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Pgrw-closed map in a Topological Space

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Abstract: *The aim of this paper is to introduce pgrw-closed maps and pgrw*-closed maps and to obtain some of their properties. In section 3 pgrw-closed map is defined and compared with other closed maps. In section 4 composition of pgrw-maps is studied. In section 5 pgrw*-closed maps are defined.*
Keywords: *pgrw-closed set, pgrw-closed maps, pgrw*-closed maps.*

I. Introduction

Different mathematicians worked on different versions of generalized closed maps and related topological properties. Generalized closed mappings were introduced and studied by Malghan [1]. wg-closed maps and rwg-closed maps were introduced and studied by Nagaveni [2]. Regular closed maps, gpr-closed maps and rw-closed maps were introduced and studied by Long [3], Gnanambal [4] and S. S. Benchallli [5] respectively.

II. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) represent the topological spaces. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. X\A or A^c denotes the complement of A in X.
We recall the following definitions and results.
Definition 2.1
A subset A of a topological space (X, τ) is called
1. a semi-open set[6] if A ⊆ cl(int(A)) and a semi-closed set if int(cl(A)) ⊆ A.
2. a pre-open set[7] if A ⊆ int(cl(A)) and a pre-closed set if cl(int(A)) ⊆ A.
3. an α-open set [8]if A ⊆ int(cl(int(A))) and an α -closed set if cl(int(cl(A)))⊆ A.
4. a semi-pre open set [9](=β-open)[10] if A⊆cl(int(cl(A))) and a semi-pre closed set (=β-closed) if int(cl(int(A)))⊆A.
5. a regular open set [10] if A = int(clA)) and a regular closed set if A = cl(int(A)).
6. δ-closed [11] if A = clδ(A), where clδ(A) = {x϶ X : int(cl(U))∩A≠ Φ, U ϵ τand x ϵ U}
7. a regular semi open [12] set if there is a regular open set U such that U ⊆ A ⊆cl(U).
8. a regular α-closed set (briefly, rα-closed)[13] if there is a regular closed set U such that U ⊂ A ⊂αcl(U).
9. a generalized closed set (briefly g-closed)[14] if cl(A)⊆U whenever A⊆U and U is open in X.
10. a regular generalized closed set(briefly rg-closed)[15] if cl(A)⊆U whenever A⊆U and U is regular open in X.
12. a generalized pre regular closed set(briefly gpr-closed)[4] if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
13. a generalized semi-pre closed set(briefly gsp-closed)[16] if spcl(A) ⊆ U whenever A⊆U and U is open in X.
14. a w-closed set [17] if cl(A) ⊆ U whenever A⊆ U and U is semi-open in X.
15. a pre generalized pre regular closed set[18] (briefly pgpr-closed) if pcl(A)⊆ U whenever A⊆U and U is rg-open in X.
16. a generalized semi pre regular closed **(**briefly gspr-closed) set [19] if spcl(A)⊆U whenever A⊆U and U is regular open in X.
17. a generalized pre closed (briefly gp-closed) set[20] if pcl(A)⊆U whenever A⊆U and U is open in X.
18. a #regular generalized closed (briefly #rg-closed) set [21] if cl(A)⊆U whenever A ⊆ U and U is rw-open.
19. a g*s-closed [22]set if scl (A)⊆ U whenever A⊆ U and U is gs open.
20. rwg-closed [2] set if cl(int(A))⊆U whenever A ⊆U and U is regular -open in X.
21. a rw-closed [5] if cl(A)⊆U whenever A ⊆U and U is regular semi-open in X.
22. αg-closed[23] if αcl(A)⊆U whenever A ⊆U and U is open in X.
23. a ωα-closed set[24] if αcl(A) ⊆ U whenever A⊆ U and U is ω-open in X.
24. an α-regular w closed set(briefly αrw -closed)[25] if αcl(A)⊆U whenever A⊆U and U is rw-open in X.
The complements of the above mentioned closed sets are the respective open sets.

Definition 2.2
A map f : (X, τ) → (Y, σ) is said to be
1. α-closed [8] if f(F) is α–closed in Y for every closed subset F of X.
2. αg-closed [23] if f(F) is αg-closed in Y for every closed subset F of X.
4. rwg-closed [2] if f(V) is rwg-closed in Y for every closed subset V of X.
6. gp-closed [20] if f(V) is gp-closed in Y for every closed subset V of X.
7. gpr-closed[4] if f(V) is gpr-closed in Y for every closed subset V of X.
8. ωα-closed [24] if f(V) is ωα-closed in Y for every closed subset V of X.
9. gspr-closed [19]if f(V) is gspr-closed in Y for every closed subset V of X.
10. ω-closed [17] if f(V) is ω-closed in Y for every closed subset V of X.
11. rω-closed [5] if f(V) is rw-closed in Y for every closed subset V of X.
12. regular-closed if f(F) is closed in Yfor every regular closed set F of X.
13. g*s-closed [22]map if for each closed set F in X,f(F) is a g*s-closed in y.
14.αrω-closed [25] if the image of every closed set in (X, τ) is αrω-closed in (Y, σ).
15. pre-closed [26] if f (V) is pre-closed in Y for every closed set V of X.
16. δ-closed [11] if for every closed set G in X, f (G) is a δ-closed set in Y.
17. #rg-closed [21] if f(F) is #rg-closed in (Y,σ) for every #rg-closed set F of (X,τ).
18. gsp-closed [16] if f(V) is gsp-closed in (Y, σ) for every closed set V of (X, τ).
19. semi-closed [27] if image of every closed subset of X is semi-closed in Y.
20. Contra-closed [28] if f(F) is open in Y for every closed set F of X.
21. Contra regular-closed if f(F) is r-open in Y for every closed set F of X.
22. Contra semi-closed [29] if f(F) is s-open in Y for every closed set F of X.
23. Semi pre-closed [30] (Beta-closed) if f(V) is semi-pre-closed in Y for every closed subset V of X.
24. g-closed [14] if f(V) is g-closed in Y for every closed subset V of X.
25. rα-closed [13] if f(V) is rα-closed in Y for every closed subset V of X.

