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Prime Difference Speed Sequence Graph with Lucas Numbers

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Abstract: *A (p,q) graph G(V,E) is said to be a prime difference speed sequence graph if there exists a bijection f: V(G) → { 0, 1, 2, ...q_2 } such that the induced mapping f: E(G) →{$\Delta_i(x)$}/ i=1, 2, 3, ...n}defined by f(uv) = |f(u) – f(v) |is a bijection. Here $\Delta_m(x) = (\Delta_m x_k) = |(x_k - x_{k+m})|$ and (x) is the Lucas sequence . The function f is called a prime difference speed sequence graph. In this paper we prove that the path graph, the fan graph, star graph, cycle graph, wheel graph and various types of graph are prime difference speed sequence graph.*
Keywords: *Graph Labeling , Prime Labeling , Difference speed sequence graphs, Lucas numbers.*
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I. Introduction

In this paper, we consider only finite simple undirected graph. The graph G has vertex set $V = (G)$ and edge set $E = E(G)$. A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1,2,...,|V|$ such that for each edge xy the labels assigned to x and y are relatively prime. Among the classes of trees known to have prime labelings are: paths, stars, caterpillars, complete binary trees, spiders (i.e., trees with one vertex of degree at least 3 and with all other vertices with degree at most 2).The complete graph K_n does not have a prime labeling for n > 4 and W_n is prime if and only if n is even.

Let G = (V(G),E(G)) be a graph with p vertices. A bijection $f : V(G) \to \{1,2,..., p\}$ is called a prime labeling if for each edge $e = uv$, $GCD (u,v) = 1$. A graph which admits prime labeling is called a prime graph.

- Path $P_n$is a prime graph for any $n \in N$.
- Cycle C_n is a prime graph for any $n \in N$.
- Complete graph K_n is a prime graph if and only if $n \leq 3$.
- Wheel W_n is a prime graph for any even n.
- Stars $K_{2,n}$and $K_{3,n}$ are prime unless $n = 3$ or $n = 7$.

The following table summarize the state of knowledge about prime labeling. In the table, P means prime labeling exists.

Summary of Prime Labelings:

stars	P
caterpillars	P
complete binary trees	P
$K_{2,n}$	P

Definition1.1
A graph with vertex set V is said to have a **prime labeling** if its vertices are labeled with distinct integers $1,2,3\ldots V$ such that for edge xy the labels assigned to x and y are relatively prime. A graph which admits prime labeling is called a prime graph.

Definition1.2
A graph is said to be a **difference labeling graph** if there is an bijection f from V to a set of positive integers S such that $xy \in E$ if and only if $|f(x)- f(y)| \in S$.Some of the difference graphs are C_n, K_n. The wheels W_n are difference graphs if and only if $n = 3,4,$ or 6.

Definition1.3
A (p,q) graph G(V,E) is said to be a **difference speed sequence graceful graph** if there exists a bijection $f: V(G) \to \{ 0, 1, 2, \ldots n \}$ such that the induced mapping $f: E(G) \to \{\Delta_i(x)\}/ i=1,2, 3, \ldots$n}defined by $f(uv) = |f(u) – f(v)|$is a bijection.(x) is the Lucas sequence.

Definition1.4
Similar to the Fibonacci numbers , each Lucas number is defined to be the sum of its immediate previous terms , thereby forming a Lucas integer sequence. The first two Lucas numbers are L_0=2 and L_1=1.
The Lucas number may thus be defined as follows:

$$L_n = \begin{cases} 0 & if\ n = 0 \\ 1 & if\ n = 1 \\ L_{n-1} + L_{n-2} & if\ n > 1 \end{cases}$$

The sequence of Lucas numbers are 0,1,3,4,7,11,18,29,47,76,123,..............

Theorem 2.1:
The Path P_n is a lucas prime difference speed sequence graph.
Proof:
Let P_n be the path of length n having $(n+1)$ vertices namely $v_1, v_2, v_3, v_4, \ldots\ldots\ldots v_n, v_{n+1}$.

Now $|v(Pn)| = n+1$ and $|E(Pn)| = n$.Define an injective function $f: V(G) \rightarrow \{ l_1, l_3, l_5, \ldots \}$ if an induced edge labeling $f_1(uv) = |f(u) - f(v)|$ is a bijection onto the set $\{l_2, l_4, l_6, l_8, \ldots\ldots\ldots\}$ with the assumption of $l_1 = 1, l_2 = 3, l_3 = 4, l_4 = 7, \ldots$

E = { $|l_3 - l_1|, |l_5 - l_3|, \ldots\ldots\ldots |l_n - l_{n-2}|$} where n=1,3,5,……

