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α-Cuts Of Interval-Valued Fuzzy Matrices With Interval-Valued Fuzzy Rows And Columns

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Abstract: *Fuzzy Matrix (FM) is a very important topic of Fuzzy algebra. In FM, the elements belong to the unit interval [0, 1]. When the elements of FM are the subintervals of the unit interval [0,1], then the FM is known as Interval-Valued Fuzzy Matrix [IVFM] . In IVFM, the membership values of rows and columns are crisp ie. Rows and columns are certain. But, in many real life situations they are also uncertain. So to model these type of uncertain problems, a new type of Interval-Valued Fuzzy Matrices (IVFMs) are called Interval-Valued Fuzzy Matrices with Interval-Valued Fuzzy Rows and Columns (IVFMFRCs). In this paper, some new elementary operators on α-cuts of IVFMFRCs are defined. Using these operators, some important theorems are proved.*

Keywords: *α-cut, α-cuts of interval valued fuzzy matrix, Fuzzy matrix, Fuzzy rows and columns, Interval valued fuzzy matrix.*

I. Introduction

Real world decision making problems are very often uncertain or vague in a number of ways. In 1965, Zadeh [9] introduced the concept of fuzzy set theory to meet those problems. In FMs, only the elements are certain. But in many real life situations we observed that rows and columns are uncertain. Fuzzy matrices were introduced by M.G.Thomson [8]. A.K.Shyamal and M.Pal introduced Fuzzy Number Matrices. Two new operators and some properties of fuzzy matrices over the new operators are given in [6]. α-cuts of Triangular Fuzzy Numbers and α-cuts of Triangular Fuzzy Number Matrices are given in [1]. Pal[3] has defined Fuzzy Matrices with Fuzzy Rows and Fuzzy Columns <FMFRCs>. The elements of FMFRCs are non-negative proper fraction. But, when the elements are the subintervals of the unit interval [0,1], then the FM is known as IVFM. In IVFM, the rows and columns are considered as scripts, but we have seen that they may also be uncertain, ie., rows and columns have same membership values. The concept of IVFMs as a generalization of fuzzy matrix was introduced and developed in 2006 by Shyamal and Pal[5] by extending the max-min operation in fuzzy algebra. In these matrices, rows and columns are also fuzzy numbers, ie., unlike Fuzzy Matrices they are also uncertain. In this paper, some new elementary operators on α-cuts of IVFMFRCs are defined. Using these operators, some important theorems are proved.

Preliminaries

Definition 1.1

Some basic operations on interval-valued fuzzy numbers are given below.

Let D denote the set of all subintervals of the interval [0,1]. Let $a = [a^-, a^+]$ and $b = [b^-, b^+]$ be two elements of D. Then

1) $a \oplus b = [a^- + b^- - a^-.b^-, a^+ + b^+ - a^+.b^+]$,

2) $a \ominus b = [a^- \ominus b^-, a^+ \ominus b^+]$, where $a \ominus b = \begin{cases} a & \text{if } a > b \\ 0 & \text{if } a \le b \end{cases}$

3) $a \vee b = [a^-, a^+] \vee [b^-, b^+] = [a^- \vee b^-, a^+ \vee b^+]$. where $a \vee b = \max\{x,y\}$

The operators, " + " and " - " used in extreme right are ordinary addition, subtraction respectively.

Two intervals $[a^-, a^+]$ and $[b^-, b^+]$ are equal if and only if $a^- = b^-$ and $a^+ = b^+$. We denote [0,0] and [1,1] as **0** and **1** respectively.

Definition 1.2 [5]

An **Interval-Valued Fuzzy Matrix** of order m×n is defined as, $A = (a_{ij})_{m\times n}$, where $a_{ij} = [a_{ij}^-, a_{ij}^+]$ is the ijth element of A, represents the membership value. All the elements of IVFM are intervals and they are members of D.

Definition 1.3 [2]

Let $A = [r_A(i)]\,[c_A(j)]\,[a_{ij}]_{m\times n}$ be an **IVFMFRC** of order m×n. Here a_{ij}, i = 1,2….m, j = 1,2,…n represents the ijth element of A, $r_A(i)$, $c_A(j)$ represents the membership values of ith row and jth column respectively for i = 1,2….m , j = 1,2,…n. $c_A(1) c_A(2) \ldots c_A(m)$

Let $A = \begin{matrix} r_A(1) \\ r_A(2) \\ \vdots \\ r_A(n) \end{matrix} \begin{bmatrix} a_{11} a_{12} \dots & a_{1n} \\ a_{21} a_{22} \dots & a_{2n} \\ \dots \dots \dots & \dots \\ a_{m1} a_{m2} \dots & a_{mn} \end{bmatrix}$ be a matrix, where $r_A(i)$, i = 1,2,….m, $c_A(j)$, j = 1,2…n a_{ij}, i = 1,2,…m, j = 1,2….n represent respectively the membership values of rows, columns and elements.

