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Inference of R = P(X<Y<Z) for n-Standby System: A Monte-Carlo Simulation Approach

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Abstract: *In interference theory of reliability, reliability expressions for multi-component systems are not simple enough to facilitate analytical estimation of reliability and its other characteristics. Here, we have shown how reliability R = P(X<Y<Z) can be estimated by Monte-Carlo simulation (MCS) for n-standbys when stress-strength both follows a particular continuous distribution.*
Keywords: *Reliability, Stress-Strength(S-S); Monte-Carlo Simulation (MCS)*

I. Introduction

The interference theory, which is the subject matter of this paper, has acquired an important place in reliability study of the systems. In it, reliability of a system is studied from the interaction of strength of the system, say X, and the stress working on it, say Y, which is the sole cause of its failure; where X and Y both are assumed to be random variables. Here, reliability, say R, of the system is defined as R = P(X > Y). So, the reliability (R) can be expressed in terms of parameters of stress (Y) and strength (X). Generally, there are two ways to estimate R, viz. (1) Non parametric estimation of R, where data consists only of the number of 'success' in a specified number of 'trials' and (2) Parametric estimation where the parameters are estimated separately by two unrelated sets of data, viz., the observations on components strengths X_i, and on the impinging stresses Y_j, (Tong [15]). In the parametric method of estimation of reliability, these parameters are estimated from separate measurements of stress and strength and substituting back in the reliability expressions, the estimated reliability is obtained. If the estimates of parameters used here are maximum likelihood estimators then from the invariance property of MLE's, the corresponding estimators of reliability are also MLE's. There exists extensive literature in estimation of R = P(X > Y) for single component system analytically, e.g., Lloyd and Lipow [6], Church and Harris [3], Mazumder [8], Singh [12] etc. However, the reliability expressions for multi-component systems are not simple enough to facilitate analytical estimation of reliability and its other characteristics. Also, due to lack of real life data of X and Y, one way out is simulation, in particular Monte-Carlo simulation (MCS). Manders et al. [7], Aldrisi [2], Stumpf and Schwartz [14], Zhang et al. [17] used MCS to estimate reliability. Further, Rezaei et al. [11] estimated reliability of stress-strength model, using MCS. Moreover, Ahmad et al. [1] obtain Bayes estimates of P (Y< X) using MCS. Similarly, Uddin et al. [16] estimated reliability for multi-component system using MCS. Moreover, Patowary et al. [9] estimated reliability for n-standby system by Monte-Carlo simulation technique. Also, Rao et al. [10] compared reliability estimates for multi-component systems evaluated by different methods such as method of moments, modified Maximum Likelihood method and Best Linear Unbiased Estimator through MCS technique. Similarly, very limited literature are available for estimation of R= P(X<Y<Z) analytically. The works of Guangming [5] is noteworthy in this context. The model consists of three random variables and so the reliability expressions are quite complicated in case of n-standby system, so the analytical estimation of R is difficult. In this paper, an attempt has been made to estimate R= P(X<Y<Z) for n standby-systems by MCS when stress-strength either follow exponential or normal distribution. In Section 2, we have described the system undertaken in this study. The technique of Reliability estimation for P(X<Y<Z) of n-standby system (n=1, 2), through Monte-Carlo simulation (MCS) technique is explained in Section 3. We have also drawn normal probability plot for each estimated reliability data sets to check the normal approximation. For distribution fitting, we have considered sample of size (k = 20). Since we have taken a small sample, when using χ^2-test, the number of classes becomes too few, due to pooling. Therefore, the goodness of fit is tested by Kolmogorov-Smirnov (K-S) one sample test (Seigel [13]). We have also tested normal approximation for sample size of 50 and 100 also for illustration purpose, using χ^2-test.

II. Description of the system

Let us consider a system with n components working under the impact of stresses. Let X_i and Z_i be the lower and upper strengths, respectively, of the i^{th} component, and let Y_i be the stress acting on it, i=1,2,3,…,n, are all assumed to be independent random variables. The i^{th} component works if the stress Y_i lie in the interval (X_i, Z_i).