$= \{l_2, l_4, l_6, \ldots\ldots\ldots l_{2n}\}$ where $l_2, l_4, l_6, \ldots\ldots\ldots l_{2n}$ are lucas numbers satisfying prime labeling (GCD$(l_i, l_{i+2}) = 1; i =$odd numbers) and difference speed sequence labeling conditions.
Therefore the Path P_n admits Lucas prime difference speed sequence graph.
Example:2.1
For path P_5, the labeling are as follows

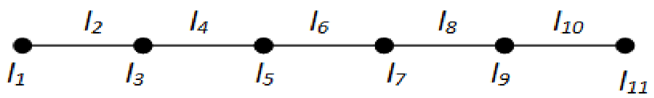


Fig 1

Therefore the Path P_5 admits lucas prime difference speed sequence graph.
For path P_6, the labeling are as follows

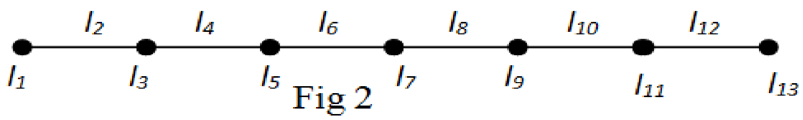


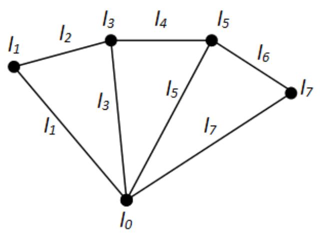
Fig 2

Therefore the Path P_6 admits lucas prime difference speed sequence graph.
Theorem 2.2:
The Fan Graph F_n is a lucas prime difference speed sequence graph.
Proof:
Let $v_1, v_2, v_3, v_4, \ldots\ldots\ldots v_n$, and u_0 be the vertices of a fan F_n.
Define $f: V(G) \rightarrow \{ l_0, l_1, l_2, l_3, \ldots\ldots l_n\}$ by $f(u_0) = l_0, f(v_i) = l_{2i-1}$, $1 \leq i \leq n+1$

$E_n = \{ |l_0 - l_{2i-1}|; 1 \leq i \leq n+1 \}$

$= \{l_{2i-1}; 1 \leq i \leq n+1 \}$

$= \{ l_1, l_3, \ldots\ldots l_{2n+1}\}$ where $l_1, l_3, \ldots\ldots\ldots l_{2n+1}$ are lucas numbers satisfying prime labeling (GCD(u,v)=1) and difference speed sequence labeling conditions.
Therefore the Fan Graph F_n admits lucas prime difference speed sequence graph.
Example 2.2:



F_3
Fig 3

Therefore the Fan Graph F_3 admits lucas prime difference speed sequence graph.

Theorem 2.3:
The Cycle Graph C_n is a lucas prime difference speed sequence graph.
Proof:
Let $u_1, u_2, u_3, u_4, \ldots\ldots u_n$ be the vertices of a cycle C_n.
Let $f(u_1) = l_0$
$f(u_i) = \{ l_1, l_3, l_5, \ldots, l_n\}$ where n=odd numbers
E = { $|l_0 - l_1|$, $|l_1 - l_3|$, ……… $|l_m - l_0|$}
= { $l_1, l_2, l_3, \ldots, l_m$} where $l_1, l_2, l_3, \ldots, l_m$ are lucas numbers satisfying prime labeling (GCD(u,v)=1) and difference speed sequence labeling conditions.
Therefore the Cycle Graph C_n admits lucas prime difference speed sequence graph.
Example 2.3:

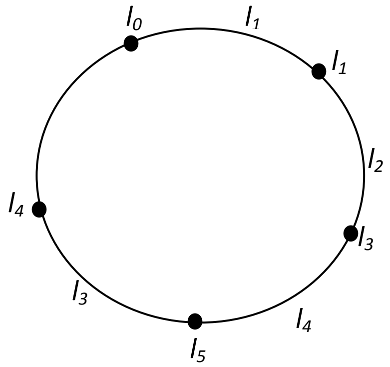


Fig 4

Therefore the Cycle Graph C_5 admits lucas prime difference speed sequence graph.
Remark:
In a cycle graph, subtracting the last vertex from the origin vertex does not satisfies the lucas prime difference speed sequence graph condition.
Theorem 2.4:
The Graph $K_{1,n}$is a lucas prime difference speed sequence graph.
Proof:
Let G = $K_{1,n}$ and Let $|V(G)| = n+1$, $|E(G)| = n$.
Define $f: V(G) \rightarrow \{ l_0, l_1, l_2, l_3, \ldots l_n\}$ by $f(u_0) = l_0$, $f(u_i) = l_i$, $1 \leq i \leq n$
E = { $|l_0 - l_1|$, $|l_0 - l_2|$, ……… $|l_0 - l_n|$}
= { $l_1, l_2, l_3, \ldots, l_n$} where $l_1, l_2, l_3, \ldots, l_n$ are lucas numbers satisfying prime labeling (GCD(l_i, l_{i+1})=1) and difference speed sequence labeling conditions.
Therefore the Graph $K_{1,n}$ admits lucas prime difference speed sequence graph.
Example2.4:

