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Efficiency of Nearest Neighbour Balanced Block Designs for Second Order Correlated Error Structure

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Abstract: *The performance of a series of Complete and Incomplete Neighbour Balanced Block Designs for Auto Regressive (AR), Moving Average (MA) and Nearest Neighbour (NN) error correlation structure is studied when generalized least squares estimation is used. We have compared the efficiency of AR(2), MA(2) and NN correlation structures. It is observed that efficiency for direct as well as neighbour effects is high, in case of complete block designs for NN correlation structure. In case of incomplete block designs MA(2) correlation structure turns out to be more efficient as compared to others models with* (ρ_1, ρ_2) *in the interval 0.1 to 0.4. It is therefore concluded that when block sizes are large and neighbouring plots are highly correlated, generalized least squares for estimation of direct and neighbour effects can be used. The gain in efficiency of NNBD and NNBIBD over regular block design is high under MA(2) models for direct and neighbour effects of treatments.*

Keywords*: Neighbour Balanced Block Design; Correlated observations; Generalized least squares; Auto Regressive; Moving Average; Nearest neighbour; Efficiency; Regular Block Design.*

I. Introduction

In many experiments, especially in agriculture, the field on a given plot may be affected by treatments on neighbouring plots as well as by the treatment applied to that plot. To diminish these undesirable effects, Rees [15] first provided designs for the test and named such designs as neighbour designs. He used technique in virus research which requires arrangement in circles of samples from a number of virus preparations in such a way that over the whole set, a sample from each virus preparation appears next to a sample from every other virus preparation. He defined a neighbour design as an arrangement of v antigens (called symbols) in b circular plates (called blocks) such that: each block has k symbols, not necessarily distinct, each symbol appears r times in the design and each symbol is a neighbour of every other symbol precisely λ times. A neighbour design with at least one block having less than v distinct symbols may called as incomplete block neighbour designs. The designs in both series are neighbour balanced in the sense that every experimental treatment has each other treatment once as a right neighbour and once as a left neighbour. When treatments are varieties, neighbour effects may be caused by differences in height, root vigor, or germination date, especially on small plots, which are used in plant breeding experiments. Treatments such as fertilizer, irrigation, or pesticide may spread to adjacent plots causing neighbour effects. Such experiments exhibit neighbour effects, because the effect of having no treatment as a neighbour is different from the neighbour effects of any treatment. In case of block design setup if each block is a single line of plots and blocks are well separated, extra parameters are needed for the effect of left and right neighbours. An alternative is to have border plots on both ends of every block. Each border plots receives an experimental treatment, but it is not used for measuring the response variable. These border plots do not add too much to the cost of one-dimensional experiments. Neighbour balanced designs, where in the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbours, are used for these situations and permit the estimation of direct and neighbor effects of treatments. The neighbour effects are also called as interference effects, indirect effects or remote treatment effects.

The effect of correlation on the usual two-way analysis of variance and on the power of usual tests has been studied by Box [4]; Anderson *et al.* [1] and Aastveit [2]. Keedwell [9] considered 2-fold perfect circuit designs, these being balanced circuit designs whose neighbour properties apply not only to immediate neighbours but also to neighbours that are two places apart. In situations where the correlation structure among the observations within a block is known, may be from the data of past similar experiments, it may be advantageous to use this information in designing an experiment and analyzing the data so as to make more precise inference about treatment effects Gill and Shukla [6]. Neighbour balanced block designs for correlated errors by Kunert [10]. Lindner *et al.* [12] considered 2-perfect k-cycle systems of order v, i.e., balanced circuit RND's (Rees Neighbour Designs) whose neighbour properties hold both for immediate neighbours and for 2-

places-apart neighbours. Optimal and highly efficient two dimensional designs have been constructed for correlated errors on the torus and in the two dimensional plane by Morgan and Nizamuddin [13]. Azais *et al.* [3] obtained a series of efficient neighbour designs with border plots that are balanced in $v-1$ blocks of size v and v blocks of size $v-1$, where v is the number of treatments. Bailey [5] has given some designs for studying one-sided neighbour effects. These neighbour balanced block designs have been developed under the assumption that the observations within a block are uncorrelated. Kunert *et al.* [11] considered two related models for interference and have shown that optimal designs for one model can be obtained from optimal designs for the other model. Martin and Eccelston [14] have given variance balanced designs under interference and dependent observations. Santharam and Ponnuswamy [18] examined the optimality and efficiency of nearest neighbor balanced block designs when error follows Auto Regressive (AR), Moving Average (MA) or Auto Regressive Moving Average (ARMA) models. Senthil Kumar and Santharam [17] Efficiency of Nearest Neighbour Balanced Block Designs using ARMA models. Senthil Kumar and Santharam [16] Efficiency of NNBD over NNBIBD using First Order Correlated Models. Iqbal *et al.* [7] constructed second order neighbour designs for $3 \le k \le 7$ in circular block using method of cyclic shifts.

