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Least Square Plane and Leastsquare Quadric Surface Approximation by Using Modified Lagrange's Method

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Abstract: *Now a days Surface fitting is applied all engineering and medical fields. Kamron Saniee ,2007 find a simple expression for multivariate LaGrange's Interpolation. We derive a least square plane and least square quadric surface Approximation from a given N+1 tabular points when the function is unique. We used least square method technique. We can apply this method in surface fitting also.*
Keywords: *Least square, quadric surface , Normal equations*

I. Introduction

Least square principle method is one the best approximation methodin numerical analysis for line and curve fitting, this method was invented by Lagrange's. A function $y = f(x)$may be given in discretedata (x_k, y_k).The best approximation in the least square is defined as that for which the constantsc_i , $i = 0,1,2,3 \dots n$are determined so that the aggregate of $w(x)E2$over given domain D is as small as possible, where $w(x) > 0$ \$ is the weight function for the function whose values are given at $N+1$ points$x_0 , x_1 \dots x_N$.

We have
$I(c_0 , c_1 \dots c_N) = \sum_{k=o}^{N} w(x_k)[f(x_k) - \sum_{i=0}^{n} \emptyset_i (x_k)]^2 = minimum$------------(1)

Where $\emptyset_i (x) = x^i$, $i = 0,1,2,3 \dots n$ and $w(x) = 1$
The necessary conditions for (1) to have a minimum value is that
$\frac{\partial I}{\partial c_i} = 0$, $i = 0,1,2,3 \dots n$.
This gives a system of $n + 1$linear equations in $n + 1$constants. These equations are called normal equations. Then we get approximated nth degree polynomial function of x.

1.Least square plane: Given a discrete data (x_k, y_k) , $k = 0,1, \dots N$, $N \geq 2$.

Consider$\emptyset(x, y) = c_0 + c_1 x + c_2 y$
Such that $I(c_0 , c_1, c_2) = \sum_{k=o}^{N}[z_k - (c_0 + c_1 x_k + c_2 y_k)]^2 = minimum$

Then Normal equations $\frac{\partial I}{\partial c_i} = 0$, $i = 0,1,2$.

ie.,

$$c_0(N+1) + c_1 \sum x_k + c_2 \sum y_k = \sum z_k$$

$$c_0 \sum x_k + c_1 \sum {x_k}^2 + c_2 \sum x_k y_k = \sum x_k z_k$$

$c_0 \sum y_k + c_1 \sum x_k y_k + c_2 \sum {y_k}^2 = \sum y_k z_k$

Solving this system of equations we getc_0 , c_1 *and* c_2 .

Example 1: Suppose that given data points $(-1, 1, -2)$, $(1, 2, 3)$, $(1, 1, 2)$
that lie on $z = f(x, y)$.These points define uniquely a linear function in two variables,

so$z_k = c_0 + c_1 x_k + c_2 y_k$, $k = 0,1,2$

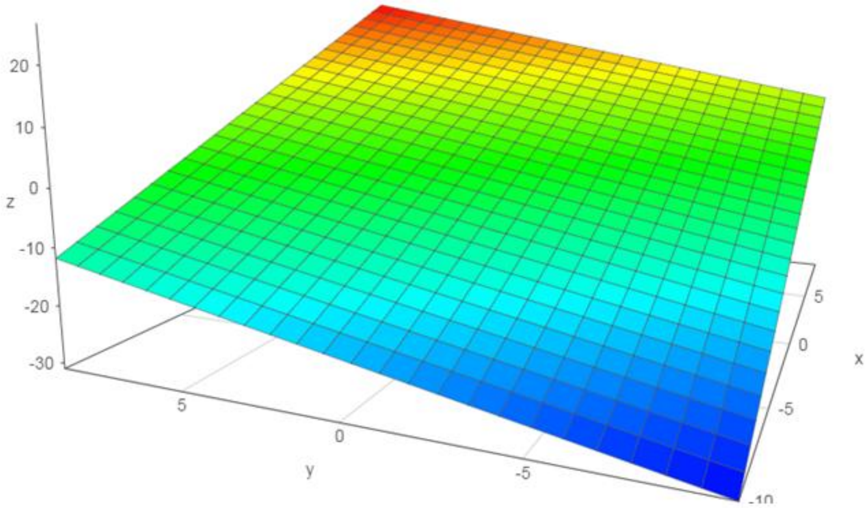
The coefficients satisfy the normal equations

$$3c_0 + 1c_1 + 4c_2 = 3$$

$$c_0 + 3\,c_1 + 2c_2 = 7$$

$$4c_0 + 2c_1 + 6c_2 = 6$$

Solving these equations we get$c_0 = -1\,, c_1 = 2\,, c_2 = 1$
Thus $z = -1 + 2x + y$.