$$R = P(X<Y<Z) \tag{2.1}$$

Whenever a stress fall lie outside these two limits, the component fails and another from standby (if there remains any) takes place of the failed component and the system continues to work. The system fails when all the components are failed. The system reliability, R_n, of the system is given by Dutta and Sriwastav [4]

$$R_n=R(1)+R(2)+ \ldots + R(n) \tag{2.2}$$

where, R(r), r = 1, 2, …,n is the marginal reliability due to r^{th} component

$$R(r)= [1 – R(1)][1 –R(2)] \ldots [P(Y_r > X_r) – P(Y_r > X_r, Y_r > Z_r)], \quad r = 1, 2, \ldots, n \tag{2.3}$$

Here, we have assumed that all the components are having the same strength distribution and are working under the same environment (stress), i.e. all X_i's , Y_i's and Z_i's are i.i.d. with probability density functions (pdf's) f(x), g(y) and h(z) respectively. In this paper, we have assumed that both stress and strength are either exponential or normal variates. Here, for simulation, the programs are developed in MATLAB, separately when stress-strength either follows exponential and normal distribution. First, a set of M = 5000 values of particular r.v. viz. either exponential or normal, are generated for a particular value of the parameter(s) of X, Y and Z. Using these values an estimate of the parameter involved is obtained. Substituting this estimate(s) in the expression of reliability we get an estimate of the reliability. This process is repeated (say) k times to give k estimates of the parameter(s) and subsequently k estimates of reliability. The whole process is repeated for different true values of the parameters; for a particular true value of the parameter(s), k is the sample size.

We have considered here the cases of n = 1, 2. Then, form Eq.(2.2) and Eq.(2.3) we can easily see that

$$R_1 = R(1) \tag{2.4}$$

$$R_2 = R_1 + (1 – R_1) R_1 \tag{2.5}$$

III. Stress-strength follows particular distribution

We have seen that in interference models system reliability is a function of stress-strength parameters. Let f(x) and h(z) be the p.d.f. of upper strength (X), the lower strength of the system respectively. Also, g(y) be the p.d.f. of the stress (Y) on the system. Here, we have considered two cases.

Case I: When f(x), h(z) and g(y) follows exponential distribution

Case II: When f(x), h(z) and g(y) follows normal distribution

3.1 Stress-strength exponentially distributed

When all X_i's, Y_i's and Z_i's are i.i.d and follow exponential distribution with densities f(x), g(y) and h(z) with means λ, μ and ν, then by Dutta and Sriwastav [4]

$$R_1 = \frac{\nu}{\nu+\mu} - \frac{\lambda\times\nu}{\lambda+\mu+\nu} \tag{3.1.1}$$

$$R_2 = (1 – R_1)\times R_1 \tag{3.1.2}$$

For MCS, first we have generated exponential r.v.s X, Y and Z of different values of μ, λ and ν of size M = 5000. The mean of these 5000 values give an estimate of μ, λ and ν for particular true values of μ, λ and ν. Substituting these estimates in Eq.(3.1.1), we get an estimate of R_1 then from Eq.(3.1.2), we get estimates of R_2 respectively. For each true value of μ, λ and ν, the complete process is repeated k times there by giving k estimates of R's. Here, we have taken (μ, λ, ν) = [(0.5, 0.7, 0.6), (0.5, 0.7, 0.8), (0.5, 0.7, 1.0), (0.5, 0.7, 1.5), (0.5, 0.7, 2.0), (0.7, 0.7, 0.6)] and k = 20. Then, in each case, we have drawn normal probability plot for the $\hat{R}_1, \hat{R}_2$ which suggest that their distribution may be normal. For illustration purpose, we have reproduced only the normal probability plots of $\hat{R}_2$, for k = 20 as well as k(= 50, 100) when (μ, λ, ν) = (0.5, 0.7, 0.6) in Fig.3.1.1 to check the normal approximation. After that, normal distribution is fitted and tested for the goodness of fit by one sample K-S test for k =20. The tabulated values of D i.e. maximum differences between empirical CDF and theoretical CDF for sample size k = 20 at 5% level of significance is 0.2940 (Seigel [13]). For test of significance between true value and estimated value (which is mean) we have used one sample t-test only between R_2 and $\hat{R}_2$ for sample size k = 20. The values are tabulated in Table 3.1.1.

From Table 3.1.1, it is observed that the calculated D for $\hat{R}_1$ and $\hat{R}_2$ for different values of true μ, λ, ν are smaller than the tabulated D = 0.2940 for k =20 at 5% level of significance. So, it is clear that normal distributions give a good fit to the values of $\hat{R}_1$ and $\hat{R}_2$ for k = 20. Also, all t-values are insignificant i.e. there is no difference between estimated reliability $\hat{R}_2$ and true reliability R_2.

