

Bipolar Intuitionistic M Fuzzy Group and Anti M Fuzzy Group.

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Abstract: The concept of a Bipolar intuitionistic M fuzzy group is a new algebraic structure of a bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group are defined and some related properties are investigated. The purpose of the study is to implement the fuzzy set theory and group theory of bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group. The relation between of a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group are established.

Keywords: M fuzzy group, anti M fuzzy group, bipolar intuitionistic fuzzy set, bipolar intuitionistic M fuzzy group, bipolar intuitionistic anti M fuzzy group.

I. Introduction

The concept of fuzzy sets was initiated by L.A. Zadeh [13] then it has become a vigorous area of research in engineering, medical science, graph theory. Rosenfeld [12] gave the idea of fuzzy subgroups. Bipolar valued fuzzy sets was introduced by K.M. Lee [5] are an extension of fuzzy sets whose membership degree range is enlarged from the interval $[0,1]$ to $[-1,1]$. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree $(0,1]$ indicates that elements somewhat satisfy the property and the membership degree $[-1,0)$ indicates that elements somewhat satisfy the implicit counter property. The author W. R. Zhang [15] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. The author Mourad Oqla [6] commenced the concept of an intuitionistic anti M fuzzy group. Chakrabarthy and R.Nanda [1] investigated note on union and intersection of intuitionistic fuzzy sets. P.S. Das, A. Rajeshkumar [2,3] were analyzed fuzzy groups and level subgroups. R. Muthuraj [8,9] introduced the concept of bipolar fuzzy subgroup of a M fuzzy group and bipolar anti M fuzzy group. He was introduced the notion of an image and pre-image of a bipolar fuzzy subset of a bipolar fuzzy subgroup of a group and also discuss some of its properties of bipolar M fuzzy subgroup under M homomorphism and M anti homomorphism. We discuss some of its properties with bipolar intuitionistic M fuzzy subgroup of M fuzzy group and anti M fuzzy group are established under M homomorphism and M anti homomorphism.

II. Preliminaries

In this paper $G = (G, *)$ is a finite groups, e is the identity element of G , and xy mean $x*y$ the fundamental definitions that will be used in the sequel.

Definition.2.1 Let G be a non empty set, A bipolar intuitionistic fuzzy set (IFS) A in G is an object of the form $A = \{x, \mu_A^+(x), \mu_A^-(x), \nu_A^+(x), \nu_A^-(x) / x \in G\}$ where $\mu_A^+ : G \rightarrow [0,1]$ and $\nu_A^+ : G \rightarrow [0,1]$, $\mu_A^- : G \rightarrow [-1,0]$ and $\nu_A^- : G \rightarrow [-1,0]$ is called degree of positive membership, degree of negative membership and the degree of positive non membership, degree of negative non membership respectively.

Definition.2.2 [8] Let G be a group. A bipolar valued intuitionistic fuzzy set (IFS) A of G is called a bipolar intuitionistic fuzzy subgroup of G , if for all $x, y \in G$

- i) $\mu_A^+(xy) \geq \min(\mu_A^+(x), \mu_A^+(y))$ and $\nu_A^+(xy) \leq \max(\nu_A^+(x), \nu_A^+(y))$
- ii) $\mu_A^-(xy) \leq \max(\mu_A^-(x), \mu_A^-(y))$ and $\nu_A^-(xy) \geq \min(\nu_A^-(x), \nu_A^-(y))$
- iii) $\mu_A^+(x^{-1}) = \mu_A^+(x)$, $\mu_A^-(x^{-1}) = \mu_A^-(x)$ and $\nu_A^+(x^{-1}) = \nu_A^+(x)$, $\nu_A^-(x^{-1}) = \nu_A^-(x)$.

Example.2.3

$$\mu_A^+(x) = \begin{cases} 0.7 & \text{if } x = 1 \\ 0.6 & \text{if } x = -1 \\ 0.4 & \text{if } x = i, -i \end{cases} ; \nu_A^+(x) = \begin{cases} 0.2 & \text{if } x = 1 \\ 0.3 & \text{if } x = -1 \\ 0.5 & \text{if } x = i, -i \end{cases} \quad \text{and} \quad \mu_A^-(x) = \begin{cases} -0.8 & \text{if } x = 1 \\ -0.5 & \text{if } x = -1 \\ -0.3 & \text{if } x = i, -i \end{cases} ; \nu_A^-(x) = \begin{cases} -0.1 & \text{if } x = 1 \\ -0.4 & \text{if } x = -1 \\ -0.6 & \text{if } x = i, -i \end{cases}$$

Definition.2.4 [7] Let G be a group. A bipolar valued IFS (or) bipolar IFS A of G is called a bipolar intuitionistic anti fuzzy subgroup of G , if for all $x, y \in G$

- i) $\mu_A^+(xy) \leq \max(\mu_A^+(x), \mu_A^+(y))$ and $\nu_A^+(xy) \geq \min(\nu_A^+(x), \nu_A^+(y))$
- ii) $\mu_A^-(xy) \geq \min(\mu_A^-(x), \mu_A^-(y))$ and $\nu_A^-(xy) \leq \max(\nu_A^-(x), \nu_A^-(y))$
- iii) $\mu_A^+(x^{-1}) = \mu_A^+(x), \mu_A^-(x^{-1}) = \mu_A^-(x)$ and $\nu_A^+(x^{-1}) = \nu_A^+(x), \nu_A^-(x^{-1}) = \nu_A^-(x)$.

Example.2.5

$$\mu_A^+(x) = \begin{cases} 0.4 & \text{if } x = 1 \\ 0.6 & \text{if } x = -1 \\ 0.7 & \text{if } x = i, -i \end{cases} ; \nu_A^+(x) = \begin{cases} 0.5 & \text{if } x = 1 \\ 0.3 & \text{if } x = -1 \\ 0.2 & \text{if } x = i, -i \end{cases} \quad \text{and} \quad \mu_A^-(x) = \begin{cases} -0.3 & \text{if } x = 1 \\ -0.5 & \text{if } x = -1 \\ -0.8 & \text{if } x = i, -i \end{cases} ; \nu_A^-(x) = \begin{cases} -0.6 & \text{if } x = 1 \\ -0.4 & \text{if } x = -1 \\ -0.1 & \text{if } x = i, -i \end{cases}$$

Definition.2.6 Let G be an M group and A be a bipolar intuitionistic fuzzy subgroup of G , then A is called a bipolar intuitionistic M fuzzy group of G , if for all $x \in G$ and $m \in M$ then,

- i) $\mu_A^+(mx) \geq \mu_A^+(x)$ and $\nu_A^+(mx) \leq \nu_A^+(x)$. ii) $\mu_A^-(mx) \leq \mu_A^-(x)$ and $\nu_A^-(mx) \geq \nu_A^-(x)$.