In this paper, neighbour balanced block designs for observations correlated within a block have been investigated for the estimation of direct as well as left and right neighbour effects of treatments. The performance of these designs for AR(2), MA(2) and NN error correlation structure is studied when generalized least squares estimation is used. We have also investigated the efficiency of Nearest Neighbour Balanced Block Design (NNBD) and Nearest Neighbour Balanced Incomplete Block Design (NNBIBD) in comparison to regular block design when the error follows second order correlated models with (ρ_1, ρ_2) in the interval -0.4 (-0.4) 0.4.

II. Model Structures and Information Matrix

Let Δ be a class of binary neighbour balanced block designs with $n = bk$ units that form b blocks each containing k units. Y_{ij} be the response from the i^{th} plot in the j^{th} block $(i = 1,2,\ldots,k;\ j = 1,2,\ldots,b)$. The layout includes border plots at both ends of every block, i.e. at 0^{th} and $(k+1)^{th}$ position and observations for these units are not modeled. The following fixed effects additive model is considered for analyzing a neighbour balanced block design under correlated observations:

$$Y_{ij} = \mu + \tau_{(i,j)} + l_{(i-1,j)} + \gamma_{(i+1,j)} + \beta_j + e_{ij} \qquad (2.1)$$

where μ is the general mean, $\tau_{(i,j)}$ is the direct effect of the treatment in the i^{th} plot of j^{th} block, β_j is the effect of the j^{th} block, $l_{(i-1,j)}$ is the left neighbour effect due to the treatment in the $(i-1)^{th}$ plot of j^{th} block, $\gamma_{(i+1,j)}$ is the right neighbour effect due to the treatment in the $(i+1)^{th}$ plot in j^{th} block, e_{ij} are error terms distributed with mean zero and a variance-covariance structure $\Omega = I_b \otimes \Lambda$ (I_b is an identity matrix of order b and $\otimes$ denotes the kronecker product). Assuming no correlation among the observations between the blocks and correlation structure between plots within a block to be the same in each block, Λ is the correlation matrix of k observations within a block. If the errors within a block follow an Second Order Auto Regressive model (AR(2)) then $\Omega = I_b \otimes M_k$ where M_k is a $k \times k$ matrix given by

$$M_k = \begin{bmatrix} r_0 & r_1 & r_2 & \ldots & r_{k-1} \\ r_1 & r_0 & r_1 & \ldots & r_{k-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{k-1} & r_{k-2} & r_{k-3} & \ldots & r_0 \end{bmatrix}$$

The elements of M_k are

$$r_0 = (1-\rho_2)/(1+\rho^2)\left\{(1-\rho_2)^2 - \rho_1^2\right\}$$
$$r_1 = \left\{\rho_1^2/(1-\rho_2)\right\} r_0$$
$$r_2 = \left\{\rho_1^2/(1-\rho_2) + \rho_2\right\} r_0$$

For $k \geq 3$, $r_k = \{\rho_1 r_{k-1} + \rho_2 r_{k-2}\} r_0$

If the errors within a block follow Second Order Moving Average model (MA(2)) the $\Omega = I_b \otimes N_k$ where N_k is a $k \times k$ matrix given by

$$N_k = \begin{bmatrix} 1+\rho_1^2+\rho_2^2 & \rho_1+\rho_1\rho_2 & \rho_2 & 0 & \dots & 0 \\ \rho_1+\rho_1\rho_2 & 1+\rho_1^2+\rho_2^2 & \rho_1+\rho_1\rho_2 & \rho_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1+\rho_1^2+\rho_2^2 \end{bmatrix}$$