We have also used χ^2-test to test the goodness of fit for k = 50 and 100. For illustration purpose, we have given only the case of R_2 for one value of μ, λ and ν. For example, for $\mu = 0.5$, $\lambda = 0.2$ and $\nu = 0.6$; mean $(\hat{R}_2) = 0.7000$, SD $(\hat{R}_2) = 0.0037$, $\chi^2 = 4.7813$ (d.f. = 4) when k = 50. Similarly, mean $(\hat{R}_2) = 0.7034$, SD $(\hat{R}_2) = 0.0042$, $\chi^2 = 1.5466$ (d.f. = 4), when k = 100. It is observed that calculated χ^2 for k = 50 and 100 is less than the tabulated $\chi^2 = 9.488$ for d.f. = 4 at 5% level of significance. So from χ^2-test also we find that the data fits the normal distribution well for k =50 and 100. The results are given in Table 3.1.1.

Table 3.1.1: Stress-Strength exponentially distributed

True μ	True λ	True ν	True R_1	Mean $\hat{R}_1$	SD $\hat{R}_1$	D for $\hat{R}_1$	True R_2	Mean $\hat{R}_2$	SD $\hat{R}_2$	D for $\hat{R}_2$	t for $\hat{R}_2$
0.5	0.7	0.6	0.3121	0.3134	0.0032	0.1095	0.5268	0.5286	0.0044	0.1096	0.6157
0.5	0.7	0.8	0.3353	0.3357	0.0036	0.1090	0.5583	0.5587	0.0048	0.1098	0.3427
0.5	0.7	1.0	0.3485	0.3490	0.0037	0.1506	0.5755	0.5761	0.0049	0.1517	0.5827
0.5	0.7	1.5	0.3611	0.3603	0.0046	0.1105	0.5918	0.5907	0.0058	0.1096	0.8272
0.5	0.7	2.0	0.3625	0.3616	0.0048	0.1204	0.5936	0.5925	0.0062	0.1194	0.8188
0.7	0.7	0.6	0.2515	0.2513	0.0028	0.0778	0.4394	0.4394	0.0042	0.0773	0.0391
0.8	0.7	0.6	0.2286	0.2281	0.0039	0.1284	0.4049	0.4042	0.0060	0.1275	0.5558
0.5	0.6	0.6	0.3580	0.3581	0.0028	0.1515	0.5878	0.5879	0.0036	0.2000	0.2000
0.5	0.4	0.6	0.3855	0.3853	0.0036	0.1038	0.6223	0.6222	0.1047	0.0045	0.0608
0.5	0.2	0.6	0.4531	0.4541	0.0036	0.1412	0.7010	0.7020	0.0039	0.1423	1.1131

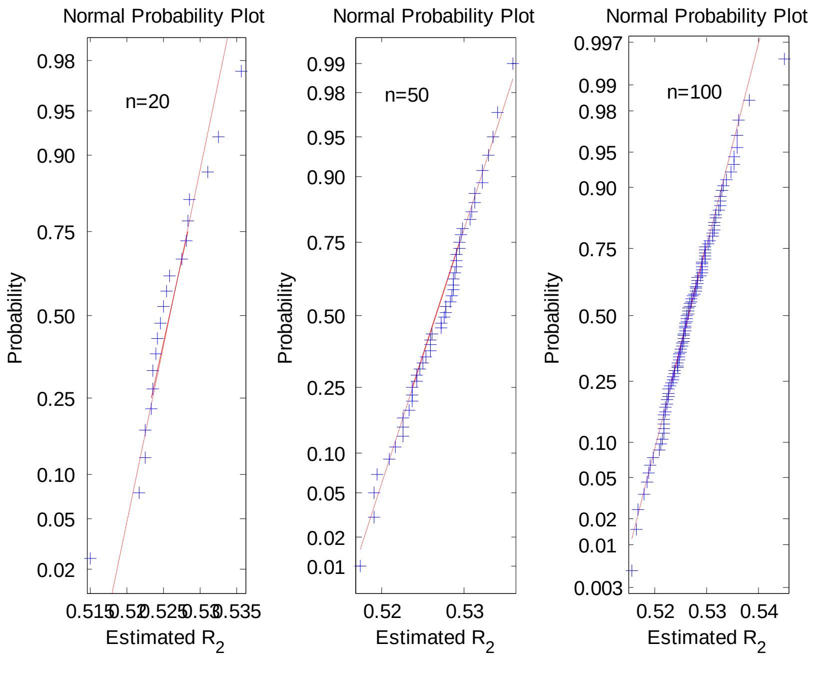

Fig.3.1.1: Normal Probability plot $\hat{R}_2$ for n=20, 50 and 100

3.2 Stress-strength normally distributed

When all X_i's, Y_i's and Z_i's are i.i.d and follow Normal densities f(x), g(y) and h(z) with means θ, μ and β and standard deviations σ, λ and ρ then by Dutta and Sriwastav [4]