Example.2.7

Consider $1 \in M$

$$\mu_A^+(x) = \begin{cases} 0.7 & \text{if } x = 1 \\ 0.6 & \text{if } x = -1 \\ 0.4 & \text{if } x = i, -i \end{cases} ; \nu_A^+(x) = \begin{cases} 0.2 & \text{if } x = 1 \\ 0.3 & \text{if } x = -1 \\ 0.5 & \text{if } x = i, -i \end{cases} \quad \text{and} \quad \mu_A^-(x) = \begin{cases} -0.8 & \text{if } x = 1 \\ -0.5 & \text{if } x = -1 \\ -0.3 & \text{if } x = i, -i \end{cases} ; \nu_A^-(x) = \begin{cases} -0.1 & \text{if } x = 1 \\ -0.4 & \text{if } x = -1 \\ -0.6 & \text{if } x = i, -i \end{cases}$$

Definition.2.8 Let G be an M group and A be a bipolar intuitionistic anti fuzzy subgroup of G , then A is called a bipolar intuitionistic anti M fuzzy group of G , if for all $x \in G$ and $m \in M$ then,

- i) $\mu_A^+(mx) \leq \mu_A^+(x)$ and $\nu_A^+(mx) \geq \nu_A^+(x)$. ii) $\mu_A^-(mx) \geq \mu_A^-(x)$ and $\nu_A^-(mx) \leq \nu_A^-(x)$.

Example.2.9

Consider $1 \in M$

$$\mu_A^+(x) = \begin{cases} 0.4 & \text{if } x = 1 \\ 0.6 & \text{if } x = -1 \\ 0.7 & \text{if } x = i, -i \end{cases} ; \nu_A^+(x) = \begin{cases} 0.5 & \text{if } x = 1 \\ 0.3 & \text{if } x = -1 \\ 0.2 & \text{if } x = i, -i \end{cases} \quad \text{and} \quad \mu_A^-(x) = \begin{cases} -0.3 & \text{if } x = 1 \\ -0.5 & \text{if } x = -1 \\ -0.8 & \text{if } x = i, -i \end{cases} ; \nu_A^-(x) = \begin{cases} -0.6 & \text{if } x = 1 \\ -0.4 & \text{if } x = -1 \\ -0.1 & \text{if } x = i, -i \end{cases}$$

Theorem.2.10 If A and B are bipolar intuitionistic M fuzzy group of G , then $A \cap B$ is a bipolar intuitionistic M fuzzy group of G .

Proof Consider $m \in M$ and $x \in A \cap B$ implies $x \in A, x \in B$

Consider $\mu_{A \cap B}^+(mx) = \min(\mu_A^+(mx), \mu_B^+(mx)) \geq \min(\mu_A^+(x), \mu_B^+(x)) = \mu_{A \cap B}^+(x)$.

Therefore $\mu_{A \cap B}^+(mx) \geq \mu_{A \cap B}^+(x)$.

Consider $\nu_{A \cap B}^+(mx) = \max(\nu_A^+(mx), \nu_B^+(mx)) \leq \max(\nu_A^+(x), \nu_B^+(x)) = \nu_{A \cap B}^+(x)$.

Therefore $\nu_{A \cap B}^+(mx) \leq \nu_{A \cap B}^+(x)$.

Consider $\mu_{A \cap B}^-(mx) = \max(\mu_A^-(mx), \mu_B^-(mx)) \leq \max(\mu_A^-(x), \mu_B^-(x)) = \mu_{A \cap B}^-(x)$.

Therefore $\mu_{A \cap B}^-(mx) \leq \mu_{A \cap B}^-(x)$.

Consider $\nu_{A \cap B}^-(mx) = \min(\nu_A^-(mx), \nu_B^-(mx)) \geq \min(\nu_A^-(x), \nu_B^-(x)) = \nu_{A \cap B}^-(x)$.

Therefore $\nu_{A \cap B}^-(mx) \geq \nu_{A \cap B}^-(x)$.

Therefore $A \cap B$ is a bipolar intuitionistic M fuzzy group of G

Theorem.2.11 If A is a bipolar intuitionistic M fuzzy group of G, then $\overline{\overline{A}} = A$ is also a bipolar intuitionistic M fuzzy group of G.

Proof Let $m \in M$ and $x \in A$

Consider $\mu_{\overline{\overline{A}}}^+(mx) = \nu_{\overline{\overline{A}}}^+(mx) = \mu_{\overline{\overline{A}}}^+(mx) \geq \mu_{\overline{\overline{A}}}^+(x)$. Therefore $\mu_{\overline{\overline{A}}}^+(mx) \geq \mu_{\overline{\overline{A}}}^+(x)$.

Consider $\nu_{\overline{\overline{A}}}^+(mx) = \mu_{\overline{\overline{A}}}^+(mx) = \nu_{\overline{\overline{A}}}^+(mx) \leq \nu_{\overline{\overline{A}}}^+(x)$. Therefore $\nu_{\overline{\overline{A}}}^+(mx) \leq \nu_{\overline{\overline{A}}}^+(x)$.

Consider $\mu_{\overline{\overline{A}}}^-(mx) = \nu_{\overline{\overline{A}}}^-(mx) = \mu_{\overline{\overline{A}}}^-(mx) \leq \mu_{\overline{\overline{A}}}^-(x)$. Therefore $\mu_{\overline{\overline{A}}}^-(mx) \leq \mu_{\overline{\overline{A}}}^-(x)$.