The NN correlation structure, the Λ is a matrix with diagonal entries as 1 and off-diagonal entries as ρ. Model (2.1) can be rewritten in the matrix notation as follows:

$$Y = \mu 1 + \Delta'\tau + \Delta_1' l + \Delta_2' \gamma + D'\beta + e \tag{2.2}$$

where Y is $n \times 1$ vector of observations, 1 is $n \times 1$ vector of ones, Δ' is an $n \times v$ incidence matrix of observations versus direct treatments, τ is $v \times 1$ vector of direct treatment effects, Δ_1' is a $n \times v$ matrix of observations versus left neighbour treatment, Δ_2' is a $n \times v$ matrix of observations versus right neighbour treatment, l is $v \times 1$ vector of left neighbour effects, γ is $v \times 1$ vector of right neighbour effects, D' is an $n \times b$ incidence matrix of observations versus blocks, β is $b \times 1$ vector of block effects and e is $n \times 1$ vector of errors. The joint information matrix for estimating the direct and neighbour (left and right) effects under correlated observations estimated by generalized least squares is obtained as follows:

$$C = \begin{bmatrix} \Delta(I_b \otimes \wedge^*)\Delta' & \Delta(I_b \otimes \wedge^*)\Delta_1' & \Delta(I_b \otimes \wedge^*)\Delta_2' \\ \Delta_1(I_b \otimes \wedge^*)\Delta' & \Delta_1(I_b \otimes \wedge^*)\Delta_1' & \Delta_1(I_b \otimes \wedge^*)\Delta_2' \\ \Delta_2(I_b \otimes \wedge^*)\Delta' & \Delta_2(I_b \otimes \wedge^*)\Delta_1' & \Delta_2(I_b \otimes \wedge^*)\Delta_2' \end{bmatrix} \tag{2.3}$$

with

$$\wedge^* = \wedge^{-1} - \left(1_k' \wedge^{-1} 1_k\right)^{-1} \wedge^{-1} 1_k 1_k' \wedge^{-1}$$

The above $3v \times 3v$ information matrix (C) for estimating the direct effects and neighbour effects of treatments in a block design setting is symmetric, non-negative definite with row and column sums equal to zero. The information matrix for estimating the direct effects of treatments from (2.3) is as follows:

$$C_\tau = C_{11} - C_{12} C_{22}^{-1} C_{21} \tag{2.4}$$

where

$$C_{11} = \Delta(I_b \otimes \wedge^*)\Delta'$$
$$C_{12} = \begin{bmatrix} \Delta(I_b \otimes \wedge^*)\Delta_1' & \Delta(I_b \otimes \wedge^*)\Delta_2' \end{bmatrix}$$

and

$$C_{22} = \begin{bmatrix} \Delta_1(I_b \otimes \wedge^*)\Delta_1' & \Delta_1(I_b \otimes \wedge^*)\Delta_2' \\ \Delta_2(I_b \otimes \wedge^*)\Delta_1' & \Delta_2(I_b \otimes \wedge^*)\Delta_2' \end{bmatrix}$$

Similarly, the information matrix for estimating the left neighbour effect of treatments (C_l) and right neighbour effect of treatments (C_γ) can be obtained.

2.1 Construction of Design

Tomer *et al*. [19] has constructed neighbour balanced block design with parameters v (prime or prime power), $b = v(v-1), r = (v-1)(v-m), k = (v-m),\quad m = 1,2,\ldots,v-4$ and $\lambda = (v-m)$ using Mutually Orthogonal Latin Squares (MOLS) of order v. This series of design bas been investigated under the correlated error structure. It is seen that the design turns out to be pair-wise uniform with $\alpha = 1$ and also variance balanced for estimating direct (V_1) and neighbour effects $(V_2 = V_3)$.

Example:

Let $v = 5$ and $m = 0$. The following is a neighbour balanced pair-wise uniform complete block design with parameters $v = 5$, $b = 20$, $r = 20$, $k = 5$, $\lambda = 5$ and $\alpha = 1$:

2	3	4	5	1	2	3
3	4	5	1	2	3	4
4	5	1	2	3	4	5
5	1	2	3	4	5	1
1	2	3	4	5	1	2
3	4	5	1	2	3	4
4	5	1	2	3	4	5
5	1	2	3	4	5	1
1	2	3	4	5	1	2
2	3	4	5	1	2	3
4	5	1	2	3	4	5
5	1	2	3	4	5	1
1	2	3	4	5	1	2
2	3	4	5	1	2	3
3	4	5	1	2	3	4
5	1	2	3	4	5	1
1	2	3	4	5	1	2
2	3	4	5	1	2	3
3	4	5	1	2	3	4
4	5	1	2	3	4	5
1	2	3	4	5	1	2
2	3	4	5	1	2	3
3	4	5	1	2	3	4
4	5	1	2	3	4	5
5	1	2	3	4	5	1

The information matrices for estimating the direct and neighbouring effects of treatments for AR(2) with $\rho = 0.1\,(\rho_1 = \rho_2 = \rho)$ is obtained given as below:

$$C_\tau = 15.47944\left[I - \frac{J}{5}\right] \quad \text{and} \quad C_l = C_\gamma = 15.45525\left[I - \frac{J}{5}\right]$$