Definition 1.4

Let A = $[r_A(i)]\,[c_A(j)]\,[a_{ij}]_{m\times n}$ and B = $[r_B(i)]\,[c_B(j)]\,[b_{ij}]_{m\times n}$ be two IVMFRCs of order m×n. Then the following operators are defined

1) $A \oplus B = [\,[r_A(i)]\,[c_A(j)]\,[a_{ij}]_{m\times n}] \oplus [\,[r_B(i)][c_B(j)][b_{ij}]_{m\times n}]$
2) $A \vee B = [\,[r_A(i)]\,[c_A(j)]\,[a_{ij}]_{m\times n}] \vee [\,[r_B(i)]\,[c_B(j)]\,[b_{ij}]_{m\times n}\,]$
3) $A \ominus B = [\,[r_A(i)]\,[c_A(j)]\,[a_{ij}]_{m\times n}] \ominus [\,[r_B(i)]\,[c_B(j)]\,[b_{ij}]_{m\times n}]$
4) $A \geq B$ iff $[r_A(i)] \geq [r_B(i)]$, $[c_A(j)] \geq [c_B(j)]$, $[a_{ij}]_{m\times n} \geq [b_{ij}]_{m\times n}$

Definition 1.5

The **Upper α-cut** of an IVFMFRC A = $[r_A(i)]\,[c_A(j)]\,[a_{ij}]_{m\times n}$ is defined as

$A^{(\alpha)} = [r_A^{(\alpha)}(i)][c_A^{(\alpha)}(j)]\,[a_{ij}^{(\alpha)}]_{m\times n}$

Here $r_A(i)$ and $c_A(j)$ represents the membership values of i[th] row and j[th] column respectively for i = 1,2,…m, j = 1,2,….n.

Here, a_{ij} , i = 1,2,…m, j = 1,2…n represents the ij[th] elements of A.

$a_{ij}^{(\alpha)} = [\,a_{ij}^{-(\alpha)}, a_{ij}^{+(\alpha)}\,] = [1,1]$ if $a_{ij}^{-(\alpha)}, a_{ij}^{+(\alpha)} \geq \alpha$
$[0,1]$ if $a_{ij}^{-(\alpha)} < \alpha$ and $a_{ij}^{+(\alpha)} \geq \alpha$
$[0,0]$ if $a_{ij}^{-(\alpha)}, a_{ij}^{+(\alpha)} < \alpha$

$r_A^{(\alpha)}(i) = [\,r_A^{-(\alpha)}(i), r_A^{+(\alpha)}(i)\,] = [1,1]$ if $r_A^{-(\alpha)}(i), r_A^{+(\alpha)}(i) \geq \alpha$
$[0,1]$ if $r_A^{-(\alpha)}(i) < \alpha$ and $r_A^{+(\alpha)}(i) \geq \alpha$
$[0,0]$ if $r_A^{-(\alpha)}(i), r_A^{+(\alpha)}(i) < \alpha$

$c_A^{(\alpha)}(j) = [\,c_A^{-(\alpha)}(j), c_A^{+(\alpha)}(j)\,] = [1,1]$ if $c_A^{-(\alpha)}(j), c_A^{+(\alpha)}(j) \geq \alpha$
$[0,1]$ if $c_A^{-(\alpha)}(j) < \alpha$ and $c_A^{+(\alpha)}(j) \geq \alpha$
$[0,0]$ if $c_A^{-(\alpha)}(j), c_A^{+(\alpha)}(j) < \alpha$

Definition 1.6

The **Lower α-cut** of an IVFMFRC
A = $[r_A(i)]\,[c_A(j)]\,[a_{ij}]_{m\times n}$ is defined as
$A_{(\alpha)} = [r_{A\,(\alpha)}(i)][\,c_{A\,(\alpha)}(j)][\,a_{ij(\alpha)}]_{m\times n}$

Here $r_A(i)$ and $c_A(j)$ represents the membership values of i[th] row and j[th] column respectively for i = 1,2,…m, j = 1,2,….n.

Here, a_{ij} , i = 1,2,…m, j = 1,2…n represents the ij[th] elements of A.

$a_{ij(\alpha)} = [\,a_{ij-(\alpha)}, a_{ij+(\alpha)}] = [\,a_{ij-(\alpha)}, a_{ij+(\alpha)}]$ if $a_{ij-(\alpha)}, a_{ij+(\alpha)} \geq \alpha$
$= [\,0, a_{ij+(\alpha)}\,]$ if $a_{ij-(\alpha)} < \alpha$, $a_{ij+(\alpha)} \geq \alpha$
$= [\,0,0\,]$ if $a_{ij-(\alpha)}, a_{ij+(\alpha)} < \alpha$

$r_{A\,(\alpha)}(i) = [r_{A-(\alpha)}(i), r_{A+(\alpha)}(i)] = [r_{A-(\alpha)}(i), r_{A+(\alpha)}(i)]$ if $r_{A-(\alpha)}(i), r_{A+(\alpha)}(i) \geq \alpha$
$= [\,0, r_{A+(\alpha)}(i)]$ if $r_{A-(\alpha)}(i) < \alpha$, $r_{A+(\alpha)}(i) \geq \alpha$
$= [0,0]$ if $r_{A-(\alpha)}(i)$, $r_{A+(\alpha)}(i) < \alpha$