Consider $\nu_{\overline{\overline{A}}}^-(mx) = \mu_{\overline{\overline{A}}}^-(mx) = \nu_{\overline{\overline{A}}}^-(mx) \geq \nu_{\overline{\overline{A}}}^-(x)$. Therefore $\nu_{\overline{\overline{A}}}^-(mx) \geq \nu_{\overline{\overline{A}}}^-(x)$.

Therefore $\overline{\overline{A}} = A$ is a bipolar intuitionistic M fuzzy group of G.

Theorem.2.12 Union of any two bipolar intuitionistic M fuzzy group is also a bipolar intuitionistic M fuzzy group if either is contained in the other.

Proof Let A and B be a bipolar intuitionistic M fuzzy group of G .

To prove that $A \cup B$ is a bipolar intuitionistic M fuzzy group of G if $A \subseteq B$ (or) $B \subseteq A$

If $A \subseteq B \Rightarrow A \cup B = B$ (or) $B \subseteq A \Rightarrow A \cup B = A$

Let $m \in M$ & $x \in A \cup B$

Consider $\mu_{A \cup B}^+(mx) = \max(\mu_A^+(mx), \mu_B^+(mx)) \geq \max(\mu_A^+(x), \mu_B^+(x)) = \mu_{A \cup B}^+(x)$.

Therefore $\mu_{A \cup B}^+(mx) \geq \mu_{A \cup B}^+(x)$.

Consider $\nu_{A \cup B}^+(mx) = \min(\nu_A^+(mx), \nu_B^+(mx)) \leq \min(\nu_A^+(x), \nu_B^+(x)) = \nu_{A \cup B}^+(x)$.

Therefore $\nu_{A \cup B}^+(mx) \leq \nu_{A \cup B}^+(x)$.

Consider $\mu_{A \cup B}^-(mx) = \min(\mu_A^-(mx), \mu_B^-(mx)) \leq \min(\mu_A^-(x), \mu_B^-(x)) = \mu_{A \cup B}^-(x)$.

Therefore $\mu_{A \cup B}^-(mx) \leq \mu_{A \cup B}^-(x)$.

Consider $\nu_{A \cup B}^-(mx) = \max(\nu_A^-(mx), \nu_B^-(mx)) \geq \max(\nu_A^-(x), \nu_B^-(x)) = \nu_{A \cup B}^-(x)$.

Therefore $\nu_{A \cup B}^-(mx) \geq \nu_{A \cup B}^-(x)$.

Hence union of any two bipolar intuitionistic M fuzzy group is also a bipolar intuitionistic M fuzzy group if either is contained in the other.

Theorem.2.13 If A is a bipolar intuitionistic anti M fuzzy group of G, then $\overline{A} = A$ is also a bipolar intuitionistic anti M fuzzy group of G.

Proof

Consider $m \in M$ and $x \in A$

Consider $\mu_{\overline{A}}^+(mx) = \nu_A^+(mx) = \mu_A^+(mx) \leq \mu_A^+(x)$. Therefore $\mu_{\overline{A}}^+(mx) \leq \mu_{\overline{A}}^+(x)$.

Consider $\nu_{\overline{A}}^+(mx) = \mu_A^+(mx) = \nu_A^+(mx) \geq \nu_A^+(x)$. Therefore $\nu_{\overline{A}}^+(mx) \geq \nu_{\overline{A}}^+(x)$.

Consider $\mu_{\overline{A}}^-(mx) = \nu_A^-(mx) = \mu_A^-(mx) \geq \mu_A^-(x)$. Therefore $\mu_{\overline{A}}^-(mx) \geq \mu_{\overline{A}}^-(x)$.

Consider $\nu_{\overline{A}}^-(mx) = \mu_A^-(mx) = \nu_A^-(mx) \leq \nu_A^-(x)$. Therefore $\nu_{\overline{A}}^-(mx) \leq \nu_{\overline{A}}^-(x)$.

Therefore $\overline{A} = A$ is a bipolar intuitionistic anti M fuzzy group of G.

Theorem.2.14 Union of any two bipolar intuitionistic anti M fuzzy group is also a bipolar intuitionistic anti M fuzzy group if either is contained in the other.

Proof

Let A and B be a bipolar intuitionistic anti M fuzzy group of G. To prove that $A \cup B$ is also a bipolar intuitionistic anti M fuzzy group of G if $A \subseteq B$ (or) $B \subseteq A$

If $A \subseteq B \Rightarrow A \cup B = B$ (or) $B \subseteq A \Rightarrow A \cup B = A$.

Consider $m \in M$ and $x \in A \cup B$.

Consider $\mu_{A \cup B}^+(mx) = \max(\mu_A^+(mx), \mu_B^+(mx)) \leq \max(\mu_A^+(x), \mu_B^+(x)) = \mu_{A \cup B}^+(x)$.

Therefore $\mu_{A \cup B}^+(mx) \leq \mu_{A \cup B}^+(x)$.

Consider $\nu_{A \cup B}^+(mx) = \min(\nu_A^+(mx), \nu_B^+(mx)) \geq \min(\nu_A^+(x), \nu_B^+(x)) = \nu_{A \cup B}^+(x)$.

Therefore $\nu_{A \cup B}^+(mx) \geq \nu_{A \cup B}^+(x)$.

Consider $\mu_{A \cup B}^-(mx) = \min(\mu_A^-(mx), \mu_B^-(mx)) \geq \min(\mu_A^-(x), \mu_B^-(x)) = \mu_{A \cup B}^-(x)$.

Therefore $\mu_{A \cup B}^-(mx) \geq \mu_{A \cup B}^-(x)$.

Consider $\nu_{A \cup B}^-(mx) = \max(\nu_A^-(mx), \nu_B^-(mx)) \leq \max(\nu_A^-(x), \nu_B^-(x)) = \nu_{A \cup B}^-(x)$.

Therefore $\nu_{A \cup B}^-(mx) \leq \nu_{A \cup B}^-(x)$.

Therefore union of any two bipolar intuitionistic anti M fuzzy group is also a bipolar intuitionistic anti M fuzzy group if either is contained in the other.