The following results are from [31]
Theorem: Every pgpr-closed set is pgrw-closed.
Theorem: A pre-closed set is pgrw-closed.
Corollary: Every α- closed set is pgrw- closed.
Corollary: Every closed set is pgrw-closed.
Corollary: Every regular closed set is pgrw-closed.
Corollary: Every δ- closed set is pgrw- closed.
Theorem: Every #rg- closed set is pgrw- closed**.**
Theorem: Every αrw-closed set is pgrw-closed.
Theorem: Every pgrw-closed set is gp-closed
Theorem: Every pgrw- closed set is gsp-closed.
Corollary: Every pgrw- closed set is gspr- closed.
Corollary: Every pgrw- closed set is gpr- closed.
Theorem: If A is regular open and pgrw-closed, then A is pre-closed.
Theorem: If A is open and gp-closed, then A is pgrw-closed.
Theorem: If A is both open and g-closed, then A is pgrw -closed.
Theorem: If A is regular- open and gpr-closed, then it is pgrw-closed.
Theorem: If A is both semi-open and w-closed, then it is pgrw-closed.
Theorem: If A is open and αg-closed, then it is pgrw -closed**.**

III. Pgrw-CLOSED MAP

Definition 3.1: A map f **:**(X, , τ) → (Y, σ) is said to be a pre generalized regular weakly-closed map(pgrw-closed map) if the image of every closed set in (X, τ) is pgrw-closed in (Y, σ).

Example 3.2: X={a,b,c} , τ ={X, ϕ, {a}} and Y={a,b,c}, σ ={Y, ϕ, {a},{b,c}}.
Closed sets in X are X ,ϕ,{b,c}. pgrw-closed sets in Y are Y,ϕ,{a},{b},{c},{a,b},{a,c},{b,c}.
A map f :X→Y is defined by f(a)=b, f(b)=c,f(c)=a. Image of every closed set in X is pgrw-closed in Y. So f is a pgrw-closed map.

Theorem 3.3: Every closed map is a pgrw-closed map.
Proof: f: (X, τ) → (Y, σ) is a closed map.
⇒ ∀ closed set A in X f(A) is closed in Y.
⇒ ∀ closed set A in X f(A) is pgrw- closed in Y.
⇒ f is a pgrw-closed map.
The converse is not true.

Example: 3.4: In the example 3.2 f is a pgrw-closed map and as {b,c} is closed in X and f({b,c})={a,c}is not closed in Y, f is not a closed map.

Theorem 3.5: Every pre-closed (regular-closed, α-closed,δ-closed ,#rg-closed, pgpr-closed, αrw-closed) map is pgrw–closed.

Proof: f: (X, τ) → (Y, σ) is a pre-closed map.

⇒ ∀ closed set A in X f(A) is pre- closed in Y.

⇒ ∀ closed set A in X f(A) is pgrw- closed in Y.

⇒ f is a pgrw-closed map.

Similarly remaining statements can be proved.

The converse is not true.

Example 3.6: X={a,b,c} , τ ={X, ϕ, {a},{b},{a,b}} and Y={a,b,c,d} , σ ={Y, ϕ, {a}, {b}, {a,b}, {a,b,c}}. Closed sets in X are X, ϕ,{c},{b,c},{a,c}.Pgrw-closed sets in Y are Y, ϕ, {c}, {d}, {b,c}, {c,d},{a,d},{b,d},{b,c,d},{a,c,d},{a,b,d}. Pre-closed sets in Y are Y, ϕ, {c}, {d},{c,d}, {a,c,d},{b,c,d}. pgpr-closed sets in Y are Y, ϕ, {c}, {d},{c,d}, {a,c,d},{b,c,d}. δ-closed sets in Y are Y, ϕ, {c,d}, {a,c,d},{b,c,d}. A map f :X→Y is defined by f(a)=b, f(b)=c, f(c)=d. f is a pgrw-closed map.{a,c} is closed in X. f({a,c})={b,d} which is neither a pre-closed nor a pgpr-closed set. So f is neither a pre-closed map nor a pgpr-closed map. {c} is closed in X. f({c})={d} is not δ-closed. So f is not a δ-closed map.

