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Matrix Representation of Generalized k-Fibonacci Sequence

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Abstract: *In this paper we will present some properties of generalized k-Fibonacci sequence* $U_{k,n+2} = kU_{k,n+1} + U_{k,n}, U_{k,0} = q, U_{k,1} = kq$ *by matrix methods (Multiplication and Addition of Matrices) such as the nth power for the matrix representation of generalized k-Fibonacci sequence, Cassini's Identity of generalized k-Fibonacci sequence and some identities will be presented on the relations between k-Fibonacci and generalized k-Fibonacci sequence.*
Keywords–*k-Fibonacci numbers, Generalized k-Fibonacci numbers, Cassini's identity and Matrix methods.*

I. Introduction

Many authors have studied k-Fibonacci numbers by different ways to discuss the different properties of these numbers in [4, 6, 8, 9]. The well-known Fibonacci numbers and k-Fibonacci numbers are defined as

$$F_{n+1} = F_n + F_{n-1},\ n \geq 1,\ F_0 = 0,\ F_1 = 1 \tag{1.1}$$

$$F_{k,n+1} = kF_{k,n} + F_{k,n-1},\ n \geq 1,\ F_{k,0} = 0,\ F_{k,1} = 1 \tag{1.2}$$

In addition to this many researchers from time to time studied the Fibonacci numbers in terms of matrices. In 1960 Charles H. King introduced and studied the matrix for classical Fibonacci numbers in his Master thesis which is known as Q-matrix which is to be discussed in the koshy's book [10] and Q-matrix is given as

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \text{ then } Q^n = \begin{pmatrix} F_{n+1} & F_n \\ F & F_{n-1} \end{pmatrix}$$

He showed with the help of matrices and determinants that

$F_{n+1}F_{n-1} - F_n^2 = (-1)^n$ which is known as Cassini's Identity.

After that in 1983 1983 Sam Moore introduced M- matrix for classical Fibonacci numbers for this case one can see [10] and M-matrix is defined as

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \text{ then } Q^n = \begin{pmatrix} F_{2n-1} & F_n \\ F_{2n} & F_{2n+1} \end{pmatrix}$$

In [7] Silvester derived a number of properties of the Fibonacci sequence by considering a matrix representation if

$$\text{if } A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ then } A^n \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} u_n \\ u_{n+1} \end{bmatrix}$$

In [3] many properties have been presented about Fibonacci and Lucas sums with the help of two cross two matrices which are given as

$$S = \begin{bmatrix} \frac{1}{2} & \frac{5}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \text{ and } K = \begin{bmatrix} 0 & 5 \\ 1 & 0 \end{bmatrix}$$

In [5] authors discussed the matrix representations of Jacobsthal and Jacobsthal-Lucas numbers. In which they considered Jacobsthal F-matrix and Jacobsthal M-matrix and these are defined as

$$F = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } M = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \text{ then } F^n = \begin{bmatrix} J_{n+1} & 2J_n \\ J_n & 2J_{n-1} \end{bmatrix} \text{ and } M^n = \begin{bmatrix} J_{2n+1} & 2J_{2n} \\ J_{2n} & 2J_{2n-1} \end{bmatrix}$$

where J_n is the nth Jacobsthal number. In [1] authors derived results for k-Fibonacci and k-Lucas sequences and obtained a Binet's form of these sequences by matrix diagonalization. In doing so they considered a matrix F which is called a generating matrix.

$$F = \begin{pmatrix} 0 & 1 \\ 1 & k \end{pmatrix} \text{ then } F^n = \begin{bmatrix} F_{k,n-1} & F_{k,n} \\ F_{k,n} & F_{k,n+1} \end{bmatrix}$$

In [2] Godase gave the matrix properties of generalized k-Fibonacci Like sequence to do so he considered a two cross two matrix P and proved an one of the important result as follows

$$F = \begin{pmatrix} 0 & 1 \\ 1 & k \end{pmatrix} \text{ then } \begin{bmatrix} M_{k,n+1} & M_{k,n} \\ M_{k,n} & M_{k,n-1} \end{bmatrix} = P^n \begin{bmatrix} m+k & 2 \\ 2 & m-k \end{bmatrix}$$

where $M_{k,n} = mF_{k,n} + L_{k,n}$, $n \ge 0$

II. Two Cross Two Matrix Representation of a Generalized k-Fibonacci Sequence

So in the present paper we are going to study a generalized k-Fibonacci sequence by some matrix methods after using a two cross two matrix representation for the generalized k-Fibonacci sequence. Hence the generalized k-Fibonacci is defined as