Similarly for MA(2) & NN structures,

$$C_\tau = 15.36396\left[I - \frac{J}{5}\right] \quad \text{and} \quad C_l = C_\gamma = 16.81641\left[I - \frac{J}{5}\right]$$

$$C_\tau = 9.12109\left[I - \frac{J}{5}\right] \quad \text{and} \quad C_l = C_\gamma = 10.44325\left[I - \frac{J}{5}\right]$$

These matrices have been worked out using R package. For, $m = 1$ the resulting design will be a neighbour balanced pair-wise uniform incomplete block design with parameters $v = 5, b = 20$, $r = 16$, $k = 4$, $\lambda = 4$ and $\alpha = 1$.

III. Efficiency of Neighbour Balanced Pair-Wise Uniform Block Designs:

In this section, a quantitative measure of efficiency of the NNBD has been derived when error structure follows AR(2) and MA(2) models. The comparison of universally optimal neighbour balanced design for v treatments in $(v-1)$ complete blocks of Azais *et al.* [3] considering observations to be correlated within the blocks. We compare the average variance of an elementary treatment contrast $\hat{\tau}_s - \hat{\tau}_{s'}$ in both cases. The average variance of an elementary treatment contrast Kempthorne, [8] for direct effects of the neighbour balanced design of Azais *et al.* [3] estimated by generalized least squares methods, is given by

$$V_A = \frac{2\sigma^2}{v-1}\sum_{s=1}^{v-1}\theta_s^{-1}$$

where θ_s's are the $(v-1)$ non-zero eigen values of C_τ for Azais *et al.* [3], σ^2 is the variance of an observation. The efficiency factor (E_τ) for direct effects of the neighbour balanced pair-wise uniform block design is thus given as:

$$E_\tau = \frac{(v-1)\sum_{s=1}^{v-1}\theta_s^{-1}}{(v-m)\sum_{s=1}^{v-1}\delta_s^{-1}}$$

δ_s's are the $(v-1)$ non-zero eigen values of C_τ. Similarly the efficiency (E_l) and (E_γ) for neighbour effects (left and right) of treatments is obtained. The ranges of correlation coefficient (ρ) for different correlation structures investigated are $|\rho| \le 0.40$ for AR(2), MA(2) and NN correlation structures. For these ranges, the matrix of correlation coefficients among observations within a block is positive definite. For $\rho = 0$, the efficiency is that of totally balanced designs obtained by Tomer *et al.* [19].

In Tables 1, 2 and 3 the parameters of neighbour balanced pair-wise uniform block design for $v = 5$ $(m = 0)$ and $v = 6$ $(m = 0, 1, 2)$ along with the efficiency for direct and neighbour effects (left and right) has been shown. The efficiency values have been reported under the AR(2), MA(2) and NN correlation structures with (ρ_1, ρ_2) in the interval -0.4 (-0.4) 0.4. It is seen that efficiency for direct as well as neighbour effects is high, in case of complete block designs i.e., $(m = 0)$ for NN correlation structure. In case of incomplete block designs $(m = 1, 2, \ldots, v-4)$ MA(2) correlation structure turns out to be more efficient as compared to others models with (ρ_1, ρ_2) in the interval 0.1 to 0.4. It is therefore concluded that when block sizes are large and neighbouring plots are highly correlated, generalized least squares for estimation of direct and neighbour effects can be used.

IV. Efficiency of NNBD and NNBIBD in comparison to Regular Block Design

In this section, a quantitative measure of efficiency of NNBD is also derived when the errors follow AR(2) and MA(2) models. If errors within a block follow an AR(2) then

$$V_{AR(2)}(\tau_i - \hat{\tau}_j) = 2r^{-1}\sigma_\varepsilon^2, r_0\left[1 + \frac{\alpha}{r_0} - \frac{2}{t(t-1)}\sum_{i=1}^{t-1}(t-i)r_i\right]$$

where $r_0 = (1-\rho_2)/(1+\rho_2)\left[(1-\rho_2)^2 - \rho_1^2\right]$

$$r_1 = \{\rho_1/(1-\rho_2)\}r_0$$

$$r_2 = \left\{\left[\rho_1^2/(1-\rho_2)\right] + \rho_2\right\}r_0$$

for $k \ge 3$, $r_k = \{\rho_1 r_{k-1} + \rho_2 r_{k-2}\}r_0$

If errors follow second order moving average model MA(2), then

$$V_{MA(2)}\left(\tau_i - \widehat{\tau}_j\right) = 2r^{-1}\sigma_\varepsilon^2, r_0\left[1 + \frac{\alpha}{r_0} - \frac{2}{t(t-1)}\sum_{i=1}^{t-1}(t-i)r_i\right]$$