Example 3.7: In the example 3.2 regular closed sets in Y are Y, ϕ, {a}, {b,c}, α-closed sets in Y are Y, ϕ, {a}, {b,c} ,#rg-closed sets in Y are Y, ϕ, {a}, {b,c} and αrw-closed sets are Y, ϕ, {a}, {b,c}. f is a pgrw closed map, but f({b,c})={a,c} is neither a regular closed set nor α-closed nor #rg-closed nor αrω-closed . So f is neither a regular closed map nor α-closed nor #rg-closed nor αrw-closed.

Theorem 3.8: If f: (X, τ) → (Y, σ) is a contra-r-closed and pgrw-closed map, then f is pre-closed.

Proof: f: (X, τ) → (Y, σ) is a contra-r-closed and pgrw-closed map.

⇒ ∀ closed set A in X f(A) is regular open and pgrw-closed in Y.

⇒ ∀ closed set A in X f(A) is pre- closed in Y.

⇒ f is a pre-closed map.

Theorem 3.9: If f: (X, τ) → (Y, σ) is a pgrw–closed map, then f is a gp-closed(gsp-closed, gspr-closed, gpr-closed) map.

Proof: f: (X, τ) → (Y, σ) is a pgrw-closed map.

⇒ ∀ closed set A in X f(A) is a pgrw-closed set in Y.

⇒ ∀ closed set A in X f(A) is a gp-closed set in Y.

⇒ f is a gp-closed map. Similarly the other results follow.

The converse is not true.

Example 3.10: X={a,b,c}, τ **=**{X,ϕ,{a},{b,c}}. Y={a,b,c}, σ ={Y, ϕ ,{a}}.

Pgrw-closed sets in Y are Y, ϕ, {b}, {c},{b,c}. gp-closed sets in Y are ,ϕ,{b},{c},{a,b},{b,c},{a,c}. gpr-closed sets in Y are all subsets of Y. A map f: (X, τ) → (Y, σ) is defined by f(a)=b, f(b)=a, f(c)=c. f is gp-closed and gpr-closed. f({b,c})={a,c} is not pgrw-closed . So f is not pgrw-closed.

Example 3.11:X={a,b,c,d} , τ ={X, ϕ, {a},{b},{a,b},{a,b,c}} and Y={a,b,c},
σ ={Y, ϕ, {a}, {b}, {a,b}}. Closed sets in X are X, ϕ,{b,c,d},{a,c,d},{c,d},{d}.
gsp-closed sets in Y are all subsets of Y.gspr-closed sets in Y are all subsets of Y. pgrw-closed sets in Y are Y, ϕ, {c}, {a,c}, {b,c}. A map f :X→Y is defined by f(a)=b, f(b)=a, f(c)=c, f(d)=a. f is gsp-closed and gspr-closed, but f is not pgrw-closed.

Theorem 3.12: If f: (X, τ) → (Y, σ) is a contra closed and gp-closed map, then f is pgrw-closed.

Proof: f: (X, τ) → (Y, σ) is a contra closed and gp-closed map.

⇒∀ closed set V in X f(V) is open and gp-closed in Y .

⇒ ∀ closed set V in X f(V) is pgrw-closed in Y.

⇒ f is a pgrw-closed map.

Theorem 3.13: If f: (X, τ) → (Y, σ) is a contra-closed and αg-closed map, then f is pgrw-closed.

Proof: f: (X, τ) → (Y, σ) is a contra closed and αg -closed map.

⇒∀ closed set V in X f(V) is open and αg -closed in Y .

⇒ ∀ closed set V in X f(V) is pgrw-closed in Y.

⇒ f is a pgrw-closed map.

Theorem 3.14: If f: (X, τ) → (Y, σ) is a contra regular-closed and gpr-closed map, then f is a pgrw-closed map.

Proof: f: (X, τ) → (Y, σ) is a contra regular-closed and gpr-closed map.

⇒∀ closed set V in X f(V) is regular-open and gpr-closed in Y .

⇒ ∀ closed set V in X f(V) is pgrw-closed in Y.

⇒ f is a pgrw-closed map.

