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2DFMT in the Range $[0,0,0,0]$ to $[\infty,\infty,\frac{1}{a},\frac{1}{b}]$ & its Application with Some Function

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Abstract: *In the present paper, the two dimensional Fourier-Mellin Transform (2DFMT) in the range of* [0,0,0,0] *to*$[\infty,\infty,\frac{1}{a},\frac{1}{b}]$ *is defined by using two dimensional Fourier-Laplace transform (2DFLT). Then, we have obtained the solution of 2DFMT of some function in the above defined range.*
Keywords: *Fourier Transform, Mellin Transform, Two Dimensional Fourier-Mellin Transform (2DFMT), Integral Transform.*

I. Introduction

Mathematicians have long had techniques to solve quadratic equations, geometric questions and for physical conditions of speed or position. Fourier transform (FT) is named in the honor of Joseph Fourier (1768-1830), one of greatest names in the history of mathematics. Fourier Transform is a linear operator that maps a functional space to another functions space and decomposes a function into another function of its frequency components. Fourier transform's have many applications in Applied Mathematics, Physics, Chemistry and Engineering such as in medical images, communication, image analysis, data analysis, in image segmentation, signal and noise of fMRI [2]. Fourier transform has established itself as the most efficient tool for deriving closed-form option pricing formulas in various model classes [1]. Fourier transforms have been applied to different remote sensing applications. Lillo-Saavedra et al. used Fourier transforms to fuse panchromatic and multispectral data obtained from Landsat ETM+ sensor [3].

The Mellin transform is a basic tool for analyzing the behavior of many important functions in mathematics and mathematical physics. Mellin transform is a natural analytical tool to study the distribution of products and quotients of independent random variables. Using Mellin transform agricultural land classification is possible. It has application to derive different properties in statistics and probability densities of single continuous random variable and also used in deriving densities for algebraic combination of random variables. Dirichlet Boundary Value Problem is solved by Mellin Transform [6]. Mellin transform method is applied to fractional differential equations with a right-sided derivative and variable potential [4]. Mellin transform also use to establish the means, variances, skewness and kurtosis of fuzzy numbers and applied them to the random coefficient autoregressive (RCA) time series models [5].

In the present work, the two dimensional Fourier-Mellin Transform (2DFMT) of some signals in the range of [0,0,0,0] to$[\infty,\infty,\frac{1}{a},\frac{1}{b}]$ are defined .

Preliminary Result

Let $f(t,l,w,z)$ be the function of four variable in t,l,w & z. Then, Two Dimensional Fourier-Laplace Transform is defined for $f(t,l,w,z)$, for all $t,l,w,z \geq 0$ [9,10] as-

$$FL\{f(t,l,w,z)\} = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty f(t,l,w,z)\, e^{-i(st+ul)} e^{-pw-vz} dt\, dl\, dw\, dz, \qquad (1.1)$$

where, s,u,p,v are parameters and $s,u,p,v > 0$.

Put, $w = -\log ax$, $z = -\log by$

$dw = -\frac{dx}{x}$, $dz = -\frac{dy}{y}$

And $w = 0 \Rightarrow x = \frac{1}{a}$ and $w = \infty \Rightarrow x = 0$

Also, $z = 0 \Rightarrow y = \frac{1}{b}$ and $z = \infty \Rightarrow z = 0$,

Then (1.1) is-

$$FL\{f(t,l,w,z)\} = \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} f(t,l,x,y)\, e^{-i(st+ul)} a^p b^v x^p y^v dt\, dl\, \left(-\frac{dx}{x}\right)\left(-\frac{dy}{y}\right)$$

$$FL\{f(t,l,x,y)\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} f(t,l,x,y)\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$= FM\{f(t,l,x,y)\}$ in the range $[0,0,0,0]$ to$[\infty,\infty,\frac{1}{a},\frac{1}{b}]$. (1.2)

where, $K(t,l,x,y,s,u,p,v) = e^{-i(st+ul)}x^{p-1}y^{v-1}$ and $t(0 < t < \infty), x(0 < x < \infty), x(0 < x < \frac{1}{a}), y(0 < y < \frac{1}{b})$.