$c_{A\,(\alpha)}(j) = [c_{A-(\alpha)}(j), c_{A+(\alpha)}(j)] = [c_{A-(\alpha)}(j), c_{A+(\alpha)}(j)]$ if $c_{A-(\alpha)}(j)$, $c_{A+(\alpha)}(j) \geq \alpha$
$= [\,0, c_{A+(\alpha)}(j)\,]$ if $c_{A-(\alpha)}(j) < \alpha, c_{A+(\alpha)}(j) < \alpha$
$= [0,0]$ if $c_{A-(\alpha)}(j)$, $c_{A+(\alpha)}(j) < \alpha$

Example 1.7

Consider the IVFMFRCs as follows

$$A = \begin{array}{c} \\ [0.4,0.8] \\ [0.2,0.7] \\ [0.6,0.7] \end{array} \begin{array}{ccc} [0.6,0.9] & [0.7,1] & [.2,0.5] \\ \hline [0.2,0.7] & [0.2,0.7] & [0.0,0.4] \\ [0.1,0.5] & [0.1,0.6] & [0.0,0.2] \\ [0.3,0.6] & [0.4,0.5] & [0.1,0.3] \end{array}$$

Then by taking α = 0.5, we get

$$A^{\alpha} = \begin{array}{c} \\ [0,1] \\ [0,1] \\ [1,1] \end{array} \begin{array}{ccc} [1,1] & [1,1] & [0,1] \\ \hline [0,1] & [0,1] & [0,0] \\ [0,1] & [0,1] & [0,0] \\ [0,1] & [0,1] & [0,0] \end{array}$$

and

$$A_{\alpha} = \begin{array}{c} \\ [0,0.8] \\ [0,0.7] \\ [0.6,0.7] \end{array} \begin{array}{ccc} [0.6,0.9] & [0.7,1] & [0,0.5] \\ \hline [0,0.7] & [0,0.7] & [0,0] \\ [0,0.5] & [0,0.6] & [0,0] \\ [0,0.6] & [0,0.5] & [0,0] \end{array}$$

2. Operator on IVFMFRCs

Definition 2.1

Let $A = [r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n}$ and $B = [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n}$ be two IVMFRCs, then, A∨B is defined as

$A \vee B = D = [r_D(i)]\ [c_D(j)][d_{ij}]_{m\times n}$

where, $r_D(i) = r_A(i) \vee r_B(i) = [\ r_A^-(i) \vee r_B^-(i)\ ,\ r_A^+(i) \vee r_B^+(i)\ ]$

$c_D(j) = c_A(j) \vee c_B(j) = [\ c_A^-(j) \vee c_B^-(j)\ ,\ c_A^+(j) \vee c_B^+(j)\ ]$

and $d_{ij} = a_{ij} \vee b_{ij} = [\ a_{ij}^- \vee b_{ij}^-\ ,\ a_{ij}^+ \vee b_{ij}^+\ ]$ for all i , j.

Theorem 2.2

If A and B are two IVFMFRCs, then $(A \vee B)^{(\alpha)} = A^{(\alpha)} \vee B^{(\alpha)}$

Proof :

Let $A = [r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n}$ and $B = [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n}$ be two IVMFRCs, then

$A \vee B = D = [r_D(i)]\ [c_D(j)][d_{ij}]_{m\times n}$

where, $r_D(i) = r_A(i) \vee r_B(i) = [\ r_A^-(i) \vee r_B^-(i)\ ,\ r_A^+(i) \vee r_B^+(i)\ ]$

$c_D(j) = c_A(j) \vee c_B(j) = [\ c_A^-(j) \vee c_B^-(j)\ ,\ c_A^+(j) \vee c_B^+(j)\ ]$

and $d_{ij} = a_{ij} \vee b_{ij} = [\ a_{ij}^- \vee b_{ij}^-\ ,\ a_{ij}^+ \vee b_{ij}^+\ ]$ for all i , j.

Here the order of A and B must be equal.

Let E_{ij} and F_{ij} be the ij[th] element of $(A \vee B)^{(\alpha)}$ and $A^{(\alpha)} \vee B^{(\alpha)}$

Therefore, $E_{ij} = (A \vee B)^{(\alpha)}$ and $F_{ij} = A^{(\alpha)} \vee B^{(\alpha)}$

Case 1:

$A \geq B \geq \alpha$

ie., $[r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n} \geq [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n} \geq \alpha$

$\Rightarrow [r_A(i)] \geq [r_B(i)] \geq \alpha\ ,\ [c_A(j)] \geq [c_B(j)] \geq \alpha\ ,\ a_{ij} \geq b_{ij} \geq \alpha$

$E_{ij} = (A \vee B)^{(\alpha)}$

$= \left([r_D(i)][c_D(j)][d_{ij}]_{m\times n}\right)^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)}]_{m\times n}$

$= [r_A^{-(\alpha)}(i), r_A^{+(\alpha)}(i)]\ c_A^{-(\alpha)}(j), c_A^{+(\alpha)}(j)\ ]\ [a_{ij}^{-(\alpha)}, a_{ij}^{+(\alpha)}\ ]$

$= [1,1]\ [1,1]\ [1,1]_{m\times n}$ (2.2.1)

$F_{ij} = A^{(\alpha)} \vee B^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)} \vee \left([r_B(i)][c_B(j)][b_{ij}]_{m\times n}\right)^{(\alpha)}$

$= (\ [r_A^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)}]_{m\times n}\ ) \vee (\ [r_B^{(\alpha)}(i)]\ [c_B^{(\alpha)}(j)]\ [b_{ij}^{(\alpha)}]_{m\times n}\ )$