Theorem 3.15: If f: (X, τ) → (Y, σ) is a contra semi-closed and w-closed map, then f is a pgrw-closed map.
Proof: f: (X, τ) → (Y, σ) is a contra semi-closed and w-closed map.
⇒∀ closed set V in X f(V) is a semi-open and w-closed set in Y .
⇒ ∀ closed set V in X f(V) is pgrw-closed in Y.
⇒ f is a pgrw-closed map.
Theorem 3.16: If f: (X, τ) → (Y, σ) is a contra closed and g-closed map, then f is pgrw-closed.
Proof: f: (X, τ) → (Y, σ) is a contra closed and g-closed map.
⇒∀ closed set V in X f(V) is an open and g-closed set in Y .
⇒ ∀ closed set V in X f(V) is pgrw-closed in Y.
⇒ f is a pgrw-closed map.
The following examples illustrate that the pgrw-closed map and rw-closed map (g^*s-closed map, rα-closed map and wα-closed map, β-closed map, semi-closed map) are independent.
Example 3.17: To show that pgrw-closed map and rw-closed map are independent.
i)X={a,b,c} , τ ={X, ϕ, {a}, {a,c}}. Y={a,b,c,d} , σ ={Y, ϕ, {a,b},{c,d}.
pgrw-closed sets in Y are all subsets of Y. rw-closed sets in Y are Y,ϕ,{a,b},{c,d}. A map f:(X, τ) → (Y, σ) is defined by f(a)=b, f(b)=c, f(c)=a. f is pgrw-closed , but f is not rw-closed.
ii) X={a,b,c},τ ={X, ϕ, {a}},Y={a,b,c}, σ ={Y, ϕ, {a}, {b}, {a,b}}.
pgrw-closed sets in Y are Y, ϕ , {c}, {b,c}, {a,c}. rw-closed sets in Y areY,ϕ,{c},{a,b},{b,c},{a,c}}. A map f:(X, τ) → (Y, σ) is defined by f(a)=c, f(b)=a, f(c)=b. f is not pgrw-closed but f is rw-closed.
Example 3.18: To show that pgrw-closed map and g^*s-closed map are independent.
i)X={a,b,c,d} , τ ={X, ϕ, {a}, {a,c}}. Y={a,b,c} , σ ={Y, ϕ, {a},{b,c}}.
pgrw-closed sets in Y are all subsets of Y. g*s-closed sets in Y are Y,ϕ,{a}, {b,c}. A map f :X→Y is defined by f(a)=c, f(b)=a, f(c)=b,f(d)=b. f is pgrw-closed , but f is not g*s-closed.
ii)X={a,b,c,d} , τ ={X, ϕ, {b,c},{b,c,d} {a,b,c }}. Y={a,b,c,d} , σ ={Y, ϕ, {a}, {b},{a,b}, {a,b,c}}. pgrw-closed sets in Y are Y,ϕ,{c}, {d}, {b,c}, {c,d},{a,d}, {b,d}, { b,c,d}, {a,c,d}, {a,b,d}. g^*s-closed sets in Y are Y, ϕ,{a},{b},{c},{d},{b,c},{c,d},{a,d},{a,c},{b,c,d},{a,c,d}. A map f :X→Y is defined by f(a)=c, f(b)=d, f(c)=b, f(d)=a. f is not pgrw-closed , but f is g^*s-closed.
Example 3.19: To show that pgrw-closed map and rα-closed map are independent.
i) X={a,b,c}, τ={X,ϕ,{a},{a,c}}and Y={a,b,c}, σ ={Y, ϕ, {a},{b,c}}. Pgrw-closed sets in Y are Y,ϕ,{a}, {b}, {c}, {a,b},{b,c}, {a,c}. rα-closed sets in Y are Y,ϕ ,{a},{b,c}. A map f :X→Y is defined by f(a)=c, f(b)=a, f(c)=b. f is a pgrw-closed map, but f is not rα-closed .
ii)X={a,b,c,d} , τ ={X, ϕ, {a}, {c,d}, {a,c,d}}. Y={a,b,c,d}, σ={Y,ϕ,{a}, {b}, {a,b}, {a,b,c}}. Pgrw-closed sets in Y areY, ϕ ,{c},{d},{b,c},{c,d},{a,d},{b,d},{b,c,d},{a,c,d},{a,b,d}.
rα-closed sets in Y are Y,ϕ,{a},{b}, {b,c,d},{a,c,d},{a,d},{b,d},{a,c},{b,c}.
A map f :X→Y is defined by f(a)=c, f(b)=a, f(c)=c , f(d)=d. f is not a pgrw-closed map, but f is rα-closed.
Example 3.20: To show that pgrw-closed map and wα-closed map are independent.
i)X={a,b,c,d}, τ={X,ϕ,{a,b}, {c,d}}. Y={a,b,c,d}σ ={Y, ϕ,{b,c},{b,c,d},{a,b,c}}
Pgrw-closed sets in Y are Y,ϕ ,{a},{b},{c},{d},{a,b},{a,c}, {c,d}, {a,d},{b,d}, {a,c,d}, {a,b,d}.wα-closed sets in Y are Y,ϕ,{a},{d},{a,d},{a,c,d},{a,b,d}. A map f :X→Y is defined by f(a)=a, f(b)=c, f(c)=d,f(d)=a. f is pgrw-closed , but f is not wα-closed.
ii) X={a,b,c}, τ ={X,ϕ,{a},{b}, {a,b}}. Y={a,b,c,d} σ ={Y, ϕ,{ a},{b}, {a,b},{a,b,c}}
pgrw-closed sets in Y are Y,ϕ,{c},{d},{b,c}, ,{c,d},{a,d},{b,d}, {b,c,d},{a,c,d},{a,b,d}.
wα-closed sets in Y are Y,ϕ,{a},{b},{c},{a,b},{b,c},{a,c},{a,b,c},{a,b,d}. A map f :X→Y is defined by f(a)=c, f(b)=a, f(c)=b. f is not pgrw-closed, but wα-closed.
Example 3.21: To show that pgrw-closed map and β-closed map are independent.
i) X={a,b,c,d},σ={X,ϕ,{a},{b},{a,b},{a,b,c} } and Y={a,b,c,d}, σ ={Y,ϕ,{a},{b},{a,b},{a,b,c}} pgrw-closed sets in Y are Y, ϕ, {c}, {d}, {b,c}, {c,d}, {a,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d}. β-closed sets in Y are Y, ϕ, ,{a},{b},{c},{d},{b,c}, {a,c}, {b,d}, {c,d}, {a,d}, {b,c,d},{a,c,d}. A map f :X→Y is defined by f(a)=b, f(b)=a, f(c)=b, f(d)=d. f is pgrw-closed map, but not β -closed.
ii) A map f :X→Y is defined by f(a)=c, f(b)=c,f(c)=d,f(d)=a in the above example. f is β-closed, but not pgrw-closed.
Example 3.22: To show that pgrw-closed map and semi-closed map are independent.
i) X={a,b,c}, τ ={X,ϕ,{a}} and Y={a,b,c,d}, σ={Y,ϕ,{a},{b},{a,b},{a,b,c}}
Pgrw-closed sets in Y are Y,ϕ,{c},{d},{b,c}, ,{c,d},{a,d},{b,d}, {b,c,d},{a,c,d},{a,b,d}.
Semi-closed sets in Y are Y, ϕ, {a},{b},{c},{d},{b,d},{a,c},{b,c},{c,d}, {a,d},{b,c,d},{a,c,d}.
A map f :X→Y is defined by f(a)=b, f(b)=a,f(c)=d. f is pgrw-closed, but f is not semi-closed.
ii) A map f :X→Y is defined by f(a)=d,f(b)=a,f(c)=a in the above example. f is semi-closed, but not pgrw-closed.