In the present paper, we have solved some functions using Two Dimensional Fourier-Mellin Transform in the range $[0,0,0,0]$ to$[\infty,\infty,\frac{1}{a},\frac{1}{b}]$.

1. Application of 2DFMT with Some Function in the Range $[0,0,0,0]$ to$[\infty,\infty,\frac{1}{a},\frac{1}{b}]$

1.1. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform in the given defined range of $f(t,l,x,y)$ then $FM\{1\} = -[supv]^{-1}$

Proof: We have from (1.2) as-

$$FM\{1\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} 1\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= a^p b^v \left\{ \int_0^\infty e^{-ist}\ dt \int_0^\infty e^{-iul}\ dl \int_0^{\frac{1}{a}} x^{p-1}\ dx\ \int_0^{\frac{1}{b}} y^{v-1}\, dy \right\}$$

$$= a^p b^v \left\{ \left[\frac{e^{-ist}}{-is}\right]_0^\infty \left[\frac{e^{-iul}}{-iu}\right]_0^\infty \left[\frac{x^p}{p}\right]_0^{\frac{1}{a}} \left[\frac{y^v}{v}\right]_0^{\frac{1}{b}} \right\}$$

$$= a^p b^v \left\{ \left[\frac{-1}{-is}\right] \left[\frac{-1}{-iu}\right] \frac{1}{p} \left[\frac{1}{a}\right]^p \frac{1}{v} \left[\frac{1}{b}\right]^v \right\}$$

$$FM\{1\} = \frac{-1}{supv}$$

$$FM\{1\} = -[supv]^{-1}$$

1.2. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then $FM\{tlxy\} = \frac{1}{ab}[(p+1)(v+1)s^2u^2]^{-1}$

Proof: We have from (1.2) as-

$$FM\{tlxy\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} tlxy\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= a^p b^v \left\{ \int_0^\infty te^{-ist}\ dt \int_0^\infty le^{-iul}\ dl \int_0^{\frac{1}{a}} x^p\ dx \int_0^{\frac{1}{b}} y^v\, dy \right\}$$

$$= a^p b^v \left\{ \left[\left(t\frac{e^{-ist}}{-is}\right)_0^\infty - \int_0^\infty \frac{e^{-ist}}{-is} dt \right] \left[\left(l\frac{e^{-iul}}{-iu}\right)_0^\infty - \int_0^\infty \frac{e^{-iul}}{-iu} dl \right] \left[\frac{x^{p+1}}{p+1}\right]_0^{\frac{1}{a}} \left[\frac{y^{v+1}}{v+1}\right]_0^{\frac{1}{b}} \right\}$$

$$= a^p b^v \left\{ \left[\frac{-1}{s^2}\right] \left[\frac{-1}{u^2}\right] \frac{1}{p+1} \left[\frac{1}{a}\right]^{p+1} \frac{1}{v+1} \left[\frac{1}{b}\right]^{v+1} \right\}$$

$$FM\{tlxy\} = \frac{1}{ab}[(p+1)(v+1)s^2u^2]^{-1}$$

1.3. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$$FM\{t^n l^n x^n y^n\} = \frac{1}{(-1)^{n+1} a^n b^n}[(p+n)(v+n)]^{-1}(su)^{-(n+1)}\ [n\Gamma n]^2$$

