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On Independent Equitable Cototal Dominating set of graph

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Abstract : *A subset D of $V(G)$ is an independent set if no two vertices in D are adjacent. A dominating set D which is also an independent dominating set. An independent dominating set D of vertex set $V(G)$ is called independent equitable cototal dominating set, if it satisfied the following condition:*

i) *For every vertex* $u \in D$ there exist a vertex $v \in V - D$ *such that* $uv \in E(G)$ *and* $|\deg(u) - \deg(v)| \leq 1$.

ii) $< V - D >$ *contains no isolated vertex.*

The minimum cardinality of independent equitable cototal dominating set is called independent equitable cototal domination number of a graph and it is denoted by $\gamma_{ic}^{e}(G)$. In this paper, we initiate the study of new degree equitable domination parameter.

Keywords: *Domination number, Equitable domination number, Cototal domination number, independent equitable cototal domination number.*

I. Introduction

All graphs considered here are simple, finite, connected and nontrivial. Let $G = (V(G), E(G))$ be a graph, where *V(G)* is the vertex set and *E(G)* be the edge set of *G*. A subset $D \subseteq V$ is said to be a *dominating set* of G if every vertex $V - D$ is adjacent to at least one vertex in *D*. The minimum cardinality of a minimal dominating set is called the *domination number* of *G* [2]. A subset *D* of $V(G)$ is an independent set if no two vertices in *D* are adjacent. A dominating set *D* which is also an independent dominating set. The independent domination number $i(G)$ is the minimum cardinality of an independent domination set [2,3]. A subset *D* of *V* is called an *equitable dominating set* if for every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$, where *deg(u) and deg(v)* denotes the degree of a vertex *u* and *v* respectively. The minimum cardinality of such a dominating set is denoted by γ^{e} and is called the *equitable domination number* [7].

A dominating set *D* is said to be a *cototal dominating set* if the induced subgraph $< V - D >$ has no isolated vertex. The cototal domination number $\gamma_{cl}(G)$ of *G* is the minimum cardinality of a cototal dominating set of *G* [6].

Analogously, we introduce new concept on independent equitable cototal dominating set as follows.

Definition 1.

An independent dominating set *D* of vertex set $V(G)$ is called i*ndependent equitable cototal dominating set*, if it satisfied the following condition:

1. For every vertex $u \in D$ there exist a vertex $v \in V - D$ such that $uv \in E(G)$ *and* $|\deg(u) - \deg(v)| \leq 1$.
2. $< V - D >$ contains no isolated vertex.

The minimum cardinality of independent equitable cototal dominating set is called *independent equitable cototal domination number* of a graph and it is denoted by $\gamma_{ic}^{e}(G)$.

Example:

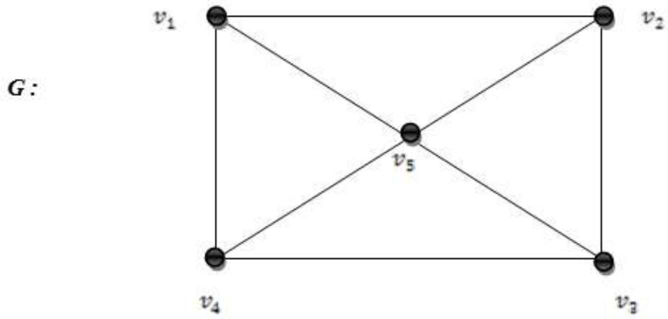


Fig.1.

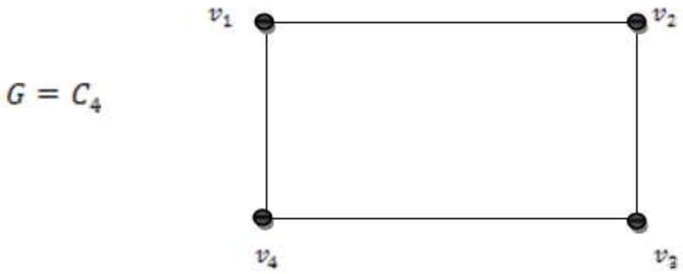
The dominating set $D = \{ v_5 \}$ which is also an independent dominating set.