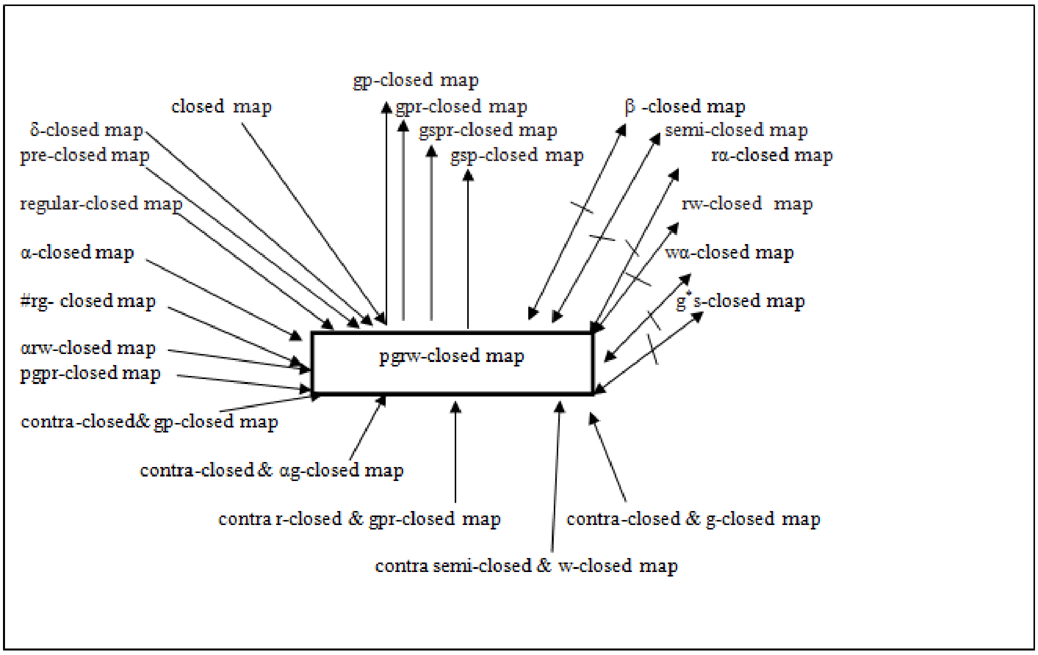
Theorem 3.23: If a map f: (X, τ) → (Y, σ) is pgrw-closed and A is a closed subset of X, then f_A: (A, τ_A) → (Y, σ) is pgrw-closed.

Proof: A is a closed set of X. Let F be a closed set of (A, τ_A). Then F = A ∩ E for some closed set E of (X, τ) and so F is a closed set of (X, τ). Since f is a pgrw–closed map, f (F) is pgrw-closed set in (Y, σ). But for every F in A , f_A(F)= f(F) and ∴ f_A: (A, τ_A) → (Y, σ) is pgrw-closed.

Theorem 3.24: If a map f:(X, τ) → (Y, σ) is pgrw-closed , then pgrwcl(f(A)) ⊆f(cl(A)) for every subset A of X.