$$R_1 = \int_{-\infty}^{\infty} \Phi\left(\frac{y-\theta}{\sigma}\right)\frac{1}{\lambda\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\lambda}\right)^2} dy - \int_{-\infty}^{\infty} \Phi\left(\frac{y-\theta}{\sigma}\right)\Phi\left(\frac{y-\beta}{\rho}\right)\frac{1}{\lambda\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\lambda}\right)^2} \qquad (3.2.1)$$

$$R_2 = R_1 + (1 - R_1)\times R_1 \qquad (3.2.2)$$

As in Section 3.1, similarly for MCS, we have generated r.v.s X, Y and Z of size M (= 5000) which are normally distributed for different values of means for X, Y, Z i.e. θ, μ and β and standard deviations (SD) σ, λ and ρ by MATLAB. Then, the mean and SDs of generated r.v.s X, Y and Z give the estimates of θ, μ, β σ, λ and ρ, respectively. Substituting these estimates, in Eq.(3.2.1), we get an estimate $\hat{R}_1$ of R_1 and from Eq.(3.2.2), $\hat{R}_2$ of R_2 respectively. Obviously, no closed form expression for R_1 and R_2, can be obtained and it is bound to go for numerical integration. Using the Gauss-Hermite quadrature formula, we have evaluated estimated $\hat{R}_1$ and $\hat{R}_2$ for different true values of θ, μ, β σ, λ and ρ. The complete process is repeated k times thereby we get a set of $\hat{R}_1$ and $\hat{R}_2$ of size k. Here, we have also taken k = 20 as in Section 3.2. Moreover, in each case, we have drawn normal probability plot of $\hat{R}_1, \hat{R}_2$ which suggest that their distribution may be normal. For illustration purpose, we have reproduced only the normal probability plots of $\hat{R}_2$, for k = 20 as well as k(= 50, 100) when (θ, μ, β, σ, λ and ρ) = (1, 1, 1, 1, 3, 1) in Fig.3.2.2. Finally, K-S test is applied in each case for the goodness of fit for k =20. Also, as in section 3.1, for test of significance between true value $R_2$and estimated value $\hat{R}_2$, we have used one sample t-test for sample size k = 20. The values are tabulated in Table 3.2.2. In Table 3.2.2, it is observed that the calculated D for $\hat{R}_1$ and $\hat{R}_2$ for different values of true θ, μ, β, σ, λ and ρ are smaller than the tabulated D = 0.2940 for k =20 at 5% level of significance. So, it is apparent that the normal distributions give a good fit to the values of $\hat{R}_1$ and $\hat{R}_2$ for k = 20. In addition, all t-values are insignificant i.e. there is no difference between estimated reliability and true reliability.

Similarly, we have also used χ^2-test to test the goodness of fit for k = 50 and 100. For illustration purpose, we have given only the case of R_2 for one value of μ, λ and ν. For example, for (θ, μ, β, σ, λ and ρ) = (1, 1, 1, 1, 3, 1); mean$(\hat{R}_2)$ = 0.4300, SD$(\hat{R}_2)$ = 0.0032, χ^2 = 5.912 (d.f. = 4) when k = 50. Similarly, mean$(\hat{R}_2)$ = 0.6725, SD$(\hat{R}_2)$ = 0.0040, χ^2 =3.66 (d.f. = 4), when k = 100. It is found that calculated χ^2 for k = 50 and 100 is less than the tabulated χ^2 = 9.488 for d.f. = 4 at 5% level of significance. So from χ^2-test, it is also uncover that the data fits the normal distribution well for k =50 and 100. All numerical results are tabulated in Table 3.2.2.

