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Analyze the certainty range (minimum and Maximum) of FAP by Statistical Approach

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Abstract: *In this paper, we proposed two efficient algorithms to minimizing the fuzzy cost. First we obtained the optimal value of fuzzy assignment problem using one's termination method. Next, we separate the Fuzzy assignment cost into lower and upper bound of the α cut interval and we analyzed the range of certainty of FAP by statistical approach. A numerical example is included.*
Keywords: *α - cut, Triangular Fuzz Number (TFN), T- test, One Termination Method*

I. Introduction

The introduction of the paper should explain the nature of the problem, previous work, purpose, and the contribution of the paper. The contents of each section may be provided to understand easily about the paper. Most of the time the decision maker's decision is not good because of uncertainty nature of the problem. In this situation fuzzy concept is most helpful to analyze the uncertainty optimization problem. Already researchers [1, 2, 3, 4, 6, 7] have proposed the algorithms of fuzzy assignment problem to obtain optimal cost. But [5,8,9,10] are considered the costs and times are to be fuzzy variables and applied the statistical interpretation and take the coefficients of fuzzy numbers from Pascal triangles and develop a new procedure to solve fuzzy assignment problem. In this paper we analyzed the optimal cost using the testing of hypothesis. Here, we considered the assignment costs are Triangular Fuzzy Numbers (TNFs) and proposed the two algorithms (i) One's Termination method (ii) new algorithm namely fuzzy partition method to obtain the interval of fuzzy optimal assignment cost using α cuts. We analyzed the cost of significance using testing of hypothesis. This paper organized as follows: In section 2, Preliminaries and definitions are summarized.Section3; mathematically formulation of Fuzzy interval assignment problem is discussed. In section 4: New method is proposed. In section 5, Numerical Example and further Statistical implementation is presented.

II. Preliminaries And Definitions

Definition 2.1:
The α – cut of a fuzzy set $\tilde{A}$ is defined by $\tilde{A}$ = {x: $\mu_{\bar{A}}$ (x)) ≥α} where x € X

Definition 2.2:
A Fuzzy set $\tilde{A}$ is called normal fuzzy set if there exists an element x such that $\mu_{\bar{A}}$ (x) =1.

Definition 2.3:
A fuzzy number A is a triangular fuzzy number denoted by (a,b,c) and its membership function $\mu_A(x)$ is given below:

I. $$\mu_{\bar{A}}(x)=\begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ 1 & x=b \\ \frac{c-x}{c-b} & b \le x \le c \end{cases}$$

III. Mathematical Formulation of Interval Fuzzy Assignment Problem

Mathematically formulation of Interval Fuzzy Assignment problem can be stated as

$$Min\ \bar{z}=\sum_{i=1}^{n}\sum_{j=1}^{m}\left[\underline{c_{ij}^{\alpha}}\ ,\ \overline{c_{ij}^{\alpha}}\right] x_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_{ij}=1\ ,\quad \sum_{j=1}^{n} x_{ij}=1$$

$$where\quad x_{ij}=\begin{cases} 1 & if\ ith\ is\ person\ is\ assigned\ the\ jth\ work \\ 0 & otherwise \end{cases},$$

$[\bar{c}^{\alpha}{}_{ij}\ ,\ \underline{c_{ij}}^{\alpha}]$ - Denotes the cost of $i^{th\ person}$ to j^{th} job.

IV. Proposed Algorithms

4.1 One's Termination Method:

Step 1: Determine the fuzzy cost table:
(i) If the number of Row is equal to the number of Columns then go to step (3)
(ii) If the number of Row is not equal to the number of Columns then go to step (2)
Step 2: Add dummy Row or dummy Column, so that the cost table become a square matrix. The cost entries of dummy source / destinations are always zero.
Step 3: Find the maximum element of each row (column) in the assignment matrix. Then divide each element of i^{th} row and j^{th} column of the matrix, at least exists one ones in each row or column.
Step 4: O^T = Sum of the costs of all cells adjacent to one's cell is divided by the sum of the all non –zero cells.
Step 5: Assign the maximum possible to the cell having maximum (O_T).If (O_T) attains maximum values in more than one cell, choose the cell having minimum Assignment cost.
Step 6: Draw the minimum number of lines (horizontal or vertical) to cover all the ones of the matrix. If the number of assignments is equal to "n" the fuzzy optimal solution is obtained.
If the number of assignments is less than "n" the fuzzy optimal solutions is does not obtained then go to step 7.
Step 7: Draw the new revised fuzzy cost matrix as follows:
(i) Find the smallest element of the reduced matrix not covered by any of the lines.
(ii) Divided this element from all the uncovered elements and add the same to all the elements lying at the intersection on any two lines.
Step 8: Go to Step 3 to 7 and repeat the procedure until fuzzy optimum solution is obtained.