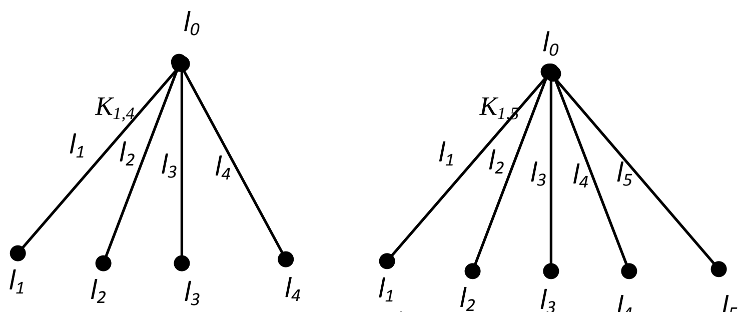


Fig 5

Therefore the Graph $K_{1,4}$and $K_{1,5}$ admits lucas prime difference speed sequence graph.
Theorem 2.5:
The Star graph is a lucas prime difference speed sequence graph.
Proof:
Let G = $K_{1,n}$ be a Star Graph with $n+1$ vertices and n edges.
Let $f(u_0) = l_0 = 0$

$f(v_i)$= { $l_1, l_2, l_3,\ldots, l_n$}
E = { $|l_0 - l_1|, |l_0 - l_2|, \ldots\ldots\ldots |l_0 - l_n|$}
= { $l_1, l_2, l_3,\ldots, l_n$} where $l_1, l_2, l_3,\ldots, l_n$ are lucas numbers satisfying prime labeling (GCD(u,v)=1) and difference speed sequence labeling conditions.Therefore the Star Graph admits lucas prime difference speed sequence graph.

Example 2.5:

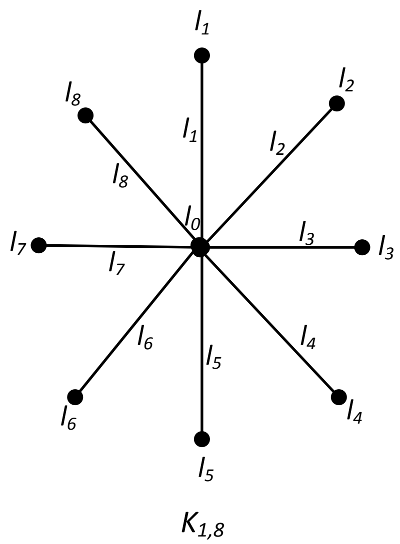


Fig 6

Therefore the Star Graph $K_{1,8}$ admits lucas prime difference speed sequence graph.
Theorem 2.6:
The Wheel graph is a lucas prime difference speed sequence graph.
Proof:
Let $u_0, v_1, v_2, v_3, v_4, \ldots, v_n$ be the vertices of wheel graph.
Define $f: V(G) \rightarrow$ { $l_0, |l_0 - l_2|, \ldots\ldots\ldots |l_0 - l_n|$} by $f(u_0) = l_0$, $f(v_i)$={$l_1, l_2, l_3, \ldots l_n$}
E_n = { $|l_0 - l_1|, |l_0 - l_2|, \ldots\ldots\ldots |l_0 - l_n|$} where n=1,2,3,…
={ $l_1, l_2, l_3,\ldots, l_n$} where $l_1, l_2, l_3,\ldots, l_n$ are lucas numbers satisfying prime labeling (GCD(u,v)=1) and difference speed sequence labeling conditions. Therefore the Wheel Graph admits lucas prime difference speed sequence graph.
Example 2.6

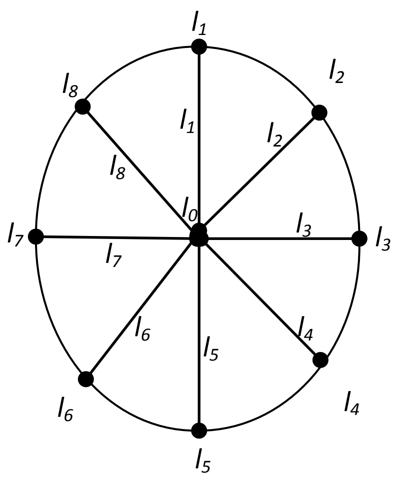


Fig 7

Therefore the above Wheel Graph admits lucas prime difference speed sequence graph.

II. Conclusion

Here we have introduced two labelings namely prime labeling and difference speed sequence labeling using lucas numbers. We have also proved it for various types of graphs like path graph, cycle graph, star graph,fan graph, wheel graph etc.,

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