Proof: Suppose f:(X, τ) → (Y, σ) is a pgrw-closed map.Let A ⊆X . As cl(A) is closed in X and f is pgrw-closed,f(cl(A)) is pgrw-closed inY . and so pgrwcl(f(cl(A))) = f(cl(A))…….(1)[32]. Next A⊆ cl(A). ∴f(A) ⊆ f(cl(A)). ∴ pgrwcl(f(A)) ⊆pgrwcl(f(cl(A))) → (ii)[32].

From (i) and (ii), pgrw-cl(f(A)) ⊆f(cl(A)) ∀ subset A of (X, τ).



In the above diagram,

A ⟶ B means'If A, then B.'

A ⟷̸ B means 'A and B are independent.'

Theorem 3.25: If a map f:(X, τ) → (Y, σ) is such that pcl(f(A))=pgrwcl(f(A))⊆f(cl(A)) ∀ A in X, then f is a pgrw-closed map.

Proof: Hypothesis: f:(X, τ) → (Y, σ) is a map such that pcl(f(A))=pgrwcl(f(A))⊆f(cl(A)) ∀ A in X. A is a closed subset of (X, τ).

⇒ A=cl(A) ⇒ f(A)=f(cl(A)).

⇒ pgrwcl(f(A)) ⊆f(A), by the hypothesis pgrwcl(f(A))⊆f(cl(A)) ∀ A in X.

⇒ f(A)= pgrwcl(f(A)) because f(A) ⊆ pgrwcl(f(A)) ∀ A in X.

=pcl(f(A)) by hypothesis. ⇒f(A) is pre-closed.

⇒ f(A) is pgrw-closed in (Y, σ). Thus ∀ closed set A in X f(A) is pgrw-closed in (Y, σ).

Hence f is a pgrw-closed map.

Theorem 3.26:A map f:(X, τ) → (Y, σ) is pgrw-closed if and only if ∀ subset S of (Y, σ) and for every open set U containing f^{-1}(S) in X, there is a pgrw-open set V of (Y, σ)such that S ⊆V and f^{-1}(V) ⊆U.

Proof:i) f : (X, τ) → (Y, σ) is a map, S is subset of Y and f^{-1}(S)⊆U , a subset of X. ⇒S∩ f(X − U)=ϕ ⇒S⊆Y-f(X-U).

ii)f : (X, τ) → (Y, σ) is a pgrw-closed map and U is an open set in X.

⇒f(X-U) is a pgrw-closed set in Y.

⇒Y-f(X-U)=V(say) is a pgrw-open set in Y.

⇒f^{-1}(V) =X – f^{-1}(f(X -U)) ⊆X− (X − U) = U.

So from (i) and (ii) if f: (X, τ) → (Y, σ) is a pgrw-closed map, then ∀ S ⊆Y and ∀ open set U containing f^{-1}(S) in X ∃ a pgrw-open set V= Y- f(X −U) such that S⊆V and f^{-1}(V)⊆U.

Conversely

Suppose $f : (X, \tau) \rightarrow (Y, \sigma)$ is a map such that $\forall S \subseteq Y$ and $\forall$ open set U containing $f^{-1}(S)$ in X, there exists a pgrw-open set V in Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.
$\forall$ $F \subseteq X$ and for any map $f : X \rightarrow Y$, $f^{-1}((f(F))^c) \subseteq F^c$. If F is a closed subset of X, then F^c is open in X. Take $S=(f(F))^c$ and $U=F^c$. Then by the hypothesis $\exists$ a pgrw-open set V in Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$. i.e. $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c \Rightarrow V^c \subseteq f(F)$ and $F \subseteq (f^{-1}(V))^c$
$\Rightarrow V^c \subseteq f(F)$ and $f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c \Rightarrow V^c \subseteq f(F)$ and $f(F) \subseteq V^c \Rightarrow V^c = f(F)$
As V is pgrw-open, V^c is pgrw-closed in Y .i.e. f(F) is pgrw-closed in Y. Thus $\forall$ closed set F in X, f(F) is pgrw-closed in Y. Hence $f : (X, \tau) \rightarrow (Y, \sigma)$ is a pgrw-closed map.

Theorem 3.27: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a surjective, continuous, pgrw-closed and open map and $cl(F) = F$, $\forall$ pgrw-closed set F in Y where X is regular, then Y is regular.

Proof: Let U be an open set in Y and $y \in U$.
f is surjective. $\therefore \exists$ a point x in $f^{-1}(U)$ such that $f(x)=y$.
f is continuous and U is open in Y. $\therefore f^{-1}(U)$ is open in X. X is a regular space. $\therefore \exists$ an open set V in X such that $x \in V \subseteq cl(V) \subseteq f^{-1}(U)$ and so $f(x) \in f(V) \subseteq f(cl(V) \subseteq f(f^{-1}(U))$.
i.e. $y \in f(V) \subseteq f(cl(V)) \subseteq U$…………(i)
f is a pgrw-closed map and cl(V) is closed in X. $\therefore$ f(cl(V)) is pgrw-closed in Y and so by the hypothesis $cl(f(cl(V)))=f(cl(V)$………(ii)
Also $V \subseteq cl(V) \Rightarrow f(V) \subseteq f(cl(V)) \Rightarrow cl(f(V) \subseteq cl(f(cl(V)))=f(cl(V)$ ………………(iii)
From (i),(ii) and (iii) we have $y \in f(V) \subseteq cl(f(V) \subseteq U$. V is open in X and f is an open map. $\therefore$ f(V) is open in Y. Thus $\forall$ open set U in Y and $\forall$ $y \in U$, $\exists$ an open set f(V) in Y such that $y \in f(V) \subseteq cl(f(V)) \subseteq U$. Hence Y is a regular space.

