable H-function.

Since the Multivariable H-function includes almost all the special functions, therefore it can define a very general class of probability model. Thus all the classical statistical distributions mentioned here and elsewhere, will be the special cases of our findings.

The multivariable H–function given in [7] is defined as follows:

$$H[z_1, \dots, z_r] = H^{0,n:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix}(a_j;\alpha_j',\dots,\alpha_j^{(r)})_{1,p}:(c_j';\gamma_j')_{1,p_1};\dots\dots;(c_j^{(r)};\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta_j',\dots,\beta_j^{(r)})_{1,q}:(d_j';\delta_j')_{1,q_1};\dots\dots;(d_j^{(r)};\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\dots\int_{L_r}\phi_1(\xi_1)\dots\phi_r(\xi_r)\psi(\xi_1,\dots,\xi_r)z_1^{\xi_1}\dots z_r^{\xi_r}d\xi_1\dots d\xi_r \quad (1)$$

where $\omega = \sqrt{(-1)}$,

$$\psi(\xi_1,\dots,\xi_r) = \frac{\prod_{j=1}^{n}\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i)}{\prod_{j=n+1}^{p}\Gamma(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i)\prod_{j=1}^{q}\Gamma(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}\xi_i)}$$

$$\phi_i(\xi_i) = \frac{\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-\delta_j^{(i)}\xi_i)\prod_{j=1}^{n_i}\Gamma(1-c_j^{(i)}+\gamma_j^{(i)}\xi_i)}{\prod_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}+\delta_j^{(i)}\xi_i)\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-\gamma_j^{(i)}\xi_i)}$$

In (3), i in the superscript (i) stands for the number of primes, e.g., $b^{(1)} = b'$, $b^{(2)} = b''$, and so on; and an empty product is interpreted as unity.

Suppose, as usual, that the parameters

$a_j$, $j = 1, \dots, p$; $c_j^{(i)}$, $j = 1, \dots, p_i$;
$b_j$, $j = 1, \dots, q$; $d_j^{(i)}$, $j = 1, \dots, q_i$; $\forall i \in \{1, \dots, r\}$

are complex numbers and the associated coefficients

$\alpha_j^{(i)}$, j = 1, ...., p; $\gamma_j^{(i)}$, j = 1,....,$p_i$;

$\beta_j^{(i)}$, j = 1, ...., q; $\delta_j^{(i)}$, j = 1,....,$q_i$; $\forall i \in \{1,....,r\}$

positive real numbers such that the left of the contour. Also

$$V_i = \sum_{j=1}^{p} \alpha_j^{(i)} + \sum_{j=1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} - \sum_{j=1}^{q_i} \delta_j^{(i)} \leq 0 \qquad (2)$$

$$\Omega_i = -\sum_{j=n+1}^{p} \alpha_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m_i+1}^{q_i} \delta_j^{(i)} + \sum_{j=1}^{n_i} \gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i} \gamma_j^{(i)} > 0 \qquad (3)$$

where the integral n, p, q, $m_i$, $n_i$, $p_i$ and $q_i$ are constrained by the inequalities $p \geq n \geq 0$, $q \geq 0$, $q_i \geq m_i \geq 1$ and $p_i \geq n_i \geq 1$ $\forall$ i $\in$ {1, 2, ..., r) and the inequalities in (2) hold for suitably restricted values of the complex variables $z_1, \ldots, z_r$. The sequence of parameters in (1) are such that none of the poles of the integrand coincide, that is, the poles of the integrand in (1) are simple. The contour $L_i$ in the complex $\xi_i$plane is of the Mellin-Barnes type which runs from $-i\infty$ to $+i\infty$ with indentations, if necessary, to ensure that all the poles of $\Gamma(d_j^{(i)} - \delta_j^{(i)}\xi_i)$, j = 1,..., $m_i$ are separated from those of $\Gamma(1 - c_j^{(i)} + \gamma_j^{(i)}\xi_i)$, i = 1, ...., $n_i$.

In the present investigation we require the following formulae:

From Erdlyi [3]:

$$\Gamma(z) = \int_0^\infty e^{-t} t^{s-1} dt, \qquad (4)$$

Re(z) > 0;

$$B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt, \qquad (5)$$

Re(x) > 0, Re(y) > 0.

## II. Probability Density Functions:

This section deals with certain classical statistical distributions associated with beta (or finite) and gamma (or infinite) distributions of one variate. The probability density function is taken in terms of the Multivariable H-function. First we find the probability density function.

Let a density function be defined by

$$f(x) = K x^{\sigma-1} (1-x)^{\rho-1} (1+bx)^{-\sigma-\rho} \times H^{0,\ n\ :(m_1,n_1);\ldots\ldots;(m_r,n_r)}_{p,\ q\ :(p_1,q_1);\ldots\ldots;(p_r,q_r)} \left[ \begin{matrix} z_1 x^{\sigma_1} (1-x)^{\rho_1} (1+bx)^{-\sigma_1-\rho_1} \\ \vdots \\ z_r \end{matrix} \right] \qquad (6)$$

$0 \leq x \leq 1$ for finite distribution or generalized beta distribution, and f(x) = 0, elsewhere. If f(x) is a probability density function, then it should satisfy the relation

$$\int_{-\infty}^{\infty} f(x)\, dx = \int_{-1}^{1} f(x)\, dx = 1 \qquad (7)$$

Putting the value of f(x) given by (6) in (7) and evaluating the resulting integral with the help of Mellin-Barnes contour integral for the Multivariable H-function given in (1) and the well known definition of beta function (5) (sec, e.g. [2], p.9, Eq. (1)), we find that.