where, $r_0 = 1 + \rho_1^2 + \rho_2^2$

$r_1 = \rho_1 + \rho_1\rho_2$

$r_2 = \rho_2$

for $k \geq 3$, $r_k = 0$ where $\alpha = \sigma_\xi^2 / \sigma_\varepsilon^2$

For NNBD the variance of an elementary contrast is given by

$$V_N = 2(t-1)^{-1}\sum_{i=1}^{t-1}\gamma_{di}^{-1}$$

Where γ_{di}'s are non-zero values of C_d. We define the efficiency of a design d relative to a regular block design as $\frac{V_{AR(2)}}{V_N}$ and $\frac{V_{MA(2)}}{V_N}$ respectively for AR(2) and MA(2) models.

The Tables 4, 5, 6 and 7 shows the efficiencies of NNBD with $t = 5, r = 20$ and $t = 6, r = 30$, (ρ_1, ρ_2) = -0.4 (-0.4) 0.4 and $\alpha = 1$. The values in the tables show that as ρ increases from -0.4 to -0.1 the gain in efficiency also increases and ρ decreases from 0.1 to 0.4 the gain in efficiency also decreases under AR(2) and MA(2) models. The gain in efficiency of NNBD over regular block design is high under MA(2) models **(**$t = 5$**,** $r = 20$ and $\alpha = 1$**)** for direct and neighbour effects of treatments.

The Tables 8, 9, 10, 11, 12 and 13 show the efficiencies of NNBIBD with $t = 5, r = 16$, $t = 6, r = 25$ and $t = 6, r = 20$, (ρ_1, ρ_2) = -0.4 (-0.4) 0.4 and $\alpha = 1$. The values in the tables show that as ρ increases from -0.4 to -0.1, the gain in efficiency also increases and ρ decreases from 0.1 to 0.4, the gain in efficiency also decreases under AR(2) and MA(2) models. The gain in efficiency of NNBIBD over regular block design is high under MA(2) models **(**$t = 6$**,** $r = 20$ and $\alpha = 1$**)** for direct and neighbour effects of treatments.

V. Conclusion

We have concluded that the efficiency for direct as well as neighbouring effects is high in the case of complete block designs for NN correlation structure. In the case of incomplete block designs, MA(2) correlation structure turned out to be more efficient as compared to other models with (ρ_1, ρ_2) in the interval 0.1 to 0.4. The gain in efficiency of NNBD and NNBIBD over regular block design is high under MA(2) models for direct and neighbour effects of treatments.

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Tables:

Table 1. Efficiency of neighbour balanced pair-wise uniform block designs AR(2) model

Parameters					**Correlation Structure**			
					AR(2)			
v	b	m	r	$k=\lambda$	(ρ_1,ρ_2)	E_τ	E_l	E_γ
5	20	0	20	5	(-0.4,-0.4)	0.56427	0.56808	0.54146
					(-0.3,-0.3)	0.64066	0.64761	0.64341
					(-0.2,-0.2)	0.78467	0.71141	0.60989
					(-0.1,-0.1)	0.76701	0.76929	0.76893
					(0,0)	0.80000	0.80000	0.80000
					(0.1,0.1)	0.80582	0.81691	0.79696
					(0.2,0.2)	0.79178	0.80068	0.80757
					(0.3,0.3)	0.72334	0.79433	0.73051
					(0.4,0.4)	0.88824	0.69126	0.70063
6	30	0	30	6	(-0.4,-0.4)	0.60734	0.59666	0.58634
					(-0.3,-0.3)	0.68867	0.68532	0.73889
					(-0.2,-0.2)	0.74974	0.75752	0.76040
					(-0.1,-0.1)	0.80637	0.79854	0.80464
					(0,0)	0.83333	0.83333	0.83333
					(0.1,0.1)	0.85754	0.84921	0.84619
					(0.2,0.2)	0.83448	0.82871	0.88098
					(0.3,0.3)	0.81760	0.80621	0.76668
					(0.4,0.4)	0.72861	0.71027	0.68055
6	30	1	25	5	(-0.4,-0.4)	0.83447	0.82390	0.81067
					(-0.3,-0.3)	0.87554	0.87287	0.87532
					(-0.2,-0.2)	0.92940	0.92201	0.93455
					(-0.1,-0.1)	0.97469	0.97316	0.98455
					(0,0)	1.00000	1.00000	1.00000
					(0.1,0.1)	1.03364	1.03548	1.04214
					(0.2,0.2)	1.04870	1.05409	1.04523
					(0.3,0.3)	1.03975	1.01356	1.01522
					(0.4,0.4)	0.94793	1.15515	1.70573
6	30	2	20	4	(-0.4,-0.4)	0.93088	0.49412	0.87094
					(-0.3,-0.3)	1.03841	1.00006	1.00628
					(-0.2,-0.2)	1.13443	1.13032	1.13469
					(-0.1,-0.1)	1.21476	1.24003	1.23816
					(0,0)	1.25000	1.25000	1.25000
					(0.1,0.1)	1.32514	1.34697	1.32511
					(0.2,0.2)	1.38279	1.40407	1.41581
					(0.3,0.3)	1.38420	1.41755	1.40362
					(0.4,0.4)	1.43294	1.44810	1.42910