Theorem 3.28: If $f: (X,\tau) \rightarrow (Y,\sigma)$ is a continuous, pgrw–closed and bijective map and X, a normal space, then for every pair of disjoint closed sets A and B in (Y,σ), there exist disjoint pgrw-open sets C and D in Y such that $A \subseteq pint(C)$ and $B \subseteq pint(D)$.

Proof: A and B be disjoint closed sets in (Y,σ). If f is continuous, then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed sets of (X,τ). If X is a normal space , then $\exists$ disjoint-open sets U and V in X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Now f is a pgrw-closed map, $A \subseteq Y$ and U, an open set containing $f^{-1}(A)$ in X. $\Rightarrow \exists$ a pgrw-open set C in Y such that $A \subseteq C$ and $f^{-1}(C) \subseteq U$ by theorem 3.26. Similarly for B and V $\exists$ a pgrw-open set D in Y such that $B \subseteq D$ and $f^{-1}(D) \subseteq V$.
To prove $C \cap D = \phi$: If f is an injective map, then $U \cap V = \phi$. $\Rightarrow f(U) \cap f(V) = \phi$. And $f^{-1}(C) \subseteq U$ and $f^{-1}(D) \subseteq V$. $\Rightarrow f(f^{-1}(C)) \subseteq f(U)$ and $f(f^{-1}(D)) \subseteq f(V)$.
$\Rightarrow C \subseteq f(U)$ and $D \subseteq f(V)$, f being surjective $f(f^{-1}(G))=G$, $\forall$ G in Y.
$\Rightarrow C \cap D \subseteq f(U) \cap f(V)=\phi \Rightarrow C \cap D = \phi$.
Next A is a closed set in Y and $A \subseteq C$, a pgrw-open set.
$\Rightarrow$A is rw-closed and $A \subseteq C$, a pgrw-open set
$\Rightarrow A \subseteq pint(C)$ [32 (4.4)]. Similarly $B \subseteq pint(D)$.

IV. Composition Of Maps

Remark 4.1: The composition of two pgrw-closed maps need not be a pgrw-closed map.

Example 4.2: $X=\{a,b,c\}$, $\tau = \{X,\phi,\{a\},\{a,b\}\}$. The closed sets in X are X, ϕ, {b,c}, {c}.
$Y=\{a,b,c\}$, $\sigma=\{Y, \phi ,\{a\}\}$, the closed sets in Y are Y,ϕ, {b,c}. pgrw-closed sets in Y are Y, ϕ, {b},{c},{b,c}. $Z=\{a,b,c\}$, $\eta=\{Z, \phi,\{a\},\{c\},\{a,c\}\}$. pgrw-closed sets in Z are Z, ϕ,{b},{a,b},{b,c}. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y,\sigma) \rightarrow (Z, \eta)$ be the identity maps. Then f and g are pgrw-closed maps. The composition $g \circ f:(X, \tau) \rightarrow (Z, \eta)$ is not pgrw-closed , because {c} is closed in X and $g \circ f(\{c\})=g(f\{c\})=g(\{c\})=\{c\}$ is not pgrw-closed in Z.

Theorem 4.3: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a closed map and $g: (Y,\sigma) \rightarrow (Z, \eta)$ is a pgrw-closed map, then the composition $g \circ f:(X, \tau) \rightarrow (Z, \eta)$ is a pgrw-closed map.

Proof: $f: (X, \tau) \rightarrow (Y, \sigma)$ is a closed map and $g: (Y,\sigma) \rightarrow (Z, \eta)$ is a pgrw-closed map.
$\Rightarrow \forall$ closed set F in X f(F) is closed set in (Y, σ) and g(f(F)) is a pgrw-closed set in (Z, η).
$\Rightarrow \forall$ closed set F in X $g \circ f(F) = g(f(F))$ is a pgrw-closed set in (Z, η).
$\Rightarrow g \circ f:(X, \tau) \rightarrow (Z, \eta)$ is a pgrw-closed map.

Remark 4.4: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a pgrw-closed map and $g:(Y, \sigma) \rightarrow (Z, \eta)$ is a closed map, then the composition gof need not be a pgrw-closed map.