Definition 2.1. *For the integers* $n \ge 0$ and $k \ge 0$ *and for fixed positive integer* q *the generalized k-Fibonacci sequence is recurrently defined by*

$$U_{k,n+2} = kU_{k,n+1} + U_{k,n}, U_{k,0} = q, U_{k,1} = kq \tag{2.1}$$

and the two cross two matrix representation for the generalized k-Fibonacci sequence is given as

$$U = \begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix} \tag{2.2}$$

III. Main Results

Theorem 3.1. *For any positive integer* n *the nth power of a matrix U is given by*

$$U^n = q^{-1} \begin{pmatrix} U_{k,2n} & U_{k,2n-1} \\ U_{k,2n-1} & U_{k,2n-2} \end{pmatrix} \tag{3.1}$$

Proof: To prove the result we shall use induction on n.

For $n = 1$, $U^n = q^{-1} \begin{pmatrix} U_{k,2} & U_{k,1} \\ U_{k,1} & U_{k,0} \end{pmatrix} = \begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}$, that is true using initial conditions of the sequence. Suppose that (3.1) is true for n. Now we show that (3.1) is true for $n+1$ then

$$U^{n+1} = q^{-1} \begin{pmatrix} U_{k,2n} & U_{k,2n-1} \\ U_{k,2n-1} & U_{k,2n-2} \end{pmatrix} \begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}$$

$$U^{n+1} = q^{-1} \begin{bmatrix} k(kU_{k,2n} + U_{k,2n-1}) + U_{k,2n} & U_{k,2n+1} \\ k(kU_{k,2n-1} + U_{k,2n-2}) + U_{k,2n-1} & U_{k,2n} \end{bmatrix}$$

$$U^{n+1} = q^{-1} \begin{bmatrix} kU_{k,2n+1} + U_{k,2n} & U_{k,2n+1} \\ kU_{k,2n} + U_{k,2n-1} & U_{k,2n} \end{bmatrix}$$

$$U^{n+1} = q^{-1} \begin{bmatrix} U_{k,2n+2} & U_{k,2n+1} \\ U_{k,2n+1} & U_{k,2n} \end{bmatrix}$$

Hence the result.

Theorem 3.2. *For any positive integer* n *the nth power of a matrix U is given by*

$$U^n = \begin{pmatrix} F_{k,2n+1} & F_{k,2n} \\ F_{k,2n} & F_{k,2n-1} \end{pmatrix} \tag{3.2}$$

Proof: To prove the result we shall use induction on n.

For $n = 1$, $U^n = q^{-1} \begin{pmatrix} F_{k,3} & F_{k,2} \\ F_{k,2} & F_{k,1} \end{pmatrix} = \begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}$, that is true using initial conditions of the sequence. Suppose that (3.2) is true for n. Now we show that (3.2) is true for $n+1$ then

$$U^{n+1} = \begin{pmatrix} F_{k,2n+1} & F_{k,2n} \\ F_{k,2n} & F_{k,2n-1} \end{pmatrix} \begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}$$

$$U^{n+1} = \begin{pmatrix} k^2F_{k,2n+1} + F_{k,2n+1} + kF_{k,2n} & kF_{k,2n+1} + F_{k,2n} \\ k^2F_{k,2n} + F_{k,2n} + kF_{k,2n-1} & kF_{k,2n} + F_{k,2n-1} \end{pmatrix}$$

$$U^{n+1} = \begin{pmatrix} kF_{k,2n+2} + F_{k,2n+1} & F_{k,2n+2} \\ kF_{k,2n+1} + F_{k,2n} & F_{k,2n+1} \end{pmatrix}$$

$$U^{n+1} = \begin{pmatrix} F_{k,2n+3} & F_{k,2n+2} \\ F_{k,2n+2} & F_{k,2n+1} \end{pmatrix}$$

as required.

Theorem 3.3. *For any positive integer* n

$$U_{k,2n} = qF_{k,2n+1} \tag{3.3}$$

Proof: By equating the equations (3.1) and (3.2) we can get the desired result.