Proof: We have from (1.2) is

$$FM\{t^n l^n x^n y^n\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} t^n l^n x^n y^n\ e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= a^p b^v \left\{ \int_0^\infty t^n e^{-ist}\ dt \int_0^\infty l^n e^{-iul}\ dl \int_0^{\frac{1}{a}} x^{p+n-1}\ dx \int_0^{\frac{1}{b}} y^{v+n-1}\, dy \right\}$$

$$= a^p b^v \left\{ \left[\left(t^n \frac{e^{-ist}}{-is}\right)_0^\infty - n\int_0^\infty t^{n-1} \frac{e^{-ist}}{-is} dt \right] \left[\left(l^n \frac{e^{-iul}}{-iu}\right)_0^\infty - n\int_0^\infty l^{n-1} \frac{e^{-iul}}{-iu} dl \right] \left[\frac{x^{p+n}}{p+n}\right]_0^{\frac{1}{a}}\ \left[\frac{y^{v+n}}{v+n}\right]_0^{\frac{1}{b}} \right\}$$

$$= a^p b^v \left\{ \left[\frac{n}{is}\int_0^\infty t^{n-1} e^{-ist}\ dt\right] \left[\frac{n}{iu}\int_0^\infty l^{n-1} e^{-ist}\ dt\right] \frac{1}{p+n} \left[\frac{1}{a}\right]^{p+n} \frac{1}{v+n} \left[\frac{1}{b}\right]^{v+n} \right\}$$

$$= a^p b^v \left[\frac{1}{(is)^n} n(n-1)(n-2) \ldots \ldots 3.2.1 \int_0^\infty e^{-ist}\, dt \right]$$
$$\left[\frac{1}{(iu)^n} n(n-1)(n-2) \ldots \ldots 3.2.1 \int_0^\infty e^{-iul}\, dl \right] \frac{1}{p+n} \left[\frac{1}{a}\right]^{p+n} \frac{1}{v+n} \left[\frac{1}{b}\right]^{v+n}$$

$$FM\{t^n l^n x^n y^n\} = \frac{1}{a^n b^n (p+n)(v+n)(is)^{n+1}(iu)^{n+1}}\ [n!]^2$$

$$FM\{t^n l^n x^n y^n\} = \frac{1}{(-1)^{n+1} a^n b^n}[(p+n)(v+n)]^{-1}(su)^{-(n+1)}\ [n\Gamma n]^2$$

1.4. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then $FM\{e^{at+bl}xy\} = \frac{1}{ab}[(p+1)(v+1)(is-a)(iu-b)]^{-1}$

Proof: - We have from (1.2) as-

$$FM\{e^{at+bl}xy\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} e^{at+bl}xy\ \ e^{-i(st+ul)}x^{p-1}y^{v-1}dt\,dl\,dx\,dy$$

$$= a^p b^v \left\{\int_0^\infty e^{-(is-a)t}\ dt \int_0^\infty e^{-(iu-b)l}\ dl \int_0^{\frac{1}{a}} x^p\ dx\ \int_0^{\frac{1}{b}} y^v\,dy\right\}$$

$$= a^p b^v \left\{\left[\frac{e^{-(is-a)t}}{-(is-a)}\right]_0^\infty \left[\frac{e^{-(iu-b)l}}{-(iu-b)}\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^{\frac{1}{a}} \left[\frac{y^{v+1}}{v+1}\right]_0^{\frac{1}{b}}\right\}$$

$$= a^p b^v \left\{\left[\frac{1}{(is-a)}\right] \left[\frac{1}{(iu-b)}\right] \frac{1}{p+1}\left[\frac{1}{a}\right]^{p+1} \frac{1}{v+1}\left[\frac{1}{b}\right]^{v+1}\right\}$$

$$FM\{e^{at+bl}xy\} = \frac{1}{ab}[(p+1)(v+1)(is-a)(iu-b)]^{-1}$$

1.5. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{cos\ at\ cos\ bl\ xy\} = \frac{-su}{ab}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (1.2) as-

$$FM\{\cos at \cos bl\, xy\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} \cos at \cos bl\, xy\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\,dl\,dx\,dy$$

$$= a^p b^v \left\{\int_0^\infty \cos at\, e^{-ist}\ dt \int_0^\infty \cos bl\, e^{-iul}\ dl \int_0^{\frac{1}{a}} x^p\ dx \int_0^{\frac{1}{b}} y^v\,dy\right\}$$