$$K^{-1} = (1+b)^{-\sigma} H^{0,\ n\ :(m_1,n_1+2);\ldots\ldots;(m_r,n_r)}_{p,\ q\ :(p_1+2,q_1+1);\ldots\ldots;(p_r,q_r)} \left[ \begin{matrix} z_1 (1+b)^{-\sigma_1} \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} \ldots\ldots:(1-\sigma,\sigma_1),(1-\rho,\rho_1),\ldots:\ldots\ldots \\ \ldots\ldots:\ldots\ldots,(1-\sigma-\rho,\sigma_1+\rho_1):\ldots\ldots \end{matrix} \right], \qquad (8)$$

provided that Re (σ) + $\sigma_1$> 0, Re (ρ) + $\rho_1$< 0 and $|\arg(z_k)| < \frac{1}{2} V_k \pi, \forall k \in [1, \ldots, r]$, where $V_k$ is given in (2).

Again let

$$f(x) = Q e^{-\xi x} x^{\sigma-1} \mathrm{H}^{0,\ n\ :(m_1,n_1);\ldots\ldots;(m_r,n_r)}_{p,\ q\ :(p_1,q_1);\ldots\ldots;(p_r,q_r)} \begin{bmatrix} z_1 x^{\sigma_1} \\ \vdots \\ z_r \end{bmatrix}, \qquad (9)$$

where $0 < x < \infty$, Re $(\xi) > 0$, Re $(\sigma) > 0$, Re $(\sigma) + \sigma_1 > 0$ and

$$Q^{-1} = \xi^{-\sigma} \mathrm{H}^{0,\ n\ :(m_1,n_1+1);\ldots\ldots;(m_r,n_r)}_{p,\ q\ :(p_1+1,q_1);\ldots\ldots;(p_r,q_r)} \left[ \begin{matrix} z_1 \xi^{-\sigma_1} \\ \vdots \\ z_r \end{matrix} \right| \begin{matrix} \ldots\ldots;(1-\sigma,\sigma_1),\ldots;\ldots\ldots \\ \ldots\ldots;\ldots\ldots;\ldots\ldots \end{matrix} \Bigg], \qquad (10)$$

then f(x) will be a probability density function for infinite distribution or generalized gamma distribution. By virtue of (7), we easily get

$$\int_0^\infty e^{-\xi x} x^{\sigma-1} \mathrm{H}^{0,\ n\ :(m_1,n_1);\ldots\ldots;(m_r,n_r)}_{p,\ q\ :(p_1,q_1);\ldots\ldots;(p_r,q_r)} \begin{bmatrix} z_1 x^{\sigma_1} \\ \vdots \\ z_r \end{bmatrix} dx = 1 \qquad (11)$$

Now, on evaluating the above integral with the help of (1) and the definition of gamma function (4) (sec, e.g. [2], p.1 Eq (1)), we obtain the expression for Q given by (10).

**Remark1:** Complex values of the parameters hold little interest for statistics, but (6) and (9) can still define probability models (Carlson [1, p.59]).

**Remark2:** Since Re $(\xi) > 0$ in (9), the convergence of the integral (11) at its upper limit of integration can be generalized under the conditions stated already.

## III. The Characteristic Function:

The characteristic function denoted by □(t) may be represented as $<e^{itx}>$, where the angle brackets denote "mathematical expectation". We may thus write characteristic function as

$$\phi(t) = <e^{itx}> = \int_{-\infty}^{\infty} e^{itx} f(x) dx \qquad (12)$$

The characteristic function for finite distribution, when f(x) is given by (6), is

$$\phi(t) = K \sum_{r=0}^{\infty} (it)^r (1/r!)\, \mathrm{H}^{0,\ n\ :(m_1,n_1+2);\ldots\ldots;(m_r,n_r)}_{p,\ q\ :(p_1+2,q_1+1);\ldots\ldots;(p_r,q_r)} \left[ \begin{matrix} z_1 (1+b)^{-\sigma_1} \\ \vdots \\ z_r \end{matrix} \right| \begin{matrix} \ldots\ldots;(1-\sigma-r,\sigma_1),(1-\rho,\rho_1),\ldots;\ldots\ldots \\ \ldots\ldots;\ldots\ldots,(1-\sigma-r-\rho,\sigma_1+\rho_1);\ldots\ldots \end{matrix} \Bigg], \qquad (13)$$

where K is given by (8).

Also, the characteristic function for infinite distribution, when f(x) is given by (9), is

$$\phi(t) = Q(\xi - it)^{-\sigma}.\mathrm{H}^{0,\ n\ :(m_1,n_1+1);\ldots\ldots;(m_r,n_r)}_{p,\ q\ :(p_1+1,q_1);\ldots\ldots;(p_r,q_r)} \left[ \begin{matrix} z_1 (\xi - it)^{-\sigma_1} \\ \vdots \\ z_r \end{matrix} \right| \begin{matrix} \ldots\ldots;(1-\sigma,\sigma_1),\ldots;\ldots\ldots \\ \ldots\ldots;\ldots\ldots;\ldots\ldots \end{matrix} \Bigg], \qquad (14)$$

where Q is given by (10).

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