4.2 Fuzzy Partition Method

Our proposed algorithm is easily understood and quickly obtained the optimal fuzzy assignment cost. So, namely we call Fuzzy Partition Method as follows:
Step 1: Formulated the Fuzzy Assignment Cost Table. If (i) If the number of row is equal to the number of column then go to step (3), (ii) If the number of row is not equal to the number of column then go to step (2)
Step 2: Add dummy row or dummy column, so that the cost table become a square matrix. The cost entries of dummy row / column are always zero.
Step3: From Step (1), Using α - cut, we obtained the Interval of Fuzzy Assignment Cost Table.
Step4: From Step (3) we split the table in lower bound and upper bound fuzzy assignment cost table
Step5: Using one's termination method we obtained the optimal fuzzy assignment cost (separately in lower and upper bound).

V. Statistical Implementation

Statistics deals with gathering, classifying and analyzing data, statistics is different from probability, and fully defined probability problems have unique and precise solutions. Statistics is concerned with the relationship between abstract probability models and actual physical systems. One of the primary tools of the statistician is knowledge of probability theory. Statistical theory split up into two branches one is descriptive and another one is inductive statistics. Descriptive statistics deals with the collection of data such as summarizing the available data by such variables as the mean, mode, SD., etc., the statistical interference uses the data draw the conclusions about the environment from which the data came. Now, the statistical concept using fuzzy assignment concept because we analyzed the significance level of each job assign to the each machine. Using T-distribution, the observed data's are collection sec (3), then we obtain the mean and variance of the interval $[x_l^{\alpha}, x_u^{\alpha}]$ and find T - Calculated value compare with the table value of T in 5% LOS. If $T_{cal} > T_{tab}$ then the null hypothesis assumptions are rejected or otherwise it's accepted. The test of statistic value is calculated as follows: $T=\frac{\bar{X}-\mu}{S/\sqrt{n}}$

The Interval of fuzzy optimal assignment cost as follows: $\bar{X}_l \pm 1.96(\sigma/\sqrt{n})$ & $\bar{X}_u \pm 1.96(\sigma/\sqrt{n})$ ------- (i)

VI. Numerical Example

A bus renting company has one bus at each of the five sheds S1, S2, S3, S4 and S5.A customer in each of the five places p1, p2, p3, p4 and p5 requires a bus for a tour. The distance in kilo meters between the sheds and places where the customers live are given in Triangular fuzzy numbers. How should the buses be assigned to the customers so as to minimize the distance traveled? Find also the minimum distance traveled by the buses?

Table 1: Fuzzy Assignment Problem

Machines/jobs	1	2	3	4	5
A	(11,14,15)	(6,7,9)	15,17,20)	(18,20,22)	(19,21,23)
B	(2,4,8)	(10,12,15)	(20,24,26)	(15,17,20)	(6,8,12)
C	(11,14,15)	(8,10,11)	(1,3,4)	(1,3,4)	(1,3,4)
D	(6,7,9)	(12,15,16)	(8,10,11)	(8,10,11)	(14,16,19)
E	(8,10,11)	(10,12,15)	(2,6,8)	(2,6,8)	(14,16,19)

Using membership function we obtained the interval of α – cut triangular fuzzy numbers.

(11, 14, 15) = [3α +11, 15 – α], (6, 7, 9) = [6 + α, 9-2α], (15, 17, 20) = [2α + 15, 20 - 3α]
(18, 20, 22) = [2α + 18, 22- 2α], (19, 21, 23) = [2α +19, 23- 2α], (2, 4, 8) = [2α+ 2, 8 - 4α]
(10, 12, 15) = [2α + 10, 15 - 3α], (20, 24, 26) = [4α + 20, 26 - 2α], (15, 17, 20) = [2α+15, 20-3α]
(6, 8, 12) = [2α +6, 12 - 4α], (11, 14, 15) = [3α +11, 15 - α], (8, 10, 11) = [2α + 8, 11 – α]
(1, 3, 4) = [2α +1, 4 – α], (1, 3, 4) = [2α +1, 4 – α], (1, 3, 4)= [2α +1, 4 – α],
(6, 7, 9)= [6 +α, 9- 2α] (12, 15, 16) = [3α +12, 16 – α], (10, 12, 15) = [2α + 10, 15- 3α],
(8, 10, 11) = [2α +8, 11- α] (14, 16, 19) = [2α + 14, 19 -3α], (8, 10, 11) = [2α + 8, 11 – α],
(10,12,15) = [2α + 10, 15 –3α] (14,16,19) =[2α + 14, 19 -3α], (2,6,8) = [4α + 2, 8 -2α],
(14, 16, 19) = [2α + 14, 19 - 3α]