$$= a^p b^v \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\cos at + a\sin at)\right]_0^\infty \left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\cos bl\, b\sin bl)\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^{\frac{1}{a}} \left[\frac{y^{v+1}}{v+1}\right]_0^{\frac{1}{b}}$$

$$= a^p b^v \left\{\left[\frac{is}{(s^2+a^2)}\right] \left[\frac{iu}{(u^2+b^2)}\right] \frac{1}{p+1}\left[\frac{1}{a}\right]^{p+1} \frac{1}{v+1}\left[\frac{1}{b}\right]^{v+1}\right\}$$

$$FM\{\cos at \cos bl\, xy\} = \frac{-su}{ab}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

1.6. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{sin\ at\ sin\ bl\ xy\} = [(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (1.2) as-

$$FM\{\sin at \sin bl\, xy\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} \sin at \sin bl\, xy\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\,dl\,dx\,dy$$

$$= a^p b^v \left\{\int_0^\infty \sin at\, e^{-ist}\ dt \int_0^\infty \sin bl\, e^{-iul}\ dl \int_0^{\frac{1}{a}} x^p\ dx \int_0^{\frac{1}{b}} y^v\,dy\right\}$$

$$= a^p b^v \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\sin at - a\cos at)\right]_0^\infty \left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\sin bl - b\cos bl)\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^{\frac{1}{a}} \left[\frac{y^{v+1}}{v+1}\right]_0^{\frac{1}{b}}$$

$$= a^p b^v \left\{\left[\frac{a}{(s^2+a^2)}\right] \left[\frac{b}{(u^2+b^2)}\right] \frac{1}{p+1}\left[\frac{1}{a}\right]^{p+1} \frac{1}{v+1}\left[\frac{1}{b}\right]^{v+1}\right\}$$

$$FM\{\sin at \sin bl\, xy\} = [(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

1.7. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{sin\ at\ cos\ bl\ xy\} = \frac{iu}{b}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (1.2) as-

$$FM\{\sin at \cos bl\, xy\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} \sin at \cos bl\, xy\, e^{-i(st+ul)}x^{p-1}y^{v-1}dt\,dl\,dx\,dy$$

$$= a^p b^v \left\{\int_0^\infty \sin at\, e^{-ist}\ dt \int_0^\infty \cos bl\, e^{-iul}\ dl \int_0^{\frac{1}{a}} x^p\ dx \int_0^{\frac{1}{b}} y^v\,dy\ \right\}$$

$$= a^p b^v \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\sin at - a\cos at)\right]_0^\infty \left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\cos bl\, b\sin bl)\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^{\frac{1}{a}} \left[\frac{y^{v+1}}{v+1}\right]_0^{\frac{1}{b}}$$

$$= a^p b^v \left\{\left[\frac{a}{(s^2+a^2)}\right] \left[\frac{iu}{(u^2+b^2)}\right] \frac{1}{p+1}\left[\frac{1}{a}\right]^{p+1} \frac{1}{v+1}\left[\frac{1}{b}\right]^{v+1}\right\}$$

$$FM\{\sin at \cos bl\, xy\} = \frac{iu}{b}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

1.8. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{cos\, at\, sin\, bl\, xy\} = \frac{is}{a}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (1.2) as-

$$FM\{\cos at \sin bl\, xy\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} \cos at \sin bl\, xy\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= a^p b^v \left\{\int_0^\infty \cos at\, e^{-ist}\ dt \int_0^\infty \sin bl\, e^{-iul}\ dl \int_0^{\frac{1}{a}} x^p\ dx \int_0^{\frac{1}{b}} y^v\, dy\right\}$$

$$= a^p b^v \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\cos at + a\sin at)\right]_0^\infty \left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\sin bl - b\cos bl)\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^{\frac{1}{a}} \left[\frac{y^{v+1}}{v+1}\right]_0^{\frac{1}{b}}$$

$$= a^p b^v \left\{\left[\frac{is}{(s^2+a^2)}\right]\left[\frac{b}{(u^2+b^2)}\right]\frac{1}{p+1}\left[\frac{1}{a}\right]^{p+1}\frac{1}{v+1}\left[\frac{1}{b}\right]^{v+1}\right\}$$

$$FM\{\cos at \sin bl\, xy\} = \frac{is}{a}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