Table 2. Efficiency of neighbour balanced pair-wise uniform block designs MA(2) model

Parameters					**Correlation Structure**			
					MA(2)			
v	b	m	r	$k=\lambda$	(ρ_1,ρ_2)	E_τ	E_l	E_γ
5	20	0	20	5	(-0.4,-0.4)	0.50455	0.50991	0.49682
					(-0.3,-0.3)	0.57578	0.59003	0.58525
					(-0.2,-0.2)	0.71456	0.69540	0.65266
					(-0.1,-0.1)	0.73357	0.65061	0.74019
					(0,0)	0.80000	0.80000	0.80000
					(0.1,0.1)	0.85091	0.86773	0.84908
					(0.2,0.2)	0.86011	0.85519	0.83159
					(0.3,0.3)	0.83086	0.85669	0.82009

					(0.4,0.4)	0.75814	0.70742	0.74459
6	30	0	30	6	(-0.4,-0.4)	0.53221	0.52322	0.50447
					(-0.3,-0.3)	0.62195	0.61417	0.61648
					(-0.2,-0.2)	0.69486	0.66319	0.68810
					(-0.1,-0.1)	0.76892	0.76855	0.76305
					(0,0)	0.83333	0.83333	0.83333
					(0.1,0.1)	0.85323	0.86888	0.86780
					(0.2,0.2)	0.87748	0.87339	0.87662
					(0.3,0.3)	0.84813	0.81497	0.84059
					(0.4,0.4)	0.77341	0.62246	0.77547
6	30	1	25	5	(-0.4,-0.4)	0.66551	0.62619	0.65638
					(-0.3,-0.3)	0.74862	0.74420	0.75138
					(-0.2,-0.2)	0.84034	0.83530	0.83880
					(-0.1,-0.1)	0.92152	0.93806	0.93681
					(0,0)	1.00000	1.00000	1.00000
					(0.1,0.1)	1.06487	1.10040	1.07081
					(0.2,0.2)	1.12050	1.08714	1.11359
					(0.3,0.3)	1.13502	1.12539	1.12761
					(0.4,0.4)	1.12955	1.11200	1.10951
6	30	2	20	4	(-0.4,-0.4)	0.79845	0.78892	0.79581
					(-0.3,-0.3)	0.86223	0.93426	0.88878
					(-0.2,-0.2)	1.05227	1.11215	1.06864
					(-0.1,-0.1)	1.15628	1.17838	1.23751
					(0,0)	1.25000	1.25000	1.25000
					(0.1,0.1)	1.35875	1.35286	1.41124
					(0.2,0.2)	1.42639	1.44671	1.47277
					(0.3,0.3)	1.46134	1.44887	1.43499
					(0.4,0.4)	1.49548	1.42926	1.50805

Table 3. Efficiency of neighbour balanced pair-wise uniform block designs NN model