Independent equitable cototal dominating set $D = \{ v_5 \}$.

Hence $\gamma_{ic}^{e}(G) = |D| = 1.$

Remark: Let G be any graph with independent dominating set D for some $u, v, w \in V(G) and\ u, w \in D$. If $N(v) \cap N(v) = \{u, w\}$ then G does not contain independent equitable cototal dominating set.

For example:



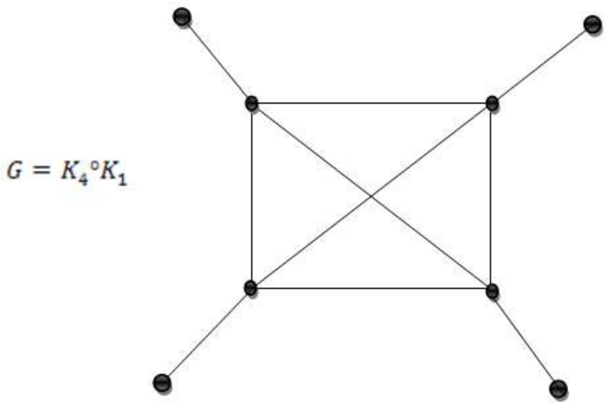
In this graph independent equitable cototal dominating set does not exist.
In general,

i) For P_n ,$n \neq 4$ independent equitable cototal dominating set does not exist.

ii) For $C_n, n \neq 3$ independent equitable cototal dominating set does not exist.

iii) $G = HoK_1$ where H is any connected graph with $\delta(G) \geq 3$ we cannot define independent equitable cototal dominating set.

Example:



Firstly, we obtain the independent equitable cototal domination number $\gamma_{ic}^{e}(G)$ of some standard class of graphs. Which are listed in the following proposition.

Proposition 1:

i) For any complete graph $G = K_n, n \geq 3$, $\gamma_{ic}^{e}(K_n) = 1$

ii) For any complete bipartite graph $G = K_{m,n}$,

$$\gamma_{ic}^{e}(K_{m,n}) = \begin{cases} 2 & \text{if } |m-n| \leq 1 \\ \text{does not exist} & \text{otherwise} \end{cases}$$

iii) For any complete bipartite graph $G = W_n$,

$$\gamma_{ic}^{e}(W_n) = \begin{cases} 1 & \text{if } n = 4 \\ \text{does not exist} & \text{otherwise} \end{cases}$$

Proof:

i) Let *G* be a complete graph of order at least 4. Let $\{v_1, v_2, v_3, v_4 \dots \dots v_n\}$ be the vertices of K_n. Let $D = \{v\}$ be independent cototal dominating set of G. Since K_n is a $(n-1) - regular$. Therefore for every vertex $u \in V - D$, $|\deg(u) - \deg(v)| = 0$. Hence *D* acts as an independent equitable cototal dominating set.

Therefore $\gamma_{ic}^{e}(G) = 1$.

ii) Let $G = K_{m,n}$ be a complete bipartite graph with partite sets of cardinality *m* & *n* respectively. We consider the following cases.

Case i) If $|m-n| \leq 1$

Let $V'(G) = m \text{ and } V''(G) = n$, $V'(G) \cup V''(G) = m + n$. By definition of complete bipartite graph no two vertices of the same partite sets are adjacent.
Since $|m-n| \leq 1$, therefore one vertex from each partite set is sufficient to dominate vertex set of G. Therefore any independent cototal dominating set acts as an independent equitable cototal dominating set of G. Hence $\gamma_{ic}^{e}(G) = 2$.

Case ii) If $|m-n| \leq 2$, then for every vertex $v \in D$ there exist a vertex $u \in V - D$ such that $|\deg(u) - \deg(v)| \geq 2$. Therefore the independent equitable cototal dominating set does not exist.

iii) Let G be wheel graph W_n. By definition of wheel graph $W_n = C_{n-1} + K_1$. We consider the following cases.

