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Electronic Spin Orbit Coupling

J.A.deWET
PO Box 514 Plettenberg Bay 6600 South Africa

Abstract: *In this note we will see how representations of the two-dimensional Unitary Group U(2) used long ago by the Author for the Many Electron Problem, gives rise to an equilateral triangle in a torus that also defines a Theta Function with Equiharmonic frequencies and whose vertices are up and down spins leading to spin-orbit coupling.*
Keywords: *Equiharmonics, Jacobi Theta Function, Coupling Constant, Lattice.*

I. Introduction

In this note we see how representations of the 2 dimensionalUnitary Group U(2),used long ago by [3] de Wet for the 'Many Electron Problem',leads to a lattice that is an equilateral triangle in a torus Fig. 1 that has been shown [4] to define a Theta Function nome q with equiharmonic frequenciesτ=sin120=sin ω or sin 60 Furthermore ω is also the angle of the tritangent to a cubic surface defining the Exceptional Lie algebra E_6 [5] that has been used by de Wet [4] to map the Standard Model. Specifically there are equilateral triangles that rotate the quarks uud, ddu into one another as shown by Fig. 1 of [4]. In this way the nome q is a quark coupling constant. It has the value q=0.06583 that is close to the constant 0.118 found by Davies et. al. [2].If we now concentrate on the complimentary angle tan 60 where $\tau = \sqrt{3}$ then we find a possible nuclear coupling constant of 0.0043 that is the same order of magnitude as suggested by Rees [8], vh, 4.
We can now study electronic spin-orbit coupling in the same light.

II. Spin-Orbit Coupling

In this section we will rely heavily on the excellent book of Mumford [7] who considered the 2d-Complex Group in some detail in Ch. 1. Specifically he considered representations of U(2) and showed on pg. 42 that if $\gamma = U(2)$ then i τ $\geq \sqrt{3}/_2$= sin ω which is the equilateral torus of Fig.1 with boundaries AB, BC defined by cos 60=1/2 and sin ω =120° of a fundamental domain. Therefore if we label the 3 apices by A=up, B=down and C= up and cyclic, then a rotation of 60 °will lead to udu then dud that imply spin-orbit coupling, because one spin must be in a different orbit by the Exclusion Principle. Then on pg.74 Mumford finds the Theta Function
Θ(0, τ) =q+q4+q9+···· (1)
Where the nome$\frac{1}{q}$ = exp ($\pi\tau$) =exp ($\pi\frac{iK}{K}$). and the dependence on z is carried by the lattice of Fig. 1[6]. The series converges very rapidly if q=0.06583.
Equation (1) is also the representation of an integer as the sum of squares ([1] p. 42) and on pages 62and 63 Mumford shows that Fig.1 is a complex torus.

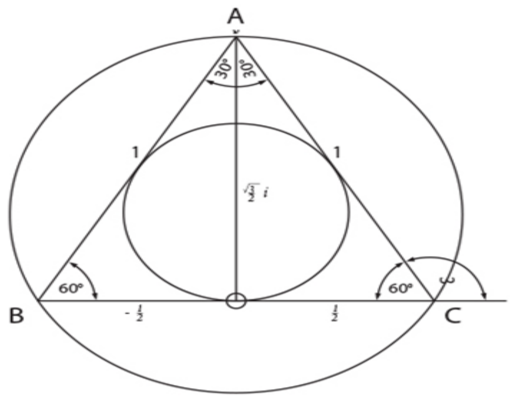

Fig 1
Spin Coupling Lattice

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Table 1

Column Head	Column Head	Column Head	Column Head	Column Head
Row Head	123	123	123	123
Row Head	456	456	456	456
Row Head	789	789	789	789
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Figures

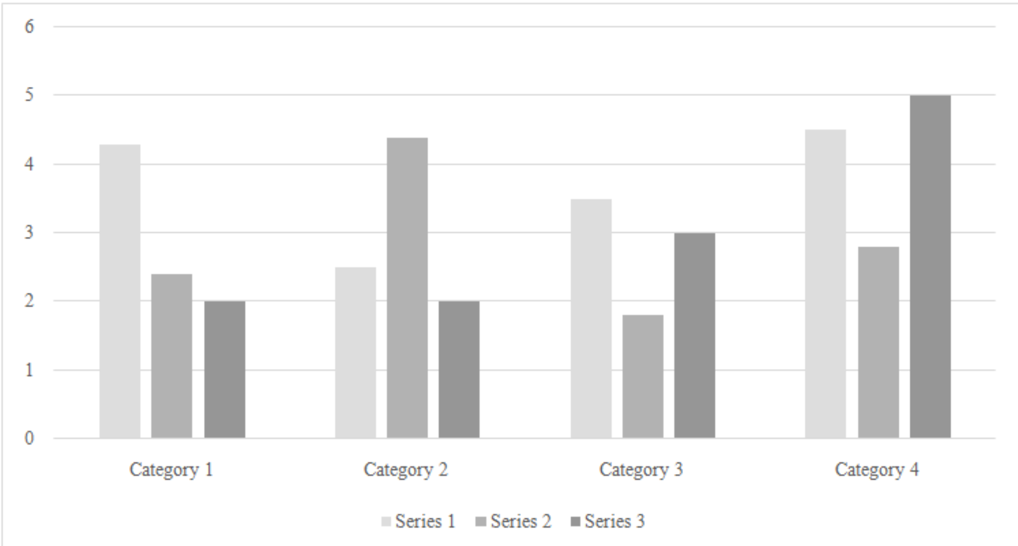

Figure 1. For more inform[Include all figures in their own section, following references (and footnotes and tables, if applicable). Include a numbered caption for each figure. Use the Table/Figure style for easy spacing between figure and caption.]ation about all elements of APA formatting, please consult the *APA Style Manual, 6th Edition*.