$= [r_A^{(\alpha)}(i) \vee r_B^{(\alpha)}(i)]\ \ [c_A^{(\alpha)}(j) \vee c_B^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)} \vee b_{ij}^{(\alpha)}]_{m\times n}$

$= [1,1]\ [1,1]\ [1,1]_{m\times n}$ (2.2.2)

From (2.2.1) and (2.2.2) , $(A \vee B)^{(\alpha)} = A^{(\alpha)} \vee B^{(\alpha)}$

Case 2 :

$A \geq \alpha \geq B$

$[r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n} \geq \alpha \geq [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n}$

ie., $r_A(i) \geq \alpha \geq r_B(i)$, $c_A(j) \geq \alpha \geq c_B(j)$, $a_{ij} \geq \alpha \geq b_{ij}$

Let $E_{ij} = (A \vee B)^{(\alpha)}$

$= \left([r_D(i)][c_D(j)][d_{ij}]_{m\times n}\right)^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)}]_{m\times n}$

$= [r_A^{(\alpha)}(i) \vee r_B^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j) \vee c_B^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)} \vee b_{ij}^{(\alpha)}]_{m\times n}$

$= [1,1]\ [1,1]\ [1,1]_{m\times n}$ (2.2.3)

$F_{ij} = A^{(\alpha)} \vee B^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)} \vee \left([r_B(i)][c_B(j)][b_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)}]_{m\times n} \vee [r_B^{(\alpha)}(i)]\ [c_B^{(\alpha)}(j)]\ [b_{ij}^{(\alpha)}]_{m\times n}$

$= [r_A^{(\alpha)}(i) \vee r_B^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j) \vee c_B^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)} \vee b_{ij}^{(\alpha)}]_{m\times n}$

$= [1\vee 0,\ 1\vee 0]\ [1\vee 0,\ 1\vee 0]\ [1\vee 0,\ 1\vee 0]$

$= [1,1]\ [1,1][1,1]$ (2.2.4)

From (2.2.3) and (2.2.4) , $(A \vee B)^{(\alpha)} = A^{(\alpha)} \vee B^{(\alpha)}$

Case 3 :

$\alpha > A > B$

$\alpha > [r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n} \geq [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n}$

$E_{ij} = (A \vee B)^{(\alpha)}$

$= \left([r_D(i)][c_D(j)][d_{ij}]_{m\times n}\right)^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)}]_{m\times n}$

$= [r_A^{(\alpha)}(i) \vee r_B^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j) \vee c_B^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)} \vee b_{ij}^{(\alpha)}]_{m\times n}$

$= [0,0]\ [0,0]\ [0,0]_{m\times n}$ (2.2.5)

$F_{ij} = A^{(\alpha)} \vee B^{(\alpha)}$

$= [r_A^{(\alpha)}(i) \vee r_B^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j) \vee c_B^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)} \vee b_{ij}^{(\alpha)}]_{m\times n}$

$= [0\vee 0,\ 0\vee 0]\ [0\vee 0,\ 0\vee 0]\ [0\vee 0,\ 0\vee 0]$

$= [0,0]\ [0,0]\ [0,0]$ (2.2.6)

From (2.2.5) and (2.2.6) , $(A \vee B)^{(\alpha)} = A^{(\alpha)} \vee B^{(\alpha)}$

In all three cases, $(A \vee B)^{(\alpha)} = A^{(\alpha)} \vee B^{(\alpha)}$

Definition 2.3

Let Let $A = [r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n}$ and $B = [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n}$ be two IVMFRCs, then $\mathbf{A \oplus B}$ is defined as $A \oplus B = D = [r_D(i)]\ [c_D(j)][d_{ij}]_{m\times n}$

where, $r_D(i) = r_A(i) \oplus r_B(i) = [\ r_A^-(i) + r_B^-(i) - r_A^-(i)\ r_B^-(i)\ , r_A^+(i) + r_B^+(i) - r_A^+(i)\ r_B^+(i)\ ]$

$c_D(j) = c_A(j) \oplus c_B(j) = [\ c_A^-(j) + c_B^-(j) - c_A^-(j)\ c_B^-(j)\ , c_A^+(j) + c_B^+(j) - c_A^+(j)\ c_B^+(j)\ ]$

$d_{ij} = a_{ij} \oplus b_{ij} = [\ a_{ij}^- + b_{ij}^- - a_{ij}^- b_{ij}^-\ ,\ a_{ij}^+ + b_{ij}^+ - a_{ij}^+ b_{ij}^+\ ]$

Theorem 2.4

If A and B are two IVFMFRCs, then $(A \oplus B)^{(\alpha)} \geq A^{(\alpha)} \oplus B^{(\alpha)}$

Proof :

Let G_{ij} and H_{ij} be the (ij)th element of $(A \oplus B)^{(\alpha)}$ and $A^{(\alpha)} \oplus B^{(\alpha)}$.