Example 4.5: $X=\{a,b,c\}$,$\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$, $Y=\{a,b,c\}$, $\sigma=\{Y, \phi ,\{a\},\{b,c\}\}$,$Z=\{a,b,c\}$, $\eta = \{Z,\phi, \{b\},\{c\},\{b,c\}\}$. The closed sets in X are X,ϕ,{c},{a,c},{b,c}. Closed sets in Y are Y,ϕ, {b,c}, {a}. pgrw-closed sets in Y are all subsets of Y. The closed sets in Z are Z, ϕ,{a},{a,b},{a,c}. pgrw-closed sets in Z are Z, ϕ,{a},{a,b},{a,c}. Let $f :X \rightarrow Y$ be the identity map. Then f is a pgrw-closed map. A map $g :Y \rightarrow Z$ is defined by g(a)=a, g(b)=a, g(c)=b. Then g is a closed map. (g∘f)({c})={b} is not pgrw-closed. ∴ composition g∘f: (X, τ) →(Z, η) is not a pgrw-closed map.

Theorem 4.6: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) be two maps such that the composition g∘f: (X, τ) →(Z, η) is a pgrw-closed map. Then the following statements are true.
(i) If f is continuous and surjective, then g is pgrw-closed.
(ii) If g is pgrw-irresolute[35] and injective, then f is pgrw-closed.
(iii) If g is strongly pgrw-continuous and injective, then f is pgrw-closed.

Proof:(i) Let A be a closed set of Y. Since f is continuous, $f^{-1}(A)$ is a closed set in X and since g∘f is a pgrw-closed map (g∘f)(f^{-1}(A)) is pgrw-closed in Z. As f is surjective (g∘f)(f^{-1}(A)=g(A). So g(A) is a pgrw-closed set in Z. Therefore g is a pgrw-closed map.

(ii) Let B be a closed set of (X, τ). Since g∘f is pgrw-closed, (g∘f)(B) is pgrw-closed in (Z, η). Since g is pgrw-irresolute, g^{-1}(g∘f(B)) is a pgrw-closed set in(Y, σ). As g is injective,
g^{-1}(g0f)(B) =f(B) . ∴ f(B) is pgrw-closed in (Y, σ). ∴f is a pgrw-closed map.

(iii) Let C be a closed set of (X, τ). Since g∘f is pgrw-closed,(g∘f)(C) is pgrw-closed in
(Z, η). Since g is strongly pgrw–continous, g^{-1}((g∘f)(C) is a closed set in (Y, σ). As g is injective, g^{-1}(gof(C))=f(C). So f(C) is a pgrw-closed set. ∴ f is a pgrw-closed map.

V. Pgrw*-Closed Map

Definition 5.1: A map f: (X, τ) → (Y, σ) is said to be a pgrw*-closed map if ∀ pgrw-closed set A in (X, τ) the image f(A) is pgrw-closed in (Y, σ).

Example 5.2:X={a,b,c} , τ ={X, ϕ, {a}, {a,c}}. pgrw-closed sets in X are X,ϕ,{b}, {c},{b,c}. Y={a,b,c} , σ ={Y, ϕ, {a}, {b,c}}. pgrw-closed sets in Y are Y, ϕ, {a},{b}, {c}, {a,b}, {b,c}, {a,c}. A map f: (X, τ) → (Y, σ) is defined by f(a)= b , f(b)=c, f(c)=a. Then f is a pgrw*-closed map.

Theorem 5.3: Every pgrw*-closed map is a pgrw–closed map.

Proof: Let f: (X, τ) → (Y, σ) be a pgrw*-closed map. Let A be a closed set in X. Then A is pgrw-closed. As f is pgrw*-closed, f(A) is pgrw-closed in Y. Hence f is a pgrw-closed map.

The converse is not true.

Example 5.4: X={a,b,c} , τ ={X, ϕ, {a}}. Y={a,b,c} , σ ={Y, ϕ, {a},{b}, {a,b} }.
Pgrw-closed sets in X are X, ϕ , {b},{c},{b,c}. pgrw-closed sets in Y are Y, ϕ, {c},{b,c} ,{a,c}. A map f:(X, τ)→(Y, σ) is defined by f(a)= c, f(b)=c, f(c)=b. f is pgrw-closed, but not pgrw*-closed.

Theorem 5.5: If f: (X, τ) → (Y, σ) is a pgrw-closed map and g: (Y, σ) → (Z, η) is a pgrw*-closed map, then the composition g ∘ f: (X, τ) →(Z, η) is pgrw-closed.

Proof: f:(X, τ) → (Y, σ) is a pgrw-closed map and g: (Y, σ) → (Z, η) is a pgrw*-closed map.
⇒ ∀ closed set A in X, f(A) is pgrw-closed in Y and g(f(A)) is pgrw-closed in Z.
⇒ ∀ closed set A in X, g∘f(A) is pgrw-closed in Z.
⇒ g ∘ f: (X, τ) →(Z, η) is a pgrw-closed map.

Theorem 5.6: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are pgrw*-closed maps, then the composition g∘f: (X, τ) →(Z, η) is also pgrw*-closed.

Proof: f:(X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are pgrw*-closed maps.
⇒ ∀ pgrw-closed set A in X, f(A) is pgrw-closed in Y and g(f(A)) is pgrw-closed in Z.
⇒ ∀ pgrw- closed set A in X, g∘f(A) is pgrw-closed in Z.
⇒ g ∘ f: (X, τ) →(Z, η) is a pgrw*-closed map.

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