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Study on Intuitionistic Semiopen Sets

G. Sasikala[1], M. Navaneetha Krishnan[2]
[1]*Research Scholar, Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu, India.*
[1]*Assistant Professor, Department of Mathematics, Valliammai Engineering College, Tamilnadu,India.*
[2]*Associate Professor, Department of Mathematics, Kamaraj College, Tuticorin, Tamilnadu, India.*

Abstract: *The purpose of this note is to define "intuitionistic semiopen sets" in intuitionistic topological spaces and discuss its properties. Also, we define intuitionistic semiinterior and intuitionistic semiclosure of any intuitionistic set and study its properties.*
Keywords: *intuitionistic set, intuitionistic topology, intuitionistic topological space, intuitionistic openset, intuitionistic closed set, interior and closure of IS , intuitionistic points.*

I. Introduction

D. Coker [1,2,3,4] defined and studied intuitionistic topological spaces, intuitionistic open sets, intuitionistic closed sets and compactness on intuitionistic topological spaces using intuitionistic sets. Also, he defined the closure and interior operators in intuitionistic topological spaces and established their properties. In this paper, we define intuitionistic semiopen sets and intuitionistic semiclosed sets. Also, we discuss the properties of these sets. The following definitions and results are essential to proceed further.

Definition 1.1. Let X be a nonempty fixed set. An intuitionistic set (IS for short)[1] A is an object having the form $A= < X,A^1,A^2 >$ where A^1 and A^2 are subsets of X such that $A^1 \cap A^2 = \phi$. The set A^1 is called the set of member of A, while A^2 is called the set of non member of A.
Every subset A of a nonempty set X is obviously an IS having the form $< X,A,A^c >$. Several relations and operations between IS's are given below.

Definition 1.2. [1] Let X be a non empty set, $A= < X,A^1,A^2 >$ and $B= < X,B^1,B^2 >$ be IS on X and let $\{A_i : i \in j\}$ be an arbitrary family of IS in X, where $A_i = < X,A_i^1,A_i^2 >$. Then
(a) $A \subseteq B$ if and only if $A^1 \subseteq B^1$ and $B^2 \subseteq A^2$.
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $A \sqsubset B$ if and only if $A^1 \cup A^2 \supseteq B^1 \cup B^2$.
(d) $\bar{A}= < X, A^2, A^1 >$ is called the complement of A.
(e) $\cup A_i = < X, \cup A_i^1, \cap A_i^2 >$.
(f) $\cap A_i = < X, \cap A_i^1 , \cup A_i^2 >$.
(g) $A - B = A \cap B^c$.
(h) $[\,]A = < X, A^1, (A^1)^c >$.
(i) $< > A = < X, (A^2)^c,A^2 >$.
(j) $\tilde{\phi} = < X, \phi, X >$ and $\tilde{X} = < X,X,\phi >$. Clearly for every $A= < X,A^1,A^2 >$, $A \subset \tilde{X}$.

Definition 1.3. [1] An intuitionistic topology (IT for short) on a nonempty set X is a family τ of IS's satisfying the following axioms.
(a) $\tilde{\phi} , \tilde{X} \in \tau$,
(b) $G_1 \cap G_2 \in \tau$ for every $G_1, G_2 \in \tau$, and
(c) $\cup G_i \in \tau$ for every arbitrary family $\{G_i : i \in J \} \subseteq \tau$.

The pair (X, τ) is called an intuitionistic topological space (ITS for short) and any IS G in τ is called an intuitionistic open set (IOS for short) in X. The complement $\bar{A}$ of an IO set A in an ITS (X, τ) is called an intuitionistic closed set [1] (ICS for short). Also, in [1] it is stated that if (X, τ) is an ITS on X then the following families are also ITS's on X.

(a) $\tau_{0,1} = \{[\,]G \mid G \in \tau\}$.
(b) $\tau_{0,2} = \{< > G \mid G \in \tau\}$.

Now we state the definition for the closure and interior operators in ITS's.

Definition 1.4. [1] Let (X, τ) be an ITS and $A= < X,A^1,A^2 >$ be an IS in X. Then the interior and the closure of A are denoted by int(A) and cl(A), respectively and are defined as follows.
$cl(A) = \cap\{K \mid K$ is an ICS in X and $A \subseteq K\}$ and
$int(A) = \cup\{G \mid G$ is an IOS in X and $G \subseteq A\}$.
Also, it can be established that cl(A) is an ICS and int(A) is an IOS in X and closure and interior are monotonic and idempotent operators. Moreover, closure is increasing and interior is decreasing. Moreover, A is an ICS in X if and only if cl(A) = A and A is an IOS in X if and only if int(A) = A.