Table 2: The Fuzzy Interval Assignment cost table as follows:

Machines/ Jobs	1	2	3	4	5
A	[3α +11, 15 –α]	[6+α, 9-2α]	[2α+15,20-3α]	[2α+18,22-2α]	[2α+19,23-3α]
B	[2α+2,8-4α]	[2α+10,15-3α]	[4α+20,26-2α]	[2α+15,20-3α]	[2α+6,12-4α]
C	[3α+11,15-α]	[2α+8,11-α]	[2α+1,4-α]	[2α+1,4-α]	[2α+1,4-α]
D	[α+6,9-2α]	[3α+12,16-α]	[2α+10,15-3α]	[2α+8,11-α]	[2α+14,,19-3α]
E	[2α+10,11-α]	[2α+10,15-3α]	[3α+11,15-α]	[4α+2,8-2α]	[2α+14,19-3α]

Using our proposed algorithm we split up the interval fuzzy assignment problem interms of lower and upper bound fuzzy assignment problem as follows:

Table 3 : The lower bound of fuzzy Assignment problem is

Machines/ jobs	1	2	3	4	5
A	3α +11	α+6	2α+15	2α+18	2α+19
B	2α+2	2α+10	4α+20	2α+15	2α+6
C	3α+11	2α+8	2α+1	2α+1	2α+1
D	α+6	3α+12	2α+10	2α+8	2α+14
E	2α+8	2α+10	3α+11	4α+2	2α+14

α - cut of the lower bound fuzzy optimal assignment value and assignment cost as follows:
$A \to 2$, $B \to 1$, $C \to 3$, $D \to 5$, $E \to 4$ and 11α + 25

Table 4: The lower bound of fuzzy Assignment problem is

Machines/ jobs	1	2	3	4	5
A	15 –α	9-2α	20-3α	22-2α	23-3α
B	8-4α	15-3α	26-2α	20-3α	12-4α
C	15-α	11-α	4 –α	4-α	4 – α
D	9-2α	16-α	15-3α	11-α	19-3α
E	11-α	15-3α	15-α	8-2α	,19-3α

α - cut of the Upper bound fuzzy optimal assignment value and assignment cost as follows:
$A \to 2$, $B \to 5$, $C \to 3$, $D \to 1$, $E \to 4$ and 42- 11α
The Interval value of the Fuzzy Optimal Assignment cost value is: [11α + 25, 42 - 11α] ---- (1)
Now, Using T – distribution we obtain the SD and mean value of the α - cut lower and upper bound value, the data collection from (1) : 6.675 and $\overline{X}_l = 30.4$, $\overline{X}_u = 36.5$

To analyze the range of certainty value in lower and upper bound optimal fuzzy assignment cost table as follows:

Table 5: Lower and Upper bound of optimal Fuzzy Assignment Cost

α-	Lower bound Cost	α-	Lower bound Cost
0	25	0.6	31.6
0.1	26.1	0.7	32.7
0.2	27.2	0.8	33.8
0.3	28.3	0.9	34.9
0.4	29.4	1.0	35
0.5	30.5		

α-	Upper bound Cost	α-	Upper bound Cost
0	42	0.6	35.4
0.1	40.9	0.7	34.3
0.2	39.8	0.8	33.2
0.3	38.7	0.9	32.1
0.4	37.6	1.0	31
0.5	36.5		

The mean and variance of $\overline{X_l} = 30.4$, S_l = 3.343, $\overline{X_u} = 36.5$ and S_u = 3.478, From (i) we obtained the largest and smallest interval of fuzzy optimal assignment cost at 5% LOS is [28.43, 38.554] and [32.37, 34.446]

VII. Conclusion

In this paper, we proposed two efficient algorithms it is useful to solve the range of minimum and maximum interval fuzzy optimal assignment cost by one termination method. Its most useful to the industrialist quickly obtained the blocking range and helpful to take the optimal decision to the decision maker. In this method easily understood for all and quickly obtained the interval fuzzy assignment cost.

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