Case 1:

$A \geq B \geq \alpha$

ie., $[r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n} \geq [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n} \geq \alpha$

$G_{ij} = (A \oplus B)^{(\alpha)}$

$= (\ [r_D(i)]\ [c_D(j)]\ [d_{ij}]_{m\times n})^{(\alpha)}$

$= \left([r_A(i) \oplus r_B(i)][c_A^-(j) \oplus c_A^+(j)][a_{ij}^- \oplus a_{ij}^+]_{m\times n}\right)^{(\alpha)}$

$=([r_A^-(i)+r_B^-(i)-r_A^-(i)r_B^-(i), r_A^+(i)+r_B^+(i)-r_A^+(i)r_B^+(i)]\ [c_A^-(j)+c_B^-(j)-c_A^-(j)c_B^-(j), c_A^+(j)+c_B^+(j)-c_A^+(j)c_B^+(j)]\ [\ a_{ij}^- + b_{ij}^- - a_{ij}^- b_{ij}^-\ ,\ a_{ij}^+ + b_{ij}^+ - a_{ij}^+ b_{ij}^+\ ]\ )^{(\alpha)}$

$= ([r_A^-(i) + r_B^-(i) (1-r_A^-(i)) , r_A^+(i) + r_B^+(i) (1-r_A^+(i))][c_A^-(j) + c_B^-(j) (1-c_A^-(j)) , c_A^+(j) + c_B^+(j) (1-c_A^+(j))][a_{ij}^- + b_{ij}^- (1- b_{ij}^-) , a_{ij}^+ + b_{ij}^+ (1- a_{ij}^+)])^{(\alpha)}$

$\geq \left([r_A^-(i), r_A^+(i)][c_A^-(j), c_A^+(j)][a_{ij}^-, a_{ij}^+]\right)^{(\alpha)}$

$> ([r_A^-(i), r_A^+(i)]^{(\alpha)}[c_A^-(j), c_A^+(j)]^{(\alpha)}[a_{ij}^- , a_{ij}^+]_{m\times n}^{(\alpha)})$

$= [1,1][1,1][1,1]_{m\times n}$ (2.4.1)

And $H_{ij} = A^{(\alpha)} \oplus B^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)} \oplus \left([r_B(i)][c_B(j)][b_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i) \oplus r_B^{(\alpha)}(i)][c_A^{(\alpha)}(i) \oplus c_B^{(\alpha)}(i)] [a_{ij}^{(\alpha)} \oplus b_{ij}^{(\alpha)}]_{m\times n}$

$=[r_A^{-(\alpha)}(i)+r_B^{-(\alpha)}(i)-r_A^{-(\alpha)}(i)r_B^{-(\alpha)}(i), r_A^{+(\alpha)}(i)+r_B^{+(\alpha)}(i)-r_A^{+(\alpha)}(i)r_B^{+(\alpha)}(i)]$

$[c_A^{-(\alpha)}(j)+c_B^{-(\alpha)}(j)-c_A^{-(\alpha)}(j)c_B^{-(\alpha)}(j), c_A^{+(\alpha)}(j)+c_B^{+(\alpha)}(j)-c_A^{+(\alpha)}(j)c_B^{+(\alpha)}(j)]$

$[a_{ij}^- + b_{ij}^- - a_{ij}^- b_{ij}^- , a_{ij}^+ + b_{ij}^+ - a_{ij}^+ b_{ij}^+]_{m\times n}$

$= [1+1-1,1+1-1] [1+1-1,1+1-1] [1 + 1 - 1,1 + 1 - 1]_{m\times n}$

$= [1,1] [1,1] [1,1]_{m\times n}$ (2.4.2)

From (2.4.1) and (2.4.2), $H_{ij} > G_{ij}$

$(A \oplus B)^{(\alpha)} > A^{(\alpha)} \oplus B^{(\alpha)}$

Case 2 :

$A \geq \alpha > B$

ie., $[r_A(i)] [c_A(j)] [a_{ij}]_{m\times n} \geq \alpha > [r_B(i)] [c_B(j)] [b_{ij}]_{m\times n}$

$G_{ij} = (A \oplus B)^{(\alpha)}$

$= ([r_D(i)] [c_D(j)] [d_{ij}]_{m\times n})^{(\alpha)}$

$= \left([r_A^-(i) \oplus r_A^+(i)][c_A^-(j) \oplus c_A^+(j)][a_{ij}^- , a_{ij}^+]_{m\times n}\right)^{(\alpha)}$

$=([r_A^-(i)+r_B^-(i)-r_A^-(i)r_B^-(i), r_A^+(i)+r_B^+(i)-r_A^+(i)r_B^+(i)]$

$[c_A^-(j)+c_B^-(j)-c_A^-(j)c_B^-(j), c_A^+(j)+c_B^+(j)-c_A^+(j)c_B^+(j)][a_{ij}^- + b_{ij}^- - a_{ij}^- b_{ij}^- , a_{ij}^+ + b_{ij}^+ - a_{ij}^+ b_{ij}^+])^{(\alpha)}$

$= ([r_A^-(i) + r_B^-(i) (1-r_A^-(i)) , r_A^+(i) + r_B^+(i) (1-r_A^+(i))] [c_A^-(j) + c_B^-(j) (1-c_A^-(j)) , c_A^+(j) + c_B^+(j) (1- c_A^+(j))] [a_{ij}^- + b_{ij}^- (1- b_{ij}^-) , a_{ij}^+ + b_{ij}^+ (1- a_{ij}^+)])^{(\alpha)}$