Theorem 3.4. (Cassini's Identities) *For any positive integer* n

$$U_{k,2n}U_{k,2n-2} - U_{k,2n-1}^2 = q^2 \text{ and} \tag{3.4}$$

$$F_{k,2n+1}U_{k,2n-1} - F_{k,2n}^2 = 1 \tag{3.5}$$

Proof: The ongoing theorem can be simply proved by using concept of determinant to matrices U and U^n in equations (2.1), (3.1) and (3.2).

Theorem 3.5. *For any positive integer* n

$$U_{k,2n} = U_{k,n}F_{k,n+1} + U_{k,n-1}F_{k,n} \tag{3.6}$$

$$U_{k,2n-1} = U_{n,k}F_{k,n} + U_{k,n-1}F_{k,n-1} \tag{3.7}$$

Proof: $U^n = U^{\frac{n}{2}}U^{\frac{n}{2}}$

$$U^{\frac{n}{2}}U^{\frac{n}{2}} = q^{-1}\begin{pmatrix} U_{k,n} & U_{k,n-1} \\ U_{k,n-1} & U_{k,n-2} \end{pmatrix}\begin{pmatrix} F_{k,n+1} & F_{k,n} \\ F_{k,n} & F_{k,n-1} \end{pmatrix}$$

$$U^{\frac{n}{2}}U^{\frac{n}{2}} = q^{-1}\begin{pmatrix} U_{k,n}F_{k,n+1} + U_{k,n-1}F_{k,n} & U_{k,n}F_{k,n} + U_{k,n-1}F_{k,n-1} \\ U_{k,n-1}F_{k,n+1} + U_{k,n-2}F_{k,n} & U_{k,n-1}F_{k,n} + U_{k,n-2}F_{k,n-1} \end{pmatrix}$$

But $U^n = q^{-1}\begin{pmatrix} U_{k,2n} & U_{k,2n-1} \\ U_{k,2n-1} & U_{k,2n-2} \end{pmatrix}$ hence we get

$U_{k,2n} = U_{k,n}F_{k,n+1} + U_{k,n-1}F_{k,n}$ and

$U_{k,2n-1} = U_{n,k}F_{k,n} + U_{k,n-1}F_{k,n-1}$

Theorem 3.6. *For any integers* n *and* m

$$U_{k,2n+2m} = U_{k,2n}F_{k,2m} + U_{k,2n-1}F_{k,2m},\ n \ge 1,\ m \ge 0 \tag{3.7}$$

$$U_{k,2n+2m-1} = U_{k,2n}F_{k,2m} + U_{k,2n-1}F_{k,2m-1},\ n,m \ge 1 \tag{3.8}$$

Proof:

$$U^{n+m} = U^nU^m$$

$$U^{n+m} = q^{-1}\begin{pmatrix} U_{k,2n} & U_{k,2n-1} \\ U_{k,2n-1} & U_{k,2n-2} \end{pmatrix}\begin{pmatrix} F_{k,2m+1} & F_{k,2m} \\ F_{k,2m} & F_{k,2m-1} \end{pmatrix}$$

$$U^{n+m} = q^{-1}\begin{pmatrix} U_{k,2n}F_{k,2m+1} + U_{k,2n-1}F_{k,2m} & U_{k,2n}F_{k,2m} + U_{k,2n-1}F_{k,2m-1} \\ U_{k,2n-1}F_{k,2m+1} + U_{k,2n-2}F_{k,2m} & U_{k,2n-1}F_{k,2m} + U_{k,2n-2}F_{k,2m-1} \end{pmatrix}$$

But $U^{n+m} = \begin{pmatrix} U_{k,2n+2m} & U_{k,2n+2m-1} \\ U_{k,2n+2m-1} & U_{k,2n+2m-2} \end{pmatrix}$ then we get the desired result as

$U_{k,2n+2m} = U_{k,2n}F_{k,2m} + U_{k,2n-1}F_{k,2m}$

$U_{k,2n+2m-1} = U_{k,2n}F_{k,2m} + U_{k,2n-1}F_{k,2m-1}$

Theorem 3.7. *For any integers n and m*

$$U_{k,2n-2m} = U_{k,2n}U_{k,2m-2} - U_{k,2n-1}U_{k,2m-1},\ n,m \ge 1,\ n \ge m \tag{3.9}$$

$$U_{k,2n-2m-1} = U_{k,2n-1}U_{k,2m} - U_{k,2n}U_{k,2m-1},\ n,m \ge 1,\ n \ge m \tag{3.10}$$