III. Some Result Based On Bipolar Intuitionistic M Fuzzy Group And Anti M Fuzzy Group Of G.

Theorem.3.1 Let μ and ν be a bipolar intuitionistic fuzzy subset of an M fuzzy group then $\mu = (\mu^+, \mu^-)$ is a bipolar intuitionistic M fuzzy group of G if and only if $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic anti M fuzzy group of G.

Proof Let $\mu = (\mu^+, \mu^-)$ be a bipolar intuitionistic M fuzzy group of G. To prove $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic anti M fuzzy group of G.

$$\begin{aligned} \text{i) } \mu^+(xy) \geq \min\{\mu^+(x), \mu^+(y)\} &\Leftrightarrow 1 - \nu^+(xy) \geq \min\{1 - \nu^+(x), 1 - \nu^+(y)\} \\ &\Leftrightarrow \nu^+(xy) \leq 1 - \min\{1 - \nu^+(x), 1 - \nu^+(y)\} \\ &\Leftrightarrow \nu^+(xy) \leq \max\{\nu^+(x), \nu^+(y)\}. \end{aligned}$$

$$\text{Therefore } \mu^+(xy) \geq \min\{\mu^+(x), \mu^+(y)\} \Leftrightarrow \nu^+(xy) \leq \max\{\nu^+(x), \nu^+(y)\}.$$

$$\begin{aligned} \text{ii) } \mu^-(xy) \leq \max\{\mu^-(x), \mu^-(y)\} &\Leftrightarrow -1 - \nu^-(xy) \leq \max\{-1 - \nu^-(x), -1 - \nu^-(y)\} \\ &\Leftrightarrow \nu^-(xy) \geq -1 - \max\{-1 - \nu^-(x), -1 - \nu^-(y)\} \\ &\Leftrightarrow \nu^-(xy) \geq \min\{\nu^-(x), \nu^-(y)\}. \end{aligned}$$

$$\text{Therefore } \mu^-(xy) \leq \max\{\mu^-(x), \mu^-(y)\} \Leftrightarrow \nu^-(xy) \geq \min\{\nu^-(x), \nu^-(y)\}.$$

$$\begin{aligned} \text{iii) } \mu^+(x^{-1}) = \mu^+(x) &\Leftrightarrow 1 - \nu^+(x^{-1}) = 1 - \nu^+(x) \Leftrightarrow \nu^+(x^{-1}) = \nu^+(x). \text{ and} \\ \mu^-(x^{-1}) = \mu^-(x) &\Leftrightarrow -1 - \nu^-(x^{-1}) = -1 - \nu^-(x) \Leftrightarrow \nu^-(x^{-1}) = \nu^-(x). \end{aligned}$$

$$\text{iv) } \mu^+(mx) \geq \mu^+(x) \Leftrightarrow 1 - \nu^+(mx) \leq 1 - \nu^+(x) \Leftrightarrow \nu^+(mx) \leq \nu^+(x).$$

$$\text{Therefore } \mu^+(mx) \geq \mu^+(x) \text{ and } \nu^+(mx) \leq \nu^+(x).$$

$$\text{v) } \mu^-(mx) \leq \mu^-(x) \Leftrightarrow -1 - \nu^-(mx) \geq -1 - \nu^-(x) \Leftrightarrow \nu^-(mx) \geq \nu^-(x).$$

$$\text{Therefore } \mu^-(mx) \leq \mu^-(x) \text{ and } \nu^-(mx) \geq \nu^-(x).$$

Therefore $\mu = (\mu^+, \mu^-)$ is a bipolar intuitionistic M fuzzy group of G if and only if $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic anti M fuzzy group of G.

Definition.3.2 [9] Let f and g be a mapping from a group G_1 to a group G_2 . Let $\mu = (\mu^+, \mu^-)$ and $\phi = (\phi^+, \phi^-)$ and $\nu = (\nu^+, \nu^-), \psi = (\psi^+, \psi^-)$ are bipolar intuitionistic fuzzy subset in G_1 and G_2 respectively, then the image $f(\mu)$ and $g(\nu)$ is a bipolar intuitionistic fuzzy subset is defined by $f(\mu) = (f(\mu)^+, f(\mu)^-)$ and $g(\nu) = (g(\nu)^+, g(\nu)^-)$ of G_2 for all $u, v \in G_2$.

$f(\mu^+)(u) = \max\{\mu^+(x); x \in f^{-1}(u)\}$ if $f^{-1}(u) \neq \emptyset, 0$ and $g(v^+)(v) = \min\{v^+(x); x \in g^{-1}(v)\}$ if $g^{-1}(v) \neq \emptyset, 0$ and $f(\mu^-)(u) = \min\{\mu^-(x); x \in f^{-1}(u)\}$, if $f^{-1}(u) \neq \emptyset, 0$ $g(v^-)(v) = \max\{v^-(x); x \in g^{-1}(v)\}$ if $g^{-1}(v) \neq \emptyset, 0$. The preimage $f^{-1}(\phi)$ is under f and $g^{-1}(\psi)$ is under g is defined by the bipolar intuitionistic fuzzy subset of G_1 for all $x \in G_1$, $(f^{-1}(\phi)^+)(x) = \phi^+(f(x))$; $(f^{-1}(\phi)^-)(x) = \phi^-(f(x))$ and $(g^{-1}(\psi)^+)(x) = \psi^+(g(x))$; $(g^{-1}(\psi)^-)(x) = \psi^-(g(x))$.

Definition.3.3 [8] Let G_1 and G_2 be any two bipolar intuitionistic M groups then the function $f: G_1 \rightarrow G_2$ and $g: G_1 \rightarrow G_2$ is said to be an intuitionistic M homomorphism if,

- i) $f(xy) = f(x) f(y)$ for all $x, y \in G_1$
- ii) $f(mx) = m f(x)$ for all $m \in M$ and $x \in G_1$
- iii) $g(xy) = g(x) g(y)$ for all $x, y \in G_1$
- iv) $g(mx) = m g(x)$ for all $m \in M$ and $x \in G_1$.