$\geq \left([r_A^-(i), r_A^+(i)][c_A^-(j), c_A^+(j)][a_{ij}^-, a_{ij}^+]\right)^{(\alpha)}$

$> ([r_A^-(i), r_A^+(i)]^{(\alpha)}[c_A^-(j), c_A^+(j)]^{(\alpha)}[a_{ij}^- , a_{ij}^+]_{m\times n}^{(\alpha)}$

$= [1,1][1,1][1,1]_{m\times n}$ (2.4.3)

And $H_{ij} = A^{(\alpha)} \oplus B^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)} \oplus \left([r_B(i)][c_B(j)][b_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i) \oplus r_B^{(\alpha)}(i)][c_A^{(\alpha)}(i) \oplus c_B^{(\alpha)}(i)] [a_{ij}^{(\alpha)} \oplus b_{ij}^{(\alpha)}]_{m\times n}$

$=[r_A^{-(\alpha)}(i)+r_B^{-(\alpha)}(i)-r_A^{-(\alpha)}(i)r_B^{-(\alpha)}(i), r_A^{+(\alpha)}(i)+r_B^{+(\alpha)}(i)-r_A^{+(\alpha)}(i)r_B^{+(\alpha)}(i)]$

$[c_A^{-(\alpha)}(j)+c_B^{-(\alpha)}(j)-c_A^{-(\alpha)}(j)c_B^{-(\alpha)}(j), c_A^{+(\alpha)}(j)+c_B^{+(\alpha)}(j)-c_A^{+(\alpha)}(j)c_B^{+(\alpha)}(j)]$

$[a_{ij}^- + b_{ij}^- - a_{ij}^- b_{ij}^- , a_{ij}^+ + b_{ij}^+ - a_{ij}^+ b_{ij}^+]_{m\times n}$

$= [1+0-0,1+0-0] [1+0-0,1+0-0] [1 + 0 - 0,1 + 0 - 0]_{m\times n}$

$= [1,1] [1,1] [1,1]_{m\times n}$(2.4.4)

From (2.4.3) and (2.4.4), $G_{ij} > H_{ij}$

$(A \oplus B)^{(\alpha)} > A^{(\alpha)} \oplus B^{(\alpha)}$

Case 3 :

$\alpha > A > B$

ie., $\alpha > [r_A(i)] [c_A(j)] [a_{ij}]_{m\times n} > [r_B(i)] [c_B(j)] [b_{ij}]_{m\times n}$

$G_{ij} = (A \oplus B)^{(\alpha)}$

$= ([r_D(i)] [c_D(j)] [d_{ij}]_{m\times n})^{(\alpha)}$

$= \left([r_A^-(i) \oplus r_A^+(i)][c_A^-(j) \oplus c_A^+(j)][a_{ij}^- , a_{ij}^+]_{m\times n}\right)^{(\alpha)}$

$=([r_A^-(i)+r_B^-(i)-r_A^-(i)r_B^-(i), r_A^+(i)+r_B^+(i)-r_A^+(i)r_B^+(i)]$

$[c_A^-(j)+c_B^-(j)-c_A^-(j)c_B^-(j), c_A^+(j)+c_B^+(j)-c_A^+(j)c_B^+(j)][a_{ij}^- + b_{ij}^- - a_{ij}^- b_{ij}^- , a_{ij}^+ + b_{ij}^+ - a_{ij}^+ b_{ij}^+])^{(\alpha)}$

$= ([r_A^-(i) + r_B^-(i) (1-r_A^-(i)) , r_A^+(i) + r_B^+(i) (1-r_A^+(i))] [c_A^-(j) + c_B^-(j) (1-c_A^-(j)) , c_A^+(j) + c_B^+(j) (1-c_A^+(j))] [a_{ij}^- + b_{ij}^- (1- b_{ij}^-) , a_{ij}^+ + b_{ij}^+ (1- a_{ij}^+)])^{(\alpha)}$

$\geq \left([r_A^-(i), r_A^+(i)][c_A^-(j), c_A^+(j)][a_{ij}^-, a_{ij}^+]\right)^{(\alpha)}$

$> ([r_A^-(i), r_A^+(i)]^{(\alpha)} [c_A^-(j), c_A^+(j)]^{(\alpha)} [a_{ij}^-, a_{ij}^+]_{m\times n}^{(\alpha)}$

$= [0,0]\ [0,0]\ [0,0]_{m\times n}$ (2.4.5)

$H_{ij} = A^{(\alpha)} \oplus B^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)} \oplus \left([r_B(i)][c_B(j)][b_{ij}]_{m\times n}\right)^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)} \oplus \left([r_B(i)][c_B(j)][b_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i) \oplus r_B^{(\alpha)}(i)][c_A^{(\alpha)}(i) \oplus c_B^{(\alpha)}(i)]\ [a_{ij}^{(\alpha)} \oplus b_{ij}^{(\alpha)}]_{m\times n}$

$= [r_A^{-(\alpha)}(i) + r_B^{-(\alpha)}(i) - r_A^{-(\alpha)}(i) r_B^{-(\alpha)}(i), r_A^{+(\alpha)}(i) + r_B^{+(\alpha)}(i) - r_A^{+(\alpha)}(i) r_B^{+(\alpha)}(i)]$