Proof:

$$U^{n-m} = U^n (U^m)^{-1}$$

$$U^{n-m} = q^{-1}\begin{pmatrix} U_{k,2n} & U_{k,2n-1} \\ U_{k,2n-1} & U_{k,2n-2} \end{pmatrix} q \begin{pmatrix} U_{k,2m-2} & -U_{k,2m-1} \\ -U_{k,2m-1} & U_{k,2m} \end{pmatrix}^{-1}$$

$$U^{n-m} = q^{-2}\begin{pmatrix} U_{k,2n}U_{k,2m-2} - U_{k,2n-1}U_{k,2m-1} & -U_{k,2n}U_{k,2m-1} + U_{k,2n-1}U_{k,2m} \\ U_{k,2n-1}U_{k,2m-2} - U_{k,2n-2}U_{k,2m-1} & -U_{k,2n-1}U_{k,2m-1} + U_{k,2n-2}U_{k,2m} \end{pmatrix}$$

But $U^{n-m} = q^{-1}\begin{pmatrix} U_{k,2n-2m} & U_{k,2n-2m-1} \\ U_{k,2n-2m-1} & U_{k,2n-2m-2} \end{pmatrix}$

Therefore

$$U_{k,2n-2m} = U_{k,2n}U_{k,2m-2} - U_{k,2n-1}U_{k,2m-1},\ n,m \ge 1,\ n \ge m$$

$$U_{k,2n-2m-1} = U_{k,2n-1}U_{k,2m} - U_{k,2n}U_{k,2m-1},\ n,m \ge 1,\ n \ge m$$

Theorem 3.8. *For any integers n and m*

$$U_{k,2n-2m} = U_{k,2n}F_{k,2m-1} - U_{k,2n-1}F_{k,2m-1},\ n,m \ge 1,\ n \ge m \tag{3.11}$$

$$U_{k,2n-2m-1} = U_{k,2n-1}F_{k,2m+1} - U_{k,2n}F_{k,2m},\ n,m \ge 1,\ n \ge m \tag{3.12}$$

Proof: It can be proved same as theorem (3.7)

Theorem 3.9. *For any integers n and m*

$$U_{k,2n} = q\frac{U_{k,2n+2m} + U_{k,2n-2m}}{U_{k,2m} + U_{k,2m-2}},\ n,m \ge 1,\ n \ge m \tag{3.13}$$

Proof:

$$qU^{n+m} + qU^{n-m} = U^n(qU^m + qU^{-m})$$

$$qU^{n+m} + qU^{n-m} = U^n\left[\begin{pmatrix} U_{k,2m} & U_{k,2m-1} \\ U_{k,2m-1} & U_{k,2m-2} \end{pmatrix} + q^2\begin{pmatrix} U_{k,2m} & U_{k,2m-1} \\ U_{k,2m-1} & U_{k,2m-2} \end{pmatrix}^{-1}\right]$$

$$qU^{n+m} + qU^{n-m} = U^n\left[\begin{pmatrix} U_{k,2m} & U_{k,2m-1} \\ U_{k,2m-1} & U_{k,2m-2} \end{pmatrix} + \begin{pmatrix} U_{k,2m-2} & -U_{k,2m-1} \\ -U_{k,2m-1} & U_{k,2m} \end{pmatrix}^{-1}\right]$$

$$qU^{n+m} + qU^{n-m} = U^n\begin{pmatrix} U_{k,2m} + U_{k,2m-2} & 0 \\ 0 & U_{k,2m-2} + U_{k,2m} \end{pmatrix}$$

$$qU^{n+m} + qU^{n-m} = q^{-1}\begin{pmatrix} U_{k,2n} & U_{k,2n-1} \\ U_{k,2n-1} & U_{k,2n-2} \end{pmatrix}\begin{pmatrix} U_{k,2m} + U_{k,2m-2} & 0 \\ 0 & U_{k,2m-2} + U_{k,2m} \end{pmatrix}$$

$$qU^{n+m} + qU^{n-m} = q^{-1}\begin{pmatrix} U_{k,2n}(U_{k,2m} + U_{k,2m-2}) & U_{k,2n-1}(U_{k,2m-2} + U_{k,2m}) \\ U_{k,2n-1}(U_{k,2m} + U_{k,2m-2}) & U_{k,2n-2}(U_{k,2m-2} + U_{k,2m}) \end{pmatrix}$$