Definition.3.4 [8] Let G_1 and G_2 be any two bipolar intuitionistic M groups (not necessarily commutative) then the function $f: G_1 \rightarrow G_2$ and $g: G_1 \rightarrow G_2$ is said to be an intuitionistic M anti homomorphism if,

- i) $f(xy) = f(x) f(y)$ for all $x, y \in G_1$
- ii) $f(mx) = m f(x)$ for all $m \in M$ and $x \in G_1$
- iii) $g(xy) = g(x) g(y)$ for all $x, y \in G_1$
- iv) $g(mx) = m g(x)$ for all $m \in M$ and $x \in G_1$.

Theorem.3.5 Let f and g be an intuitionistic M homomorphism from an M fuzzy group of G_1 onto an M fuzzy group of G_2 . If $\mu = (\mu^+, \mu^-)$ is a bipolar intuitionistic M fuzzy group of G_1 then $f(\mu)$ the image of μ under f is a bipolar intuitionistic M fuzzy group of G_2 if and only if $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic anti M fuzzy group of G_1 then $g(\nu)$ is the image of ν under g is a bipolar intuitionistic anti M fuzzy group of G_2 .

Proof Let $f: G_1 \rightarrow G_2$ and $g: G_1 \rightarrow G_2$ be an intuitionistic M homomorphism.

Let $\mu = (\mu^+, \mu^-)$ and $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group of G_1 . To prove a bipolar intuitionistic fuzzy subset $f(\mu) = (f(\mu)^+, f(\mu)^-)$ and $g(\nu) = (g(\nu)^+, g(\nu)^-)$ on G_2 is a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group.

Let $u, v \in G_2$ since f is a intuitionistic M homomorphism and so there exist $x, y \in G_1$ such that $f(x)=u$ & $f(y)=v$ it follows that $xy \in f^{-1}(uv)$. We have to prove that g is an intuitionistic M homomorphism so there exist $x, y \in G_1$ such that $g(x)=u$ & $g(y)=v$ it follows that $xy \in g^{-1}(uv)$.

$$\begin{aligned}
 \text{i) } f(\mu)^+(uv) &= \max\{\mu^+(z) : z = xy \in f^{-1}(uv)\} \\
 &\geq \max\{\min\{\mu^+(x), \mu^+(y)\} : x \in f^{-1}(u), y \in f^{-1}(v)\} \\
 &= \min\{f(\mu)^+(u), f(\mu)^+(v)\}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Therefore } f(\mu)^+(uv) &\geq \min\{f(\mu)^+(u), f(\mu)^+(v)\} \\
 &\Leftrightarrow 1 - g(v)^+(uv) \geq \min\{(1 - g(v)^+(u)), (1 - g(v)^+(v))\} \\
 &\Leftrightarrow g(v)^+(uv) \leq 1 - \min\{(1 - g(v)^+(u)), (1 - g(v)^+(v))\} \\
 &\Leftrightarrow g(v)^+(uv) \leq \max\{g(v)^+(u), g(v)^+(v)\}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence } f(\mu)^+(uv) &\geq \min\{f(\mu)^+(u), f(\mu)^+(v)\} \\
 &\Leftrightarrow g(v)^+(uv) \leq \max\{g(v)^+(u), g(v)^+(v)\}.
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } f(\mu)^-(uv) &= \max\{\mu^-(z) : z = xy \in f^{-1}(uv)\} \\
 &\leq \max\{\max\{\mu^-(x), \mu^-(y)\} : x \in f^{-1}(u), y \in f^{-1}(v)\} \\
 &= \max\{f(\mu)^-(u), f(\mu)^-(v)\}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Therefore } f(\mu)^-(uv) &\leq \max\{f(\mu)^-(u), f(\mu)^-(v)\} \\
 &\Leftrightarrow (-1 - g(v)^-(uv)) \leq \max\{(-1 - g(v)^-(u)), (-1 - g(v)^-(v))\} \\
 &\Leftrightarrow g(v)^-(uv) \geq -1 - \max\{(-1 - g(v)^-(u)), (-1 - g(v)^-(v))\} \\
 &\Leftrightarrow g(v)^-(uv) \geq \min\{g(v)^-(u), g(v)^-(v)\}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence } f(\mu)^-(uv) &\leq \max\{f(\mu)^-(u), f(\mu)^-(v)\} \\
 &\Leftrightarrow g(v)^-(uv) \geq \min\{g(v)^-(u), g(v)^-(v)\}.
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) Now } f(\mu)^+(u^{-1}) &= \max\{\mu^+(x) : x \in f^{-1}(u^{-1})\} = \max\{\mu^+(x^{-1}) : x^{-1} \in f^{-1}(u)\} \\
 &= f(\mu)^+(u)
 \end{aligned}$$

$$\begin{aligned}
 \text{Therefore } f(\mu)^+(u^{-1}) &= f(\mu)^+(u) \Leftrightarrow (1 - g(v)^+(u^{-1})) = (1 - g(v)^+(u)) \\
 &\Leftrightarrow g(v)^+(u^{-1}) = g(v)^+(u).
 \end{aligned}$$

$$\text{Hence } f(\mu)^+(u^{-1}) = f(\mu)^+(u) \Leftrightarrow g(v)^+(u^{-1}) = g(v)^+(u).$$

$$\begin{aligned}
 \text{iv) } f(\mu)^-(u^{-1}) &= \min\{\mu^-(x) : x \in f^{-1}(u^{-1})\} = \min\{\mu^-(x^{-1}) : x^{-1} \in f^{-1}(u)\} \\
 &= f(\mu)^-(u).
 \end{aligned}$$

$$\begin{aligned}
 \text{Therefore } f(\mu)^-(u^{-1}) &= f(\mu)^-(u) \Leftrightarrow (-1 - g(v)^-(u^{-1})) = (-1 - g(v)^-(u)) \\
 &\Leftrightarrow g(v)^-(u^{-1}) = g(v)^-(u).
 \end{aligned}$$

Hence $f(\mu)^-(u^{-1}) = f(\mu)^-(u) \Leftrightarrow g(\nu)^-(u^{-1}) = g(\nu)^-(u)$

Therefore $f(\mu)$ and $g(\nu)$ is a bipolar fuzzy subgroup of G_2 .