Definition 1.5. [2] Let X be a nonempty set and $p \in X$ a fixed element in X. Then the IS $\tilde{p}$ defined by $\tilde{p} = < X,\{p\}, \{p\}^c >$ is called an intuitionistic point(IP for short) in X.

II. Intuitionistic Semiopen Set

A subset $A= < X,A^1,A^2 >$ of $\tilde{X} = < X,X, \phi >$ is said to be an intuitionistic semiopen set (in short, ISO set) in an intuitionistic topological space (X,τ) if there is an intuitionistic open set (IO set) $G \neq < X, \phi,X >$ such that $G \subset A \subset cl(G)$. Clearly, every IO set is an ISO set, $\tilde{\phi}$ and $\tilde{X}$ are ISO sets. Also, from the definition, it follows that the closure of every IO set is an intuitionistic semiopen set. The complement of every intuitionistic semiopen is said to be an intuitionistic semiclosed (ISC) set. The following Theorem 2.1 gives some properties of ISO sets.

Theorem 2.1. Let (X , τ) be an ITS and $\{A_\alpha \mid \alpha \in I\}$ be a family of ISO sets.Then $\cup\{A_\alpha \mid \alpha \in I\}$ is an ISO set.
Proof. For each α, since A_α is an ISO set, there exists an IO set G_α such that $G_\alpha \subset A_\alpha \subset cl(G_\alpha)$. Therefore, $\cup G_\alpha \subset \cup A_\alpha \subset \cup cl(G_\alpha)$, Since $G_\alpha \subset \cup G_\alpha$, $cl(G_\alpha) \subset cl(\cup G_\alpha)$ and so $\cup cl(G_\alpha) \subset cl(\cup G_\alpha)$.Hence $\cup G_\alpha \subset \cup A_\alpha \subset \cup cl(G_\alpha) \subset cl(\cup G_\alpha)$ which implies that $\cup A_\alpha$ is an ISO set.

The following Example 2.2 shows that the intersection of two ISO sets is not an ISO set.

Example 2.2. Consider (R, τ), the real line with the usual topology. Then $\{< R,G, \phi > \mid G \in \tau \}$ is an ITS. If $A = < R, (1, 2], \phi >$ and $B = < R, [2, 3), \phi >$ then A and B are intuitionistic semiopen sets but $A \cap B = < R, \{2\}, \phi >$ is not an intuitionistic semiopen set.

The following Theorem 2.3 gives a property of ISO set and Theorem 2.4 gives a characterization of intuitionistic semiopen set.

Theorem 2.3. Let (X , τ) be an ITS. If A is an intuitionistic semiopen set then $int(A) \neq \tilde{\phi}$.
Proof. Suppose that $A= < X,A^1,A^2 >$ is an intuitionistic semiopen set.Then there exists a non-empty intutionistic open set G such that $G \subset A \subset cl(G)$. Now, $G \subset A$ implies that $G = int(G) \subset int(A)$.Since $G\neq \tilde{\phi}$ $int(A) \neq \tilde{\phi}$.

Theorem 2.4. Let (X, τ) be an ITS. Then A is an intuitionistic semiopen set if and only if $A \subset cl(int(A))$.
Proof. Suppose that A is an intuitionistic semiopen set. Then there exists a non-empty intuitionistic open set G such that $G \subset A \subset cl(G)$ which implies that $cl(G) = cl(A)$ and so $A \subset cl(A) = cl(G) \subset cl(int(A))$ which implies that $A \subset cl(int(A))$. Conversely, suppose that $int(A) \subset A \subset cl(int(A))$. If $G = int(A)$ then G is an intuitionistic open set such that $G \subset A \subset cl(G)$.
Therefore, A is an ISO set.

Theorem 2.5. Let (X , τ) be an ITS. If $A \subset B \subset cl(A)$ and A is an ISO set, then B is an ISO set. In particular, if A is an ISO set, then cl(A) is an ISO set.
Proof. Given that A is an ISO set and $A \subset B \subset cl(A)$. Then $cl(B) = cl(A)$. Since $A \subset B$, we have $cl(int(A)) \subset cl(int(B))$. Since A is an ISO set, $A \subset cl(int(B))$ and so $cl(B) = cl(A) \subset cl(cl(int(B))) = cl(int(B))$. Hence $B \subset cl(int(B))$ which implies that B is an ISO set.