Since $qU^{n+m} + qU^{n-m} = \begin{pmatrix} U_{k,2n+2m} + U_{k,2n-2m} & U_{k,2n+2m-1} + U_{k,2n-2m-1} \\ U_{k,2n+2m-1} + U_{k,2n-2m-1} & U_{k,2n+2m-2} + U_{k,2n-2m-2} \end{pmatrix}$

Therefore,

$$q^{-1}U_{k,2n}(U_{k,2m} + U_{k,2m-2}) = U_{k,2n+2m} + U_{k,2n-2m}$$

Hence,

$$U_{k,2n} = q\frac{U_{k,2n+2m} + U_{k,2n-2m}}{U_{k,2m} + U_{k,2m-2}}$$

Theorem 3.10. *For any integers n and m*

$$U_{k,2n} = q\frac{U_{k,2n+2m} + U_{k,2n-2m}}{F_{k,2m+1} + F_{k,2m-1}}, \ n,m \geq 1, \ n \geq m \tag{3.14}$$

Theorem 3.11. *For any positive integer n*

$$\begin{pmatrix} U_{k,2n+1} \\ U_{k,2n} \end{pmatrix} = q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}\begin{pmatrix} F_{k,2n} \\ F_{k,2n-1} \end{pmatrix} \tag{3.15}$$

Proof: Here we shall use induction on n. Indeed the result is true for $n = 1$. Assume that the result is true for n. Now we show that the result is true for $n+1$ then

$$\begin{pmatrix} U_{k,2n+3} \\ U_{k,2n+2} \end{pmatrix} = \begin{pmatrix} kU_{k,2n+2} + U_{k,2n+1} \\ U_{k,2n+2} \end{pmatrix}$$

$$\begin{pmatrix} U_{k,2n+3} \\ U_{k,2n+2} \end{pmatrix} = \begin{pmatrix} k^2U_{k,2n+1} + kU_{k,2n} + U_{k,2n+1} \\ U_{k,2n+2} \end{pmatrix}$$

$$\begin{pmatrix} U_{k,2n+3} \\ U_{k,2n+2} \end{pmatrix} = \begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}\begin{pmatrix} U_{k,2n+1} \\ U_{k,2n} \end{pmatrix}$$

Since the result is true for n then

$$\begin{pmatrix} U_{k,2n+3} \\ U_{k,2n+2} \end{pmatrix} = q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}\begin{pmatrix} F_{k,2n} \\ F_{k,2n-1} \end{pmatrix}$$

$$\begin{pmatrix} U_{k,2n+3} \\ U_{k,2n+2} \end{pmatrix} = q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}\begin{pmatrix} k^2F_{k,2n} + F_{k,2n} + kF_{k,2n-1} \\ kF_{k,2n} + F_{k,2n-1} \end{pmatrix}$$

$$\begin{pmatrix} U_{k,2n+3} \\ U_{k,2n+2} \end{pmatrix} = q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}\begin{pmatrix} kF_{k,2n+1} + F_{k,2n} \\ F_{k,2n+1} \end{pmatrix}$$

$$\begin{pmatrix} U_{k,2n+3} \\ U_{k,2n+2} \end{pmatrix} = q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}\begin{pmatrix} F_{k,2n+2} \\ F_{k,2n+1} \end{pmatrix}$$

as required.

Corollary 3.12. *For any positive integer n*

$$\begin{pmatrix} U_{k,2n+1} \\ U_{k,2n} \end{pmatrix} = \begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}\begin{pmatrix} U_{k,2n-1} \\ U_{k,2n-2} \end{pmatrix} \tag{3.16}$$

Proof: It can be easily seen by using the concept of theorem (3.3) in theorem (3.11) and after that we get the desired result.

Theorem 3.13. *For any positive integer n*

$$\begin{pmatrix} U_{k,2n+1} \\ U_{k,2n} \end{pmatrix} = q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}^n\begin{pmatrix} F_{k,2} \\ F_{k,1} \end{pmatrix} \tag{3.16}$$