$[c_A^{-(\alpha)}(j) + c_B^{-(\alpha)}(j) - c_A^{-(\alpha)}(j) c_B^{-(\alpha)}(j), c_A^{+(\alpha)}(j) + c_B^{+(\alpha)}(j) - c_A^{+(\alpha)}(j) c_B^{+(\alpha)}(j)]$

$[a_{ij}^- + b_{ij}^- - a_{ij}^- b_{ij}^-, a_{ij}^+ + b_{ij}^+ - a_{ij}^+ b_{ij}^+]_{m\times n}$

$= [0+0-0, 0+0-0]\ [0+0-0, 0+0-0]\ [0 + 0 - 0, 0 + 0 - 0]_{m\times n}$

$= [0,0]\ [0,0]\ [0,0]_{m\times n}$ (2.4.6)

From (2.4.5) and (2.4.6) $G_{ij} = H_{ij}$

Therefore, $(A \oplus B)^{(\alpha)} = A^{(\alpha)} \oplus B^{(\alpha)}$

In all the cases, $G_{ij} \geq H_{ij}$

i.e., $(A \oplus B)^{(\alpha)} \geq A^{(\alpha)} \oplus B^{(\alpha)}$

Definition 2.5

Let $A = [r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n}$ and $B = [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n}$ be two IVMFRCs, then $A \ominus B$ is defined as

$A \ominus B = D = [r_D(i)]\ [c_D(j)][d_{ij}]_{m\times n}$

where, $r_D(i) = [r_A(i) \ominus r_B(i)] = [r_A^-(i) \ominus r_B^-(i), r_A^+(i) \ominus r_B^+(i)]$

$c_D(j) = [c_A(j) \ominus c_B(j)] = [c_A^-(j) \ominus c_B^-(j), c_A^+(i) \ominus c_B^+(j)]$

$a_{ij} = [a_{ij} \ominus b_{ij}] = [a_{ij}^- \ominus b_{ij}^-, a_{ij}^+ \ominus b_{ij}^+]$

where, $a_{ij} \ominus b_{ij} = \begin{bmatrix} 1 \text{ if } a_{ij} > b_{ij} \\ 0 \quad a_{ij} \leq b_{ij} \end{bmatrix}$

Theorem 2.6

If A and B are two IVFMFRCs, then $(A \ominus B)^{(\alpha)} \geq A^{(\alpha)} \ominus B^{(\alpha)}$.

Proof :

Let $A = [r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n}$ and $B = [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n}$ be two IVMFRCs, then $A \ominus B$ is defined as

$A \ominus B = D = [r_D(i)]\ [c_D(j)][d_{ij}]_{m\times n}$

where, $r_D(i) = [r_A(i) \ominus r_B(i)] = [r_A^-(i) \ominus r_B^-(i), r_A^+(i) \ominus r_B^+(i)]$

$c_D(j) = [c_A(j) \ominus c_B(j)] = [c_A^-(j) \ominus c_B^-(j), c_A^+(i) \ominus c_B^+(j)]$

$a_{ij} = [a_{ij} \ominus b_{ij}] = [a_{ij}^- \ominus b_{ij}^-, a_{ij}^+ \ominus b_{ij}^+]$

where, $a_{ij} \ominus b_{ij} = \begin{bmatrix} 1 \text{ if } a_{ij} > b_{ij} \\ 0 \quad a_{ij} \leq b_{ij} \end{bmatrix}$

Let K_{ij} and L_{ij} be the ij[th] element of $A^{(\alpha)} \ominus B^{(\alpha)}$ and $(A \ominus B)^{(\alpha)}$

Here, $L_{ij} = (A \ominus B)^{(\alpha)}$ and $K_{ij} = A^{(\alpha)} \ominus B^{(\alpha)}$

Case 1:

$A \geq B \geq \alpha$

ie., $[r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n} \geq [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n} \geq \alpha$

$L_{ij} = (A \ominus B)^{(\alpha)}$

$= ([r_D(i)]\ [c_D(j)]\ [d_{ij}]_{m\times n})^{(\alpha)}$

$= \left([r_A(i) \ominus r_B(i)][c_A^-(j) \ominus c_A^+(j)][a_{ij}^- \ominus a_{ij}^+]_{m\times n}\right)^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)}]_{m\times n}$

$= [r_A^{-(\alpha)}(i), r_A^{+(\alpha)}(i)]\ c_A^{-(\alpha)}(j), c_A^{+(\alpha)}(j)]\ [a_{ij}^{-(\alpha)}, a_{ij}^{+(\alpha)}]$

$= [1,1]\ [1,1]\ [1,1]_{m\times n}$ (2.6.1)

$K_{ij} = A^{(\alpha)} \ominus B^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)} \ominus \left([r_B(i)][c_B(j)][b_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i) \ominus r_B^{(\alpha)}(i)][c_A^{(\alpha)}(i) \ominus c_B^{(\alpha)}(i)]\ [a_{ij}^{(\alpha)} \ominus b_{ij}^{(\alpha)}]_{m\times n}$

$= [1\ominus 1, 1\ominus 1]\ [1\ominus 1, 1\ominus 1]\ [1\ominus 1, 1\ominus 1]$

= [0,0] [0,0][0,0] (2.6.2)