Case i) For $n = 4$, W_n is isomorphic to K_4. Therefore by (i) $\gamma_{ic}^{e}(W_n) = 1$.

Case ii) For $n \geq 5$, we can observe that $|\deg(u) - \deg(v)| \geq 2$ where u is the cototal vertex of W_n. Further $\deg(u) = n - 1$. Hence G does not contain independent equitable cototal dominating set.

II. Bounds For Independent Equitable Cototal Dominating Set

Theorem 1: For any graph *G* without isolated vertices, $1 \leq \gamma_{ic}^{e}(G) \leq \frac{n}{2}$, equality of lower bound holds if and only if $\Delta(G) = n - 1 \text{ and } \delta(G) \geq n - 2$. Further equality of upper bound holds if $G = P_4$.

Proof: Let *G* be any graph without isolated vertices, then by Proposition 1, it is easy to see that $\gamma_{ic}^{e}(G) \geq 1$.
For equality, suppose $\Delta(G) = n - 1 \; and \; \delta(G) \geq n - 2$ then *G* contains a vertex *u* which as degree $n - 1$ and a vertex of minimum degree v which as degree at least $n - 2$.
Clearly, $\{u\} = D$ is an independent equitable cototal dominating set.
Such that $|\deg(u) - \deg(v)| \leq 1$.
Conversely, Suppose $\gamma_{ic}^{e}(G) = 1$ and $\Delta(G) = n - 1 \; and \; \delta(G) \leq n - 3$. Then for every vertex $u \in D$ there is no vertex $v \in V - D$ such that $|dge(u) - \deg(v)| \leq 1$. This is a contradiction. Therefore $\delta(G) \geq n - 2$.
Now, the upper bound follows from the fact that, for any graph G contains at most $\frac{n}{2}$ independent vertices. Hence $\gamma_{ic}^{e}(G) \leq \frac{n}{2}$.
Equality case is easy to follow.
Theorem 2: For any graph *G* without isolated vertices, $\gamma_{ic}^{e}(G) \leq \beta(G)$, equality holds if $G = K_n, \; n \geq 3$ where $\beta(G)$ is the vertex independent number.
Proof: Let $\{v_1, \; v_2, \; v_3 \dots \dots . v_n\}$ be the vertex set of a graph *G*. Let *S* be a collection of all independent vertices of *G*. Such that $|S| = \beta(G)$. If for every vertex $v \in S \; there \; exist \; u \in V - D$ such that $|\deg(u) - \deg(v)| \leq 1 \; and < V - D >$ contains no isolated vertices, then *S* act as a minimal independent equitable cototal dominating set of *G*.

Hence, $\gamma_{ic}^{e}(G) \leq |S| \leq \beta(G)$
$\gamma_{ic}^{e}(G) \leq \beta(G)$.
Theorem 3: For any graph *G* without isolated vertices, $\gamma_{ic}^{e}(G) \leq n - \alpha(G)$, where $\alpha(G)$ is the vertex covering number of *G*.
Proof: Let *G* be a graph without isolated vertices. We know from famous Gallia's theorem