v) Let $m \in M$ and $u \in G_2$,

$$f(\mu)^+(mu) \geq \max\{\mu^+(x) : x \in f^{-1}(u)\} = f(\mu^+)(u).$$

Therefore $f(\mu)^+(mu) \geq f(\mu^+)(u) \Leftrightarrow (1 - g(\nu)^+)(mu) \geq (1 - g(\nu)^+)(u)$
 $\Leftrightarrow g(\nu)^+(mu) \leq g(\nu)^+(u).$

Hence $f(\mu)^+(mu) \geq f(\mu^+)(u) \Leftrightarrow g(\nu)^+(mu) \leq g(\nu)^+(u).$

vi) $f(\mu)^-(mu) \leq \min\{\mu^-(x) : x \in f^{-1}(u)\} = f(\mu^-)(u).$

Therefore $f(\mu)^-(mu) \leq f(\mu^-)(u) \Leftrightarrow (-1 - g(\nu)^-)(mu) \leq (-1 - g(\nu)^-)(u)$
 $\Leftrightarrow g(\nu)^-(mu) \geq g(\nu)^-(u).$

Hence $f(\mu)^-(mu) \leq f(\mu^-)(u) \Leftrightarrow g(\nu)^-(mu) \geq g(\nu)^-(u).$

Therefore if μ be a bipolar intuitionistic M fuzzy group of G_1 then $f(\mu)$ is a bipolar intuitionistic M fuzzy group of G_2 if and only if ν be a bipolar intuitionistic anti M fuzzy group of G_1 then $g(\nu)$ be a bipolar intuitionistic anti M fuzzy group of G_2 .

Theorem.3.6 The M homomorphic preimage of a bipolar intuitionistic M fuzzy group of G_2 is a bipolar intuitionistic M fuzzy group of G_1 if and only if M homomorphic preimage of a bipolar intuitionistic anti M fuzzy group of G_2 is a bipolar intuitionistic anti M fuzzy group of G_1 .

Proof Let $f : G_1 \rightarrow G_2$ and $g : G_1 \rightarrow G_2$ be an intuitionistic M homomorphism. let $\phi = (\phi^+, \phi^-)$ is a bipolar intuitionistic M fuzzy group of G_2 and $\psi = (\psi^+, \psi^-)$ is a bipolar intuitionistic anti M fuzzy group of G_2 , to prove a bipolar fuzzy subset $\mu = (\mu^+, \mu^-)$ and $\nu = (\nu^+, \nu^-)$ on G_1 is a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group where $\mu = f^{-1}(\phi)$ & $\nu = g^{-1}(\psi)$

i) Consider $x, y \in G_1$

$$\begin{aligned} (f^{-1}(\phi))^+(xy) &= \phi^+(f(xy)) \\ &\geq \min\{\phi^+(f(x)), \phi^+(f(y))\} \\ &= \min\{(f^{-1}(\phi))^+(x), (f^{-1}(\phi))^+(y)\}. \end{aligned}$$

Therefore $(f^{-1}(\phi))^+(xy) \geq \min\{(f^{-1}(\phi))^+(x), (f^{-1}(\phi))^+(y)\}.$

$$\begin{aligned} \Leftrightarrow (g^{-1}(\psi))^+(xy) &= \psi^+(g(xy)) \leq \max\{\psi^+(g(x)), \psi^+(g(y))\} \\ &= \max\{(g^{-1}(\psi))^+(x), (g^{-1}(\psi))^+(y)\}. \end{aligned}$$

Hence $(f^{-1}(\phi))^+(xy) \geq \min\{(f^{-1}(\phi))^+(x), (f^{-1}(\phi))^+(y)\}$
 $\Leftrightarrow (g^{-1}(\psi))^+(xy) \leq \max\{(g^{-1}(\psi))^+(x), (g^{-1}(\psi))^+(y)\}.$

ii) Let $x, y \in G_1$ $(f^{-1}(\phi))^{-}(xy) = \phi^{-}(f(xy)) \leq \max\{\phi^{-}(f(x)), \phi^{-}(f(y))\}$
 $= \max\{(f^{-1}(\phi))^{-}(x), (f^{-1}(\phi))^{-}(y)\}.$

Therefore $(f^{-1}(\phi))^{-}(xy) \leq \max\{(f^{-1}(\phi))^{-}(x), (f^{-1}(\phi))^{-}(y)\}$
 $\Leftrightarrow (g^{-1}(\psi))^{-}(xy) = \psi^{-}(g(xy))$
 $\geq \min\{\psi^{-}(g(x)), \psi^{-}(g(y))\}$
 $= \min\{(g^{-1}(\psi))^{-}(x), (g^{-1}(\psi))^{-}(y)\}.$

Hence $(f^{-1}(\phi))^{-}(xy) \leq \max\{(f^{-1}(\phi))^{-}(x), (f^{-1}(\phi))^{-}(y)\}$
 $\Leftrightarrow (g^{-1}(\psi))^{-}(xy) \geq \min\{(g^{-1}(\psi))^{-}(x), (g^{-1}(\psi))^{-}(y)\}$

iii) Consider $x \in G_1$

$$\begin{aligned} (f^{-1}(\phi))^+(x^{-1}) &= \phi^+(f(x^{-1})) \\ &= \phi^+(f(x)) \text{ as } \phi \text{ is a bipolar M fuzzy group} \\ &= (f^{-1}(\phi))^+(x). \end{aligned}$$

Therefore $(f^{-1}(\phi))^+(x^{-1}) = (f^{-1}(\phi))^+(x)$
 $\Leftrightarrow (g^{-1}(\psi))^+(x^{-1}) = \psi^+(g(x^{-1}))$
 $= \psi^+(g(x)^{-1})$ as g is an M homomorphism
 $= \psi^+(g(x))$ as ψ is a bipolar anti M fuzzy group
 $= (g^{-1}(\psi))^+(x).$