Parameters					**Correlation Structure**			
					NN			
v	b	m	r	$k=\lambda$	ρ	E_τ	E_l	E_γ
5	20	0	20	5	-0.4	0.57777	0.54370	0.55414
					-0.3	0.77230	0.76003	0.71921
					-0.2	1.12702	1.11899	1.12748
					-0.1	1.32548	1.27062	1.26528
					0	0.80000	0.80000	0.80000
					0.1	1.33665	1.39663	1.26119
					0.2	1.16767	1.20560	1.13899
					0.3	0.77862	0.78772	0.76156
					0.4	0.58218	0.63672	0.56695
6	30	0	30	6	-0.4	0.33727	0.36735	0.32629
					-0.3	0.46641	0.50243	0.45668
					-0.2	0.68408	0.68807	0.67801
					-0.1	1.37697	1.39017	1.34549
					0	0.83333	0.83333	0.83333
					0.1	1.44357	1.40101	1.40017
					0.2	0.93128	0.90471	0.87864
					0.3	0.62177	0.62254	0.58294
					0.4	0.47132	0.45645	0.45121
6	30	1	25	5	-0.4	0.15597	0.21636	0.44046
					-0.3	0.24370	0.21980	0.49732
					-0.2	0.48998	0.41370	0.82479
					-0.1	0.90261	0.95073	0.95956
					0	1.00000	1.00000	1.00000
					0.1	1.01487	1.02528	1.13842
					0.2	0.46956	0.55623	0.68783
					0.3	0.38600	0.38782	0.34714
					0.4	0.30592	0.31637	0.40601
6	30	2	20	4	-0.4	0.85435	0.90279	0.92465
					-0.3	1.08256	1.28288	1.18901
					-0.2	1.77180	1.65902	1.91532
					-0.1	1.59519	1.85066	1.90607
					0	1.25000	1.25000	1.25000
					0.1	1.54050	1.83199	1.90737
					0.2	1.85560	1.81908	1.86553
					0.3	1.24573	1.30222	1.21305
					0.4	0.94025	1.03260	0.77395

Table 4. Efficiency of NNBD using AR(2) Model ($t = 5$, $r = 20$ and $\alpha = 1$)

(ρ_1, ρ_2)	E_τ	E_l	E_γ
(-0.4, -0.4)	0.99707	0.94537	1.02110
(-0.3, -0.3)	1.10683	1.09303	1.05162
(-0.2, -0.2)	1.26307	1.40307	1.66570
(-0.1, -0.1)	2.58826	2.55587	2.57687
(0,0)	0.83954	0.85159	0.86020
(0.1,0.1)	2.42797	2.49175	2.43415
(0.2,0.2)	1.25211	1.29677	1.22696
(0.3,0.3)	0.93993	0.87836	0.93270
(0.4,0.4)	0.49401	0.30393	0.62504

Table 5. Efficiency of NNBD using AR(2) Model ($t = 6$, $r = 30$ and $\alpha = 1$)

(ρ_1, ρ_2)	E_τ	E_l	E_γ
(-0.4, -0.4)	0.53382	0.54339	0.54519
(-0.3, -0.3)	0.61517	0.61818	0.57215
(-0.2, -0.2)	0.79475	0.78659	0.79152
(-0.1, -0.1)	1.44493	1.43084	1.44203
(0,0)	0.83238	0.83238	0.84832
(0.1,0.1)	1.78390	1.80140	1.77907
(0.2,0.2)	0.93644	0.94297	0.85290
(0.3,0.3)	0.64644	0.65558	0.65871
(0.4,0.4)	0.35792	0.36716	0.37387

Table 6. Efficiency of NNBD using MA(2) Model ($t = 5$, $r = 20$ and $\alpha = 1$)

(ρ_1, ρ_2)	E_τ	E_l	E_γ
(-0.4, -0.4)	1.21164	1.14440	1.20920
(-0.3, -0.3)	1.30915	1.27530	1.22896
(-0.2, -0.2)	1.44002	1.49026	1.61606
(-0.1, -0.1)	2.15184	2.07303	2.12199
(0,0)	0.83954	0.85159	0.86020
(0.1,0.1)	2.26616	2.31200	2.25180
(0.2,0.2)	1.17302	1.17594	1.15405
(0.3,0.3)	0.77541	0.77173	0.78728
(0.4,0.4)	0.64231	0.76367	0.65257

Table 7. Efficiency of NNBD using MA(2) Model ($t = 6$, $r = 30$ and $\alpha = 1$)

(ρ_1, ρ_2)	E_τ	E_l	E_γ
(-0.4, -0.4)	0.66202	0.74614	0.68862
(-0.3, -0.3)	0.72312	0.80247	0.72799
(-0.2, -0.2)	0.88883	0.95291	0.90663
(-0.1, -0.1)	1.53897	1.58135	1.54437
(0,0)	0.83238	0.84678	0.84832
(0.1,0.1)	1.77126	1.72860	1.73183
(0.2,0.2)	0.86914	0.86297	0.83654
(0.3,0.3)	0.62122	0.64246	0.58433
(0.4,0.4)	0.51164	0.62631	0.49786

Table 8. Efficiency of NNBIBD using AR(2) Model ($t = 5$, $r = 16$ and $\alpha = 1$)