1.9. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{cos\, at\, cos\, bl\, x^n y^n\} = \frac{-su}{a^n b^n}[(p+n)(v+n)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (1.2) as-

$$FM\{\cos at \cos bl\, x^n y^n\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} \cos at \cos bl\, x^n y^n\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= a^p b^v \left\{\int_0^\infty \cos at\, e^{-ist}\ dt \int_0^\infty \cos bl\, e^{-iul}\ dl \int_0^{\frac{1}{a}} x^{p+n-1}\, dx \int_0^{\frac{1}{b}} y^{v+n-1}\, dy\right\}$$

$$= a^p b^v \left[\frac{e^{-ist}}{(s^2+a^2)}((-is)\cos at + a\sin at)\right]_0^\infty \left[\frac{e^{-iul}}{(u^2+b^2)}((-iu)\cos bl + b\sin bl)\right]_0^\infty \left[\frac{x^{p+n}}{p+n}\right]_0^{\frac{1}{a}} \left[\frac{y^{v+n}}{v+n}\right]_0^{\frac{1}{b}}$$

$$= a^p b^v \left\{\left[\frac{is}{(s^2+a^2)}\right]\left[\frac{iu}{(u^2+b^2)}\right]\frac{1}{p+n}\left[\frac{1}{a}\right]^{p+n}\frac{1}{v+n}\left[\frac{1}{b}\right]^{v+n}\right\}$$

$$FM\{\cos at \cos bl\, x^n y^n\} = \frac{-su}{a^n b^n}[(p+n)(v+n)(s^2+a^2)(u^2+b^2)]^{-1}$$

1.10. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{sin\, at\, sin\, bl\, x^n y^n\} = \frac{1}{a^{n-1} b^{n-1}}[(p+n)(v+n)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (1.2) as-

$$FM\{\sin at \sin bl\, x^n y^n\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} \sin at \sin bl\, x^n y^n\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= a^p b^v \left\{\int_0^\infty \sin at\, e^{-ist}\ dt \int_0^\infty \sin bl\, e^{-iul}\ dl \int_0^{\frac{1}{a}} x^{p+n-1}\, dx \int_0^{\frac{1}{b}} y^{v+n-1}\, dy\right\}$$

$$= a^p b^v \left\{\left[\frac{a}{(s^2+a^2)}\right]\left[\frac{b}{(u^2+b^2)}\right]\frac{1}{p+n}\left[\frac{1}{a}\right]^{p+n}\frac{1}{v+n}\left[\frac{1}{b}\right]^{v+n}\right\} \qquad \text{\{Since by previous result\}}$$

$$FM\{\sin at \sin bl\, x^n y^n\} = \frac{1}{a^{n-1} b^{n-1}}[(p+n)(v+n)(s^2+a^2)(u^2+b^2)]^{-1}$$

1.11. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{cosh\, at\, cosh\, al\, xy\} = \frac{-su}{ab}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (1.2) as-

$$FM\{\cosh at \cosh al\, xy\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} \cosh at \cosh al\, xy\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$$

$$= a^p b^v \left\{\int_0^\infty \cosh at\, e^{-ist}\ dt \int_0^\infty \cosh al\, e^{-iul}\ dl \int_0^{\frac{1}{a}} x^p\ dx \int_0^{\frac{1}{b}} y^v\, dy\ \right\}$$

$$= a^p b^v \left\{\frac{1}{2}\left[\frac{e^{-(is-a)t}}{-(is-a)} + \frac{e^{-(is+a)t}}{-(is+a)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(iu-a)l}}{-(iu-a)} + \frac{e^{-(iu+a)l}}{-(iu+a)}\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^{\frac{1}{a}} \left[\frac{y^{v+1}}{v+1}\right]_0^{\frac{1}{b}}\right\}$$

$$= a^p b^v \left\{\frac{1}{2}\left[\frac{2is}{(is)^2-a^2}\right]\frac{1}{2}\left[\frac{2iu}{(iu)^2-a^2}\right]\frac{1}{p+1}\left[\frac{1}{a}\right]^{p+1}\frac{1}{v+1}\left[\frac{1}{b}\right]^{v+1}\right\}$$

$$FM\{\cosh at \cosh al\, xy\} = \frac{-su}{ab}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