$$\alpha(G) + \beta(G) = n.$$

Hence by theorem (2) and using this result we get the required inequality.
Theorem 4: For any graph *G* without isolated vertices, $\gamma_{ic}^{e}(G) + \alpha_0(G) \leq n$, equality holds if $G = K_n, n \geq 4$.
Proof: Follows from theorem (2) and theorem (3).
For equality case, if $G = K_n, n \geq 4$ then by Proposition 1, $\gamma_{ic}^{e}(K_n) = 1$ and from the fact that $\alpha(K_n) = n - 1$, combining these two results, we get the required results.
Theorem 5: For any r-regular graph *G*, $\gamma_{ic}^{e}(G) = \gamma_{ic}(G)$.
Proof: Suppose *G* is the regular graph. Then every vertex as the some degree *r*. Let *D* be a minimum independent cototal dominating set of *G*, then $|D| = \gamma_{ic}(G)$. Let $u \in V - D$, then as *D* is an independent cototal dominating set there exist a vertex $v \in D \; and \; uv \in E(G)$. Also $\deg(u) = \deg(v) = r$. Therefore $|\deg(u) - dge(v)| = 0 < 1$. Hence *D* is degree equitable independent cototal dominating set of *G*. So that $\gamma_{ic}^{e}(G) \leq |D| \leq \gamma_{ic}(G)$. But $\gamma_{ic}(G) \leq \gamma_{ic}^{e}(G)$.
Hence $\gamma_{ic}^{e}(G) = \gamma_{ic}(G)$.
Theorem 6: For any graph without isolated vertices $\gamma_{ic}^{e}(G) \leq n - \Delta(G)$, where $\Delta(G)$is the maximum degree of G, equality holds if $G = K_n, n \geq 4$.
Proof: Let *G* be a graph containing no isolated vertices. Let $v \in V(G)$ be a vertex of maximum degree that is $\deg(v) = \Delta(G)$. Since every vertex dominates at most the vertices in its neighborhood, that is $v \in D$ dominates $\Delta(G)$ if vertices. Further if *G* contains vertex *u* of minimum degree δ. Such that

$\delta(G) \geq n - \Delta - 1$ then every vertex in $V - D$ will be degree equitable to some vertex in *D*. Further $< V - D >$ contains no isolated vertices. Hence $\gamma_{ic}^{e}(G) \leq n - \Delta(G)$.

Equality follows from Proposition 1.

Theorem 7: For any graph *G* without isolated vertices, $\frac{n}{\Delta(G)+1} \leq \gamma_{ic}^{e}(G)$.

Proof: We know that $\frac{n}{\Delta(G)+1} \leq \gamma(G)$. Further the theorem follows from the fact that

$$\gamma(G) \leq \gamma^{e}(G) \leq \gamma_{ic}^{e}(G)$$

Hence , $\frac{n}{\Delta(G)+1} \leq \gamma_{ic}^{e}(G)$.

Theorem 8: Every maximal equitable independent set is a minimal independent equitable cototal dominating set.

Proof: Let $\{v_1, v_2, v_3 \dots \dots . v_n\}$ be the vertex set of a graph *G*. Let *M* be a set of all independent vertices of *G* which are degree equitable to $V - M$. That is for every vertex $u \in M$ there exist a vertex $v \in V - M$ such that $|\deg(u) - \deg(v)| \leq 1$.

Suppose *M* is a maximal independent equitable set then obviously *M* will be minimal independent equitable dominating set.

Further, if $V - M$ contains no isolated vertices, then *M* will be equitable independent cototal dominating set of *G*. Hence every maximal equitable independent set is a minimal independent equitable cototal dominating set.

Nordhous and Gaddum Type results:

Theorem 9: For any graph *G* without isolated vertices

i) $\gamma_{ic}^{e}(G) + \gamma_{ic}^{e}(\bar{G}) \leq n + 1$

ii) $\gamma_{ic}^{e}(G) * \gamma_{ic}^{e}(\bar{G}) \leq n$.

Equality holds for $G = K_n, n \geq 4$

Proof:

i) Let *G* be a graph without isolated vertices. Suppose $\gamma_{ic}^{e}(G) + \gamma_{ic}^{e}(\bar{G}) \leq n + 1$ then either $\gamma_{ic}^{e}(G) = n$ or $\gamma_{ic}^{e}(\bar{G}) = 1$. If $\gamma_{ic}^{e}(G) = n$ then the theorem (1). It's not possible.

There fore $\gamma_{ic}^{e}(G) = 1$

By Proposition (1), *G* must be a complete graph contains no edges. Hence entire vertex set act as an independent equitable cototal dominating set. Hence $\gamma_{ic}^{e}(\bar{G}) = n$

Therefore , $\gamma_{ic}^{e}(G) + \gamma_{ic}^{e}(\bar{G}) \leq n + 1$.

ii) Follows from (i).

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