(ρ_1, ρ_2)	E_τ	E_l	E_γ
(-0.4, -0.4)	0.18527	0.28995	0.26233
(-0.3, -0.3)	0.28763	0.12989	0.16658
(-0.2, -0.2)	0.37600	0.37649	0.18700
(-0.1, -0.1)	0.71467	0.70473	0.62399
(0,0)	0.35740	0.35740	0.35740
(0.1,0.1)	0.72382	0.66743	0.75656
(0.2,0.2)	0.34895	0.35068	0.35801
(0.3,0.3)	0.29492	0.30014	0.29630
(0.4,0.4)	0.16895	0.12241	0.11859

Table 9. Efficiency of NNBIBD using AR(2) Model ($t = 6$, $r = 25$ and $\alpha = 1$)

(ρ_1, ρ_2)	E_τ	E_l	E_γ
(-0.4, -0.4)	0.07397	0.01262	0.13114
(-0.3, -0.3)	0.10149	0.11443	0.13766
(-0.2, -0.2)	0.18434	0.16134	0.07707
(-0.1, -0.1)	0.31455	0.23519	0.27502
(0,0)	0.33414	0.33164	0.33772
(0.1,0.1)	0.32705	0.31905	0.31155
(0.2,0.2)	0.15082	0.15102	0.15221
(0.3,0.3)	0.12668	0.12051	0.13172
(0.4,0.4)	0.07168	0.05185	0.03884

Table 10. Efficiency of NNBIBD using AR(2) Model ($t = 6$, $r = 20$ and $\alpha = 1$)

(ρ_1, ρ_2)	E_τ	E_l	E_γ
(-0.4, -0.4)	0.52996	0.71555	0.59596
(-0.3, -0.3)	0.56881	0.67374	0.62672
(-0.2, -0.2)	0.81717	0.56405	0.85855
(-0.1, -0.1)	1.50430	1.51538	0.36159
(0,0)	0.48303	0.48130	0.48606
(0.1,0.1)	1.33171	0.99505	1.42879
(0.2,0.2)	0.67638	0.69771	0.64563
(0.3,0.3)	0.45953	0.45152	0.42899
(0.4,0.4)	0.11869	0.08814	0.09162

Table 11. Efficiency of NNBIBD using MA(2) Model ($t = 5$, $r = 16$ and $\alpha = 1$)

(ρ_1, ρ_2)	E_τ	E_l	E_γ
(-0.4, -0.4)	0.25723	0.38273	0.35743
(-0.3, -0.3)	0.41218	0.16191	0.23084
(-0.2, -0.2)	0.43682	0.43746	0.22084
(-0.1, -0.1)	0.80030	0.74586	0.70076
(0,0)	0.35746	0.35746	0.35746
(0.1,0.1)	0.67062	0.65268	0.70637
(0.2,0.2)	0.31053	0.30814	0.31343
(0.3,0.3)	0.25911	0.24575	0.24123
(0.4,0.4)	0.14553	0.14651	0.14770

Table 12. Efficiency of NNBIBD using MA(2) Model ($t = 6$, $r = 25$ and $\alpha = 1$)

(ρ_1, ρ_2)	E_τ	E_l	E_γ
(-0.4, -0.4)	0.10080	0.01805	0.17602
(-0.3, -0.3)	0.12601	0.14248	0.17024
(-0.2, -0.2)	0.21132	0.18459	0.08900
(-0.1, -0.1)	0.33790	0.24780	0.29348
(0,0)	0.33414	0.33414	0.33414
(0.1,0.1)	0.31362	0.29660	0.29955
(0.2,0.2)	0.13776	0.14291	0.13943
(0.3,0.3)	0.11287	0.10556	0.11534
(0.4,0.4)	0.09128	0.08172	0.09062

Table 13. Efficiency of NNBIBD using MA(2) Model ($t = 6$, $r = 20$ and $\alpha = 1$)

(ρ_1, ρ_2)	E_τ	E_l	E_γ
(-0.4, -0.4)	0.67145	0.69124	0.70874
(-0.3, -0.3)	0.72723	0.76561	0.75328
(-0.2, -0.2)	0.91314	0.59420	0.94491
(-0.1, -0.1)	1.60506	1.61956	0.36742
(0,0)	0.48303	0.48130	0.48606
(0.1,0.1)	1.28310	0.78202	1.32538
(0.2,0.2)	0.63993	0.66086	0.60573
(0.3,0.3)	0.42334	0.42966	0.40811
(0.4,0.4)	0.33980	0.37711	0.25161