Proof: To prove the result we shall use induction on n. Clearly the result is true for $n = 1$. Assume that the result is true for n. Now we show that the result is true for $n+1$ then

$$q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}^n\begin{pmatrix} F_{k,2} \\ F_{k,1} \end{pmatrix} = \begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}^n\begin{pmatrix} F_{k,2} \\ F_{k,1} \end{pmatrix}$$

$$q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}^{n+1}\begin{pmatrix} F_{k,2} \\ F_{k,1} \end{pmatrix} = \begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}\begin{pmatrix} U_{k,2n+1} \\ U_{k,2n} \end{pmatrix}$$

$$q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}^{n+1}\begin{pmatrix} F_{k,2} \\ F_{k,1} \end{pmatrix} = \begin{pmatrix} kU_{k,2n+2} + U_{k,2n+1} \\ kU_{k,2n+1} + U_{k,2n} \end{pmatrix}$$

$$q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}^{n+1}\begin{pmatrix} F_{k,2} \\ F_{k,1} \end{pmatrix}=\begin{pmatrix} kU_{k,2n+2}+U_{k,2n+1} \\ kU_{k,2n+1}+U_{k,2n} \end{pmatrix}$$

$$q\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}^{n+1}\begin{pmatrix} F_{k,2} \\ F_{k,1} \end{pmatrix}=\begin{pmatrix} U_{k,2n+3} \\ U_{k,2n+2} \end{pmatrix}$$

as required.

Corollary 3.14. *For any positive integer* n

$$\begin{pmatrix} U_{k,2n+1} \\ U_{k,2n} \end{pmatrix}=\begin{pmatrix} k^2+1 & k \\ k & 1 \end{pmatrix}^{n}\begin{pmatrix} U_{k,1} \\ U_{k,0} \end{pmatrix} \tag{3.17}$$

Theorem 3.15. *For any integers* n *and* m

$$qU_{k,2n+2m+1}=U_{k,2m}U_{k,2n+1}+U_{k,2m-1}U_{k,2n},\ n\geq 0,\ m\geq 1 \tag{3.18}$$

$$qU_{k,2n+2m}=U_{k,2m-1}U_{k,2n+1}+U_{k,2m-2}U_{k,2n},\ n\geq 0,\ m\geq 1 \tag{3.19}$$

Proof: we can prove it easily by using $U^{m+n+\frac{1}{2}}=U^mU^{n+\frac{1}{2}}$

Theorem 3.16. *For any integers* n *and* m

$$qU_{k,2n+2m+1}=U_{k,2m+2}U_{k,2n-1}+U_{k,2m+1}U_{k,2n-2},\ n\geq 1,\ m\geq 0 \tag{3.20}$$

Proof: Since

$$\begin{pmatrix} U_{k,2n+1} \\ U_{k,2n} \end{pmatrix}=U\begin{pmatrix} U_{k,2n-1} \\ U_{k,2n-2} \end{pmatrix}$$

Multiplying both sides by U^m, we get

$$U^m\begin{pmatrix} U_{k,2n+1} \\ U_{k,2n} \end{pmatrix}=U^{m+1}\begin{pmatrix} U_{k,2n-1} \\ U_{k,2n-2} \end{pmatrix}$$

$$\begin{pmatrix} U_{k,2m} & U_{k,2m-1} \\ U_{k,2m-1} & U_{k,2m-2} \end{pmatrix}\begin{pmatrix} U_{k,2n+1} \\ U_{k,2n} \end{pmatrix}=\begin{pmatrix} U_{k,2m+2} & U_{k,2m+1} \\ U_{k,2m+1} & U_{k,2m} \end{pmatrix}\begin{pmatrix} U_{k,2n-1} \\ U_{k,2n-2} \end{pmatrix}$$

$$\begin{pmatrix} U_{k,2m}U_{k,2n+1}+U_{k,2m-1}U_{k,2n} \\ U_{k,2m-1}U_{k,2n+1}+U_{k,2m-1}U_{k,2n} \end{pmatrix}=\begin{pmatrix} U_{k,2m+2}U_{k,2n-1}+U_{k,2m+1}U_{k,2n-2} \\ U_{k,2m+1}U_{k,2n-1}+U_{k,2m}U_{k,2n-2} \end{pmatrix}$$

Now by using theorem (3.15), we have

$$\begin{pmatrix} qU_{k,2n+2m+1} \\ qU_{k,2n+2m} \end{pmatrix}=\begin{pmatrix} U_{k,2m+2}U_{k,2n-1}+U_{k,2m+1}U_{k,2n-2} \\ U_{k,2m+1}U_{k,2n-1}+U_{k,2m}U_{k,2n-2} \end{pmatrix}$$

Hence,

$$qU_{k,2n+2m+1}=U_{k,2m+2}U_{k,2n-1}+U_{k,2m+1}U_{k,2n-2}$$

IV. Conclusion

In the present paper properties of generalized k-Fibonacci sequence have been presented by matrix methods.

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