Hence $(f^{-1}(\phi))^+(x^{-1}) = (f^{-1}(\phi))^+(x) \Leftrightarrow (g^{-1}(\psi))^+(x^{-1}) = (g^{-1}(\psi))^+(x).$

iv) $(f^{-1}(\phi))^{-}(x^{-1}) = \phi^{-}(f(x^{-1}))$
 $= \phi^{-}(f(x))$ as ϕ is a bipolar M fuzzy group
 $= (f^{-1}(\phi))^{-}(x).$

$$\begin{aligned}
 \text{Therefore } (f^{-1}(\phi))^{-}(x^{-1}) &= (f^{-1}(\phi))^{-}(x) \\
 &\Leftrightarrow (g^{-1}(\psi))^{-}(x^{-1}) = \psi^{-}(g(x^{-1})) \\
 &= \psi^{-}(g(x)^{-1}) \text{ as } g \text{ is an M homomorphism} \\
 &= \psi^{-}(g(x)) \text{ as } \psi \text{ is a bipolar anti M fuzzy group} \\
 &= (g^{-1}(\psi))^{-}(x).
 \end{aligned}$$

$$\text{Hence } (f^{-1}(\phi))^{-}(x^{-1}) = (f^{-1}(\phi))^{-}(x) \Leftrightarrow (g^{-1}(\psi))^{-}(x^{-1}) = (g^{-1}(\psi))^{-}(x).$$

$$\begin{aligned}
 \text{v) } (f^{-1}(\phi))^{+}(mx) &= \phi^{+}(f(mx)) \\
 &\geq \phi^{+}(f(x)) \text{ as } \phi \text{ is bipolar M fuzzy group} \\
 &= (f^{-1}(\phi))^{+}(x).
 \end{aligned}$$

$$\begin{aligned}
 \text{Therefore } (f^{-1}(\phi))^{+}(mx) &\geq (f^{-1}(\phi))^{+}(x) \\
 &\Leftrightarrow (g^{-1}(\psi))^{+}(mx) = \psi^{+}(g(mx)) \\
 &= \psi^{+}(mg(x)) \text{ as } g \text{ is an M homomorphism} \\
 &\leq \psi^{+}(g(x)) \text{ as } \psi \text{ is a bipolar anti M fuzzy group} \\
 &= (g^{-1}(\psi))^{+}(x).
 \end{aligned}$$

$$\text{Hence } (f^{-1}(\phi))^{+}(mx) \geq (f^{-1}(\phi))^{+}(x) \Leftrightarrow (g^{-1}(\psi))^{+}(mx) \leq (g^{-1}(\psi))^{+}(x).$$

$$\begin{aligned}
 \text{vi) } (f^{-1}(\phi))^{-}(mx) &= \phi^{-}(f(mx)) \\
 &\leq \phi^{-}(f(x)) \text{ as } \phi \text{ is bipolar M fuzzy group} \\
 &= (f^{-1}(\phi))^{-}(x).
 \end{aligned}$$

$$\begin{aligned}
 \text{Therefore } (f^{-1}(\phi))^{-}(mx) &\leq (f^{-1}(\phi))^{-}(x) \Leftrightarrow (g^{-1}(\psi))^{-}(mx) \\
 &= \psi^{-}(g(mx)) \\
 &= \psi^{-}(mg(x)) \text{ as } g \text{ is an M homomorphism} \\
 &\geq \psi^{-}(g(x)) \text{ as } \psi \text{ is a bipolar anti M fuzzy group} \\
 &= (g^{-1}(\psi))^{-}(x).
 \end{aligned}$$

$$\text{Hence } (f^{-1}(\phi))^{-}(mx) \leq (f^{-1}(\phi))^{-}(x) \Leftrightarrow (g^{-1}(\psi))^{-}(mx) \geq (g^{-1}(\psi))^{-}(x).$$

Hence $f^{-1}(\phi) = \mu$ is a bipolar intuitionistic M fuzzy group of G_1 and $g^{-1}(\psi) = \nu$ is a bipolar intuitionistic anti M fuzzy group of G_1 .

Theorem.3.7 Let f and g be an intuitionistic M anti homomorphism from an M fuzzy group of G_1 onto an M fuzzy group of G_2 . If $\mu = (\mu^+, \mu^-)$ is a bipolar intuitionistic M fuzzy group of G_1 then $f(\mu)$ the image of μ under f is a bipolar intuitionistic M fuzzy group of G_2 if and only if

$\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic anti M fuzzy group of G_1 then $g(\nu)$ the image of ν under g is a bipolar intuitionistic anti M fuzzy group of G_2 .

Proof Let $f : G_1 \rightarrow G_2$ and $g : G_1 \rightarrow G_2$ be an intuitionistic M anti homomorphism and let $\mu = (\mu^+, \mu^-)$ and $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group of G_1 . $\mu^+ : G_1 \rightarrow [0, 1]$ & $\mu^- : G_1 \rightarrow [-1, 0]$ & $\nu^+ : G_1 \rightarrow [0, 1]$ & $\nu^- : G_1 \rightarrow [-1, 0]$ are mappings, to prove a bipolar intuitionistic fuzzy subset $f(\mu) = (f(\mu)^+, f(\mu)^-)$ and $g(\nu) = (g(\nu)^+, g(\nu)^-)$ on G_2 is a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group.

Let $u, v \in G_2$ since f is an intuitionistic M anti homomorphism so there exist $x, y \in G_1$ such that $f(x) = u$ and $f(y) = v$, it follows that $xy \in f^{-1}(uv)$ that g is intuitionistic M anti homomorphism so there exist $x, y \in G_1$ such that $g(x) = u$, $g(y) = v$ which implies $xy \in g^{-1}(uv)$

$$\begin{aligned} \text{i) Let } f(\mu)^+(uv) &\geq \max\{\mu^+(xy) : x \in f^{-1}(u), y \in f^{-1}(v)\} \\ &\geq \max\{\min\{\mu^+(x), \mu^+(y)\} : x \in f^{-1}(u), y \in f^{-1}(v)\} \\ &= \min\{f(\mu)^+(u), f(\mu)^+(v)\}. \end{aligned}$$

$$\begin{aligned} \text{Therefore } f(\mu)^+(uv) &\geq \min\{f(\mu)^+(u), f(\mu)^+(v)\} \\ &\Leftrightarrow (1 - g(\nu)^+)(uv) \geq \min\{(1 - g(\nu)^+)(u), (1 - g(\nu)^+)(v)\} \\ &\Leftrightarrow g(\nu)^+(uv) \leq 1 - \min\{(1 - g(\nu)^+)(u), (1 - g(\nu)^+)(v)\} \\ &\Leftrightarrow g(\nu)^+(uv) \leq \max\{g(\nu)^+(u), g(\nu)^+(v)\}. \end{aligned}$$