From (2.6.1) and (2.6.2) $L_{ij} > K_{ij}$

$$(A \ominus B)^{(\alpha)} > A^{(\alpha)} \ominus B^{(\alpha)}$$

Case 2:

$A \geq \alpha \geq B$

$[r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n} \geq \alpha \geq [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n}$

ie., $r_A(i) \geq \alpha \geq r_B(i)$, $c_A(j) \geq \alpha \geq c_B(j)$, $a_{ij} \geq \alpha \geq b_{ij}$

$L_{ij} = (A \ominus B)^{(\alpha)}$

$= ([r_D(i)]\ [c_D(j)]\ [d_{ij}]_{m\times n})^{(\alpha)}$

$= \left([r_A(i) \ominus r_B(i)][c_A^-(j) \ominus c_A^+(j)][a_{ij}^- \ominus a_{ij}^+]_{m\times n}\right)^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)}]_{m\times n}$

$= [r_A^{-(\alpha)}(i), r_A^{+(\alpha)}(i)]\ c_A^{-(\alpha)}(j), c_A^{+(\alpha)}(j)\ ]\ [a_{ij}^{-(\alpha)}, a_{ij}^{+(\alpha)}\ ]$

$= [1,1]\ [1,1]\ [1,1]_{m\times n}$ (2.6.3)

$K_{ij} = A^{(\alpha)} \ominus B^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)} \ominus \left([r_B(i)][c_B(j)][b_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [\ r_A^{(\alpha)}(i) \ominus r_B^{(\alpha)}(i)][c_A^{(\alpha)}(i) \ominus c_B^{(\alpha)}(i)]\ [a_{ij}^{(\alpha)} \ominus b_{ij}^{(\alpha)}]_{m\times n}$

= [1⊖0, 1⊖0] [1⊖0, 1⊖0] [1⊖0, 1⊖0]

= [1,1] [1,1][1,1] (2.6.4)

From (2.6.3) and (2.6.4),we get

$$L_{ij} = K_{ij}$$

$(A \ominus B)^{(\alpha)} = A^{(\alpha)} \ominus B^{(\alpha)}$.

Case 3:

$\alpha \geq A \geq B$

$\alpha \geq [r_A(i)]\ [c_A(j)]\ [a_{ij}]_{m\times n} \geq [r_B(i)]\ [c_B(j)]\ [b_{ij}]_{m\times n}$

ie., $\alpha \geq r_A(i) \geq r_B(i)$, $\alpha \geq c_A(j) \geq c_B(j)$, $\alpha \geq a_{ij} \geq b_{ij}$

$L_{ij} = (A \ominus B)^{(\alpha)}$

$= ([r_D(i)]\ [c_D(j)]\ [d_{ij}]_{m\times n})^{(\alpha)}$

$= \left([r_A(i) \ominus r_B(i)][c_A^-(j) \ominus c_A^+(j)][a_{ij}^- \ominus a_{ij}^+]_{m\times n}\right)^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [r_A^{(\alpha)}(i)]\ [c_A^{(\alpha)}(j)]\ [a_{ij}^{(\alpha)}]_{m\times n}$

$= [r_A^{-(\alpha)}(i), r_A^{+(\alpha)}(i)]\ c_A^{-(\alpha)}(j), c_A^{+(\alpha)}(j)\ ]\ [a_{ij}^{-(\alpha)}, a_{ij}^{+(\alpha)}\ ]$

= [0,0] [0,0] [0,0] (2.6.5)

$K_{ij} = A^{(\alpha)} \ominus B^{(\alpha)}$

$= \left([r_A(i)][c_A(j)][a_{ij}]_{m\times n}\right)^{(\alpha)} \ominus \left([r_B(i)][c_B(j)][b_{ij}]_{m\times n}\right)^{(\alpha)}$

$= [\ r_A^{(\alpha)}(i) \ominus r_B^{(\alpha)}(i)][c_A^{(\alpha)}(i) \ominus c_B^{(\alpha)}(i)]\ [a_{ij}^{(\alpha)} \ominus b_{ij}^{(\alpha)}]_{m\times n}$

= [0⊖0, 0⊖0] [0⊖0, 0⊖0] [0⊖0, 0⊖0]

= [0,0] [0,0] [0,0] (2.6.6)

From (2.6.5) and (2.6.6),we get $L_{ij} = K_{ij}$

In all the cases,

$$(A \ominus B)^{(\alpha)} \geq A^{(\alpha)} \ominus B^{(\alpha)}$$

In a similar manner, for the lower cut of IVFMFRCs, we can prove the following theorem.

Theorem 2.7

If A and B are two IVFMFRCs, then

i) $(A \vee B)_{(\alpha)} = A_{(\alpha)} \vee B_{(\alpha)}$

ii) $(A \oplus B)_{(\alpha)} = A_{(\alpha)} \oplus B_{(\alpha)}$

iii) $(A \ominus B)_{(\alpha)} = A_{(\alpha)} \ominus B_{(\alpha)}$

II. Conculsion

In this paper α-cuts of Interval-Valued Fuzzy matrix with Fuzzy Rows and Columns has been introduced. We have also given some definitions based on α-cuts with example. Some operator on α-cuts are also given. We proved some important theorems of IVFMFRCs using α-cuts.

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