1.12. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$$FM\{cosh\,at\,sinh\,al\,xy\} = \frac{is}{b}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$$

Proof: - We have from (1.2) as-

$FM\{\cosh at \sinh al\, xy\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} \cosh at \sinh al\, xy\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$

$= a^p b^v \left\{ \int_0^\infty \cosh at\, e^{-ist}\ dt \int_0^\infty \sinh al\, e^{-iul}\ dl \int_0^{\frac{1}{a}} x^p\ dx \int_0^{\frac{1}{b}} y^v\, dy \right\}$

$= a^p b^v \left\{ \frac{1}{2}\left[\frac{e^{-(is-a)t}}{-(is-a)} + \frac{e^{-(is+a)t}}{-(is+a)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(iu-a)l}}{-(iu-a)} - \frac{e^{-(iu+a)l}}{-(iu+a)}\right]_0^\infty \left[\frac{x^{p+1}}{p+1}\right]_0^{\frac{1}{a}} \left[\frac{y^{v+1}}{v+1}\right]_0^{\frac{1}{b}} \right\}$

$= a^p b^v \left\{ \frac{1}{2}\left[\frac{2is}{(is)^2-a^2}\right] \frac{1}{2}\left[\frac{2a}{(iu)^2-a^2}\right] \frac{1}{p+1}\left[\frac{1}{a}\right]^{p+1} \frac{1}{v+1}\left[\frac{1}{b}\right]^{v+1} \right\}$

$FM\{\cosh at \sinh al\, xy\} = \frac{is}{b}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$

1.13. If $FM\{f(t,l,x,y)\}(s,u,p,v)$ denotes the generalized Two Dimensional Fourier-Mellin transform of $f(t,l,x,y)$ then

$FM\{\sinh at \sinh al\, xy\} = \frac{a}{b}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$

Proof: - We have from (1.2) as-

$FM\{\sinh at \sinh al\, xy\} = a^p b^v \int_0^\infty \int_0^\infty \int_0^{\frac{1}{a}} \int_0^{\frac{1}{b}} \sinh at \sinh al\, xy\, e^{-i(st+ul)} x^{p-1} y^{v-1} dt\, dl\, dx\, dy$

$= a^p b^v \left\{ \int_0^\infty \sinh at\, e^{-ist}\ dt \int_0^\infty \sinh al\, e^{-iul}\ dl \int_0^{\frac{1}{a}} x^p\ dx \int_0^{\frac{1}{b}} y^v\, dy \right\}$

$= a^p b^v \left\{ \frac{1}{2}\left[\frac{2a}{(is)^2-a^2}\right] \frac{1}{2}\left[\frac{2a}{(iu)^2-a^2}\right] \frac{1}{p+1}\left[\frac{1}{a}\right]^{p+1} \frac{1}{v+1}\left[\frac{1}{b}\right]^{v+1} \right\}$

$FM\{\sinh at \sinh al\, xy\} = \frac{a}{b}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$.