Table 3.2.2: Stress-Strength normally distributed

True θ	True σ	True μ	True λ	True β	True ρ	True R_1	Mean $\hat{R}_1$	SD $\hat{R}_1$	D for $\hat{R}_1$	True R_2	Mean $\hat{R}_2$	SD $\hat{R}_2$	D for $\hat{R}_2$	t-value for $\hat{R}_2$
1	1	1	1	1	1	0.1662	0.1652	0.0026	0.1320	0.3048	0.3032	0.0043	0.1313	0.1106
1	1	1	1	2	1	0.3165	0.3143	0.0043	0.1258	0.5328	0.5299	0.0059	0.1250	0.1307
1	1	1	1	2.5	1	0.3826	0.3834	0.0036	0.1222	0.6188	0.6198	0.0044	0.1216	1.0264
1	1	1	1	3	1	0.4325	0.4328	0.0054	0.1103	0.6780	0.6782	0.0062	0.1121	0.1720
1	1	1	1	1	1.5	0.1854	0.1853	0.0039	0.1233	0.3364	0.3362	0.0064	0.1225	0.1204
1	1	1	1	1	3	0.2134	0.2147	0.0032	0.0982	0.3812	0.3833	0.0050	0.0986	1.8931
1	1	2	1	1	1	0.1266	0.1269	0.0029	0.1022	0.2372	0.2378	0.0051	0.1028	0.5005
1	1	2.5	1	1	1	0.0898	0.0899	0.0022	0.1262	0.1716	0.1717	0.0039	0.1262	0.1597
1	1	1	2	1	1	0.1010	0.1017	0.0023	0.1031	0.1917	0.1917	0.0041	0.1030	1.4133
1	1	1	2.5	1	1	0.0831	0.0838	0.0019	0.0743	0.1593	0.1605	0.0035	0.0740	1.5365
0.7	1	1	1	1	1	0.2098	0.2097	0.0028	0.1662	0.3755	0.3754	0.0044	0.1667	0.1237
0.5	1	1	1	1	1	0.2398	0.2401	0.0031	0.1036	0.4221	0.4226	0.0048	0.1029	0.4363
0.3	1	1	1	1	1	0.2698	0.2707	0.0047	0.1169	0.4668	0.4681	0.0068	0.1157	0.8697
1	2	1	1	1	1	0.1876	0.1886	0.0039	0.1089	0.3401	0.3417	0.0063	0.1082	1.1172
1	3	1	1	1	1	0.1962	0.1965	0.0040	0.0790	0.3539	0.3545	0.0064	0.0782	0.4245

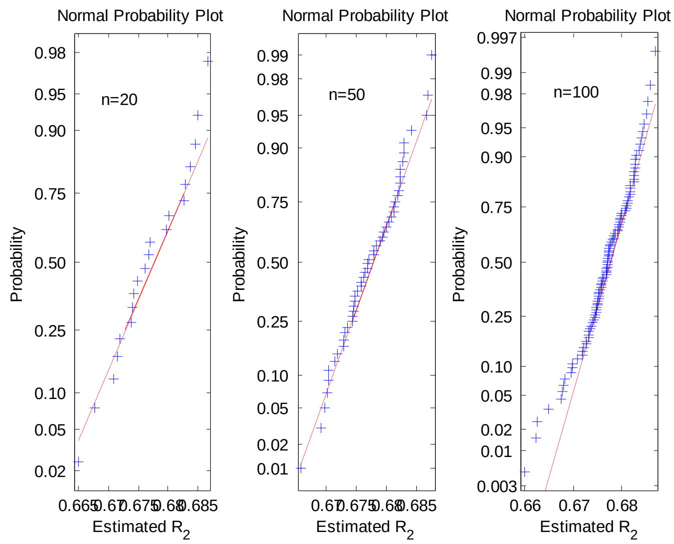

Fig. 3.2.2: Normal Probability plot $\hat{R}_2$ for n=20, 50 and 100

IV. Conclusion

In this paper, we have shown how MCS technique can be used in the situation where reliability expressions of multi-component systems are not simple enough to facilitate analytical estimation of reliability and its other characteristics. We have estimated the reliability of R = P(X<Y<Z) for n-standby system by this technique when stress-strength either follows exponential or normal distribution. Also, normal distribution is well fitted to estimate reliabilities. Once we know the distribution of estimated reliabilities, it is trouble free to study the other reliability characteristics of those samples. Also, here, we have considered only stress-strength follow exponential and normal distribution. However, there are so many distributions in the world, we may, consider other distributions also and similarly we can estimate reliability of the model R=P(X<Y<Z). Moreover, we can estimate reliability of other complex reliability models also by MCS.

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