II. Least square quadric surface

Given a discrete data $(x_k, y_k)\,, k = 0,1, \ldots N \;,\quad N \geq 5$

Consider $\emptyset(x, y) = \quad c_0 + \; c_1 x + c_2\, y + c_3\, x^2 + \; c_4 y^2 + c_5\, xy$

Such that

$$I(c_0\,, c_1, c_2\;, c_3, c_{4\,,} c_5) = \sum_{k=o}^{N} [z_k - (c_0 + \; c_1\, x_k + c_2\, y_k \; + \; c_3\, {x_k}^2 + \; c_4 {y_k}^2 + c_5\, x_k y_k)]^2 = minimum$$

The coefficients satisfy normal equations $\frac{\partial I}{\partial c_i} = 0 \;\; , i = 0,1,2,3,4,5$.

$$c_0(N+1) + c_1 \sum x_k + c_2 \sum y_k \; + c_3 \sum {x_k}^2 + c_4 \sum {y_k}^2 + c_3 \sum x_k\, y_k \; = \sum z_k$$

$$c_0 \sum x_k + \; c_1 \sum {x_k}^2 + c_2\, \sum x_k\, y_k + c_3\, \sum {x_k}^3 + \; c_4 \sum x_k {y_k}^2 + c_5\, \sum {x_k}^2 = \sum x_k z_k$$

$$c_0 \sum y_k + \; c_1 \sum x_k\, y_k + c_2\, \sum {y_k}^2 + c_3\, \sum {x_k}^2\, y_k + \; c_4 \sum {y_k}^3 + c_5\, \sum x_k {y_k}^2 = \sum y_k z_k$$

$$c_0 \sum {x_k}^2 + \; c_1 \sum {x_k}^3 + c_2\, \sum {x_k}^2\, y_k + c_3\, \sum {x_k}^4 + \; c_4 \sum {x_k}^2 {y_k}^2 + c_5\, \sum {x_k}^3\, y_k = \sum {x_k}^2 z_k$$

$$c_0 \sum {y_k}^2 + \; c_1 \sum x_k {y_k}^2 + c_2\, \sum {y_k}^3 + c_3\, \sum {x_k}^2 {y_k}^2 + \; c_4 \sum {y_k}^4 + c_5\, \sum x_k {y_k}^3 = \sum {y_k}^2 z_k$$

$$c_0 \sum x_k\, y_k + \; c_1 \sum {x_k}^2\, y_k + c_2\, \sum x_k {y_k}^2 + c_3\, \sum {x_k}^3\, y_k + \; c_4 \sum x_k {y_k}^3 + c_5\, \sum {x_k}^2 {y_k}^2 = \sum x_k\, y_k z_k$$

Solve these equations we get $c_0\,, c_1, c_2\;, c_3, c_{4\,,} c_5$.

Example 2: Suppose that given data points$(0\;,\;0,\;0)\;,(0\;,\;1\;,-4)\;,\;(1,-1\;,1)$

$(1,\ 2,-2)$,$(2,\ 1\ ,4)$, $(1,\ 3\ ,-3)$those lie on $z = f(x\ ,y)$ .These data points satisfy uniquely a degree of two variable function.

$z_k = \ c_0 + \ c_1 x_k + c_2\, x_k + c_3\, {x_k}^2 + \ c_4 {y_k}^2 + c_5\, x_k y_k$, $k = 0,1,2,3,4,5$

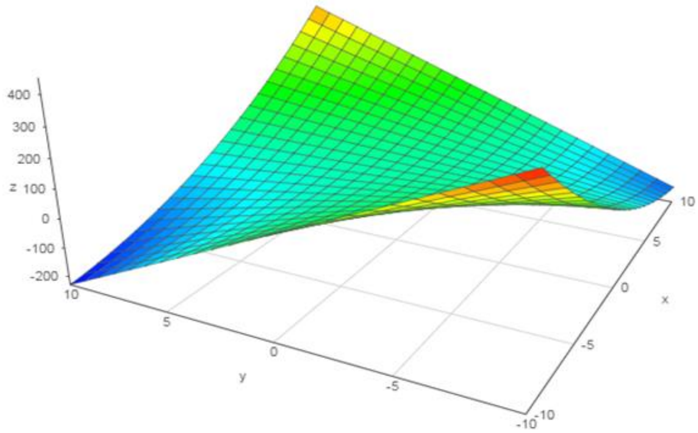
The coefficients $c_0\ , c_1, c_2\ , c_3, c_{4\ ,} c_5$ satisfy the normal equations

$$6c_0 + 5c_1 + 6c_2 + 7c_3 + 16c_4 + 6c_5 = -4$$
$$5c_0 + 7c_1 + 6c_2 + 11c_3 + 16c_4 + 8c_5 = 4$$
$$6c_0 + 6c_1 + 16c_2 + 8c_3 + 36c_4 + 16c_5 = -14$$
$$7c_0 + 11c_1 + 8c_2 + 19c_3 + 18c_4 + 12c_5 = 12$$
$$16c_0 + 16c_1 + 36c_2 + 18c_3 + 100c_4 + 36c_5 = -34$$
$$6c_0 + 8c_1 + 16c_2 + 12c_3 + 36c_4 + 18c_5 = -6$$

Solving this system of equations we get

$c_0 = 0\ , c_1 = -1\ , c_2\ = -4\ ,\ c_3 = 1\ , c_4 = 0\ , c_5\ = 3$

Thus $z = \ -x - 4y + x^2 + \ 3xy$



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