2. The Tabular form of the Results obtained are given as follows-

S.N.	$f(t,l,x,y)$	$FM\{f(t,l,x,y)\}$ in the range [0,0,0,0] to$[\infty,\infty,\frac{1}{a},\frac{1}{b}]$
1	1	$-[supv]^{-1}$
2	$tlxy$	$\frac{1}{ab}[(p+1)(v+1)s^2u^2]^{-1}$
3	$t^n l^n x^n y^n$	$\frac{1}{(-1)^{n+1}a^n b^n}[(p+n)(v+n)]^{-1}(su)^{-(n+1)}\ [n\Gamma n]^2$
4	$e^{at+bl}xy$	$\frac{1}{ab}[(p+1)(v+1)(is-a)(iu-b)]^{-1}$
5	$cos\,at\,cos\,bl\,xy$	$\frac{-su}{ab}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$
6	$sin\,at\,sin\,bl\,xy$	$[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$
7	$sin\,at\,cos\,bl\,xy$	$\frac{iu}{b}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$
8	$cos\,at\,sin\,bl\,xy$	$\frac{is}{a}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$
9	$cos\,at\,cos\,bl\,x^n y^n$	$\frac{-su}{a^n b^n}[(p+n)(v+n)(s^2+a^2)(u^2+b^2)]^{-1}$
10	$sin\,at\,sin\,bl\,x^n y^n$	$\frac{1}{a^{n-1}b^{n-1}}[(p+n)(v+n)(s^2+a^2)(u^2+b^2)]^{-1}$
11	$cosh\,at\,cosh\,al\,xy$	$\frac{-su}{ab}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$
12	$cosh\,at\,sinh\,al\,xy$	$\frac{is}{b}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$
13	$\sinh at \sinh al\,xy$	$\frac{a}{b}[(p+1)(v+1)(s^2+a^2)(u^2+b^2)]^{-1}$

II. Conclusion

In the Present paper we have converted Two Dimensional Fourier-Lapalce Transform into Two Dimensional Fourier-Mellin Transform in the finite region and using this transform we have solved some functions and obtained their solution.

References

[1]. J. Zhu: Applications of Fourier Transform to Smile Modeling Theory and Implementation Series: Springer Finance, 2nd ed. 2010, XV, 330 p. 7 illus.

[2]. Shubing Wang: Applications of Fourier Transform to Imaging Analysis, shubing@stat.wisc.edu, May 23, 2007.

[3]. Elif Sertel: Identification of Earthquake Induced Damage Areas Using Fourier Transform and SPOT HRVIR Pan Images, *Sensors* 2009, *9*, 1471-1484; doi:10.3390/s90301471.

[4]. Daniel Dziembowski, “On Mellin transform application to solution of fractional differential equations”, Scientific Research of the Institute of Mathematics and Computer Science.

[5]. S. S. Appadoo, A. Thavaneswaran and S.Mandal, “Mellin’s transform and application to some time series models”, Hindawi Publishing Corporation, ISRN Applied Mathematics Volume 2014, Article ID 976023, 12 pages.

[6]. S. M. Khairnar, R. M. Pise and J. N. Salunkhe, “Study of the Mellin integral transform with applications in statistics and probability”, Scholars Research Library, Archives of Applied Science Research, 2012, 4 (3):1294-1310.

[7]. R.M.Pise: Double fractional Mellin Integral transform in $[0,0]$ to$[\frac{1}{a},\frac{1}{b}]$ and its application, Int. Jr. of Tech. and Research Advances, issue1, Vol. 2013.

[8]. Dr. Rachana Mathur & Sarita Poonia: Application of the Mellin Type Integral Transform in the range $[0,\frac{1}{a}]$, Int. Jr. of mathematical Archieve,3(6), 2012, 2380-2385.

[9]. V.D.Sharma and P.D.Dolas: Inversion Formula For Two Dimensional Generalized Fourier-Mellin Transform And It’s Application, Int. Jr. of Advanced Scientific and Technical Research,Issue 1, Vol. 2, December 2011.

[10]. V.D. Sharma and P.D. Dolas: Representation theorem for the distributional two dimensional Fourier-Mellin Transform, Int. Jr. Matthematical Archieve, Vol. 5, issue 9, Sept. 2014.

[11]. V. D. Sharma, P. D. Dolas: Modulation & Parseval’s Theorem for Distributional Two Dimensional Fourier-Mellin Transform, Int. Jr. of Engineering Sciences & Research Technology, 5(8), August, 2016, 559-564.