$$\begin{aligned} \text{Hence } (f(\mu)^+)(uv) &\geq \min\{f(\mu)^+(u), f(\mu)^+(v)\} \\ &\Leftrightarrow g(\nu)^+(uv) \leq \max\{g(\nu)^+(u), g(\nu)^+(v)\}. \end{aligned}$$

$$\begin{aligned} \text{ii) Let } f(\mu)^-(uv) &\leq \max\{\mu^-(xy) : x \in f^{-1}(u), y \in f^{-1}(v)\} \\ &\leq \max\{\max\{\mu^-(x), \mu^-(y)\} : x \in f^{-1}(u), y \in f^{-1}(v)\} \\ &= \max\{f(\mu)^-(u), f(\mu)^-(v)\}. \end{aligned}$$

$$\begin{aligned} \text{Therefore } f(\mu)^-(uv) &\leq \max\{f(\mu)^-(u), f(\mu)^-(v)\} \\ &\Leftrightarrow (-1 - g(\nu)^-)(uv) \leq \max\{(-1 - g(\nu)^-)(u), (-1 - g(\nu)^-)(v)\} \\ &\Leftrightarrow g(\nu)^-(uv) \geq -1 - \max\{(-1 - g(\nu)^-)(u), (-1 - g(\nu)^-)(v)\} \\ &\Leftrightarrow g(\nu)^-(uv) \geq \min\{g(\nu)^-(u), g(\nu)^-(v)\}. \end{aligned}$$

$$\begin{aligned} \text{Hence } f(\mu)^-(uv) &\leq \max\{f(\mu)^-(u), f(\mu)^-(v)\} \\ &\Leftrightarrow g(\nu)^-(uv) \geq \min\{g(\nu)^-(u), g(\nu)^-(v)\}. \end{aligned}$$

$$\begin{aligned} \text{iii) Consider } f(\mu)^+(u^{-1}) &= \max\{\mu^+(x); x \in f^{-1}(u^{-1})\} \\ &= \max\{\mu^+(x^{-1}); x^{-1} \in f^{-1}(u)\} \\ &= f(\mu)^+(u). \end{aligned}$$

$$\begin{aligned} \text{Therefore } f(\mu)^+(u^{-1}) = f(\mu)^+(u) &\Leftrightarrow (1 - g(v)^+)(u^{-1}) = (1 - g(v)^+)(u) \\ &\Leftrightarrow g(v)^+(u^{-1}) = g(v)^+(u). \end{aligned}$$

$$\text{Hence } f(\mu)^+(u^{-1}) = f(\mu)^+(u) \Leftrightarrow g(v)^+(u^{-1}) = g(v)^+(u).$$

$$\begin{aligned} \text{iv) Consider } f(\mu)^-(u^{-1}) &= \min\{\mu^-(x); x \in f^{-1}(u^{-1})\} = \min\{\mu^-(x^{-1}); x^{-1} \in f^{-1}(u)\} \\ &= (f(\mu)^-)(u). \end{aligned}$$

$$\begin{aligned} \text{Therefore } f(\mu)^-(u^{-1}) = f(\mu)^-(u) &\Leftrightarrow (-1 - g(v)^-)(u^{-1}) = (-1 - g(v)^-)(u) \\ &\Leftrightarrow g(v)^-(u^{-1}) = g(v)^-(u). \end{aligned}$$

$$\text{Hence } f(\mu)^-(u^{-1}) = f(\mu)^-(u) \Leftrightarrow g(v)^-(u^{-1}) = g(v)^-(u).$$

Therefore $f(\mu)$ and $g(v)$ is a bipolar fuzzy subgroup of G_2 .

v) Consider $m \in M$ and $u \in G_2$

$$\begin{aligned} f(\mu)^+(mu) &= \max\{\mu^+(mu); x \in f^{-1}(u)\} \geq \max\{\mu^+(x); x \in f^{-1}(u)\} \\ &= f(\mu)^+(u). \end{aligned}$$

$$\begin{aligned} \text{Therefore } f(\mu)^+(mu) \geq f(\mu)^+(u) &\Leftrightarrow (1 - g(v)^+)(mu) \geq (1 - g(v)^+)(u) \\ &\Leftrightarrow g(v)^+(mu) \leq g(v)^+(u). \end{aligned}$$

$$\text{Hence } f(\mu)^+(mu) \geq f(\mu)^+(u) \Leftrightarrow g(v)^+(mu) \leq g(v)^+(u).$$

vi) Consider $m \in M$ and $u \in G_2$

$$\begin{aligned} f(\mu)^-(mu) &= \min\{\mu^-(mu); x \in f^{-1}(u)\} \leq \min\{\mu^-(x); x \in f^{-1}(u)\} \\ &= f(\mu)^-(u). \end{aligned}$$

$$\begin{aligned} \text{Therefore } f(\mu)^-(mu) \leq f(\mu)^-(u) &\Leftrightarrow (-1 - g(v)^-)(mu) \leq (-1 - g(v)^-)(u) \\ &\Leftrightarrow g(v)^-(mu) \geq g(v)^-(u). \end{aligned}$$

$$\text{Hence } f(\mu)^-(mu) \leq f(\mu)^-(u) \Leftrightarrow g(v)^-(mu) \geq g(v)^-(u).$$

Hence if μ be a bipolar intuitionistic M fuzzy group of G_1 then $f(\mu)$ is a bipolar M fuzzy group of G_2 if and only if v be a bipolar anti M fuzzy group of G_1 then $g(v)$ be a bipolar intuitionistic anti M fuzzy group of G_2 .

IV. Conclusion

The concept of a bipolar intuitionistic M fuzzy group is a new algebraic structure of a bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group are defined and some related properties are investigated. The purpose of the study is to implement the fuzzy set theory and group theory of bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group. The relation between of a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group are established. We hope that our results can also be extended to other algebraic system.

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