Theorem 2.6. Let (X,τ) be an ITS. If A is an ISO set in (X , τ), then A is an ISO set in $(X, \tau_{0,1})$.
Proof. If $A = < X,A^1,A^2 >$ is an ISO set in (X, τ), then there exists a non-empty IO set $G= < X,G^1,G^2 >$ such that $G \subset A \subset cl(G)$ which implies that $< X,G^1,G^2 > \subset < X,A^1,A^2 > \subset cl(< X,G^1,G^2 >)$. Now $< X,G^1,G^2 > \subset < X, A^1, A^2 >$ implies that $G^1 \subset A^1$ and $A^2 \subset G^2$, and $G^1 \cap G^2 = A^1 \cap A^2 = \phi$. Therefore, $G^2 \subset (G^1)^c$ and $A^2 \subset G^2$ which implies that $A^2 \subset (G^1)^c$. Therefore $H = < X,G^1, (G^1)^c >$ is $\tau_{0,1}$ open set such that
$H =< X,G^1, (G^1)^c > \subset < X,A^1,A^2 >$. Next let us prove that $< X,A^1,A^2 > \subset cl(< X,G^1, (G^1)^c >) = cl(H)$. Since

$< X,A^1,A^2 > \subset cl(< X,G^1,G^2 >) = \cap\{K_\alpha =< X,K^1_\alpha , K^2_\alpha >| K_\alpha$ is an ICS in X and $< X,G^1,G^2 > \subset < X,K^1_\alpha , K^2_\alpha >$ for every $\alpha\}$ $=\{\cap K_\alpha =< X, \cap K^1_\alpha , \cup K^2_\alpha >$ where K_α is an ICS in X and $G^1 \subset \cap K^1_\alpha , \cup K^2_\alpha \subset G^2$ for every $\alpha\}$.Therefore, $G^1 \subset \cap K^1_\alpha$ and $\cup K^2_\alpha \subset G^2$ which implies that $A^1 \subset G^1 \subset \cap K^1_\alpha$ and $\cup K^2_\alpha \subset G^2 \subset A^2$ for every α and so $\cup K^2_\alpha \subset G^2 \subset (G^1)^c$ Since $G^1 \cap G^2 = \phi$. So $< X,A^1,A^2 > \subset <X, \cap K^1_\alpha , \cup K^2_\alpha >$.Now $< X,A^1,A^2 > \subset \{\cap K_\alpha =< X, \cap K^1_\alpha , \cup K^2_\alpha >| K_\alpha$ is ICS In X and $G^1 \subset \cap K^1_\alpha , \cup K^2_\alpha \subset G^2$ for every α }, since $G^2 \subset (G^1)^c$ which implies that $< X,A^1,A^2 > \subset \{\cap K_\alpha =< X, \cap K^1_{\alpha,} \cup K^2_\alpha >| G^1 \subset \cap K^1_\alpha , \cup K^2_\alpha \subset (G^1)^c$ for every α }. $< X,A^1,A^2 > \subset \cap\{ K_\alpha =< X, K^1_{\alpha} , K^2_\alpha >| K_\alpha$ is ICS in X and $G^1 \subset K^1_\alpha , K^2_\alpha \subset (G^1)^c$ for every α } which implies that $< X,A^1,A^2 > \subset cl(< X,G^1, (G^1)^c >)$. Therefore, A is semiopen in $\tau_{0,1}$.

The following Example 2.7 shows that the converse of the above Theorem 2.6 is false.

Example 2.7. Let X = {1,2,3,4,5} and consider the family
$\tau = \{\tilde{\phi},\tilde{X},A_1,A_2,A_3,A_4\}$ where
$A_1 = < X, \{ 1, 2, 3\}, \{4\} >$, $A_2 = < X, \{3, 4\}, \{5\} >$, $A_3 = < X, \{3\}, \{4, 5\} >$,
$A_4 = < X, \{1, 2, 3, 4\},\{ \phi \} >$, $\tilde{X} = < X,X, \phi >$ and $\tilde{\phi} = < X, \phi, X >$.
Then, $\tau_{0,1} = \{ \tilde{\phi} , \tilde{X} ,A_{11},A_{12},A_{13},A_{14}\}$ where
$A_{11} = < X, \{1, 2, 3\}, \{4, 5\} >$, $A_{12} = < X, \{3,4\}, \{1, 2, 5\} >$, $A_{13} = < X, \{3\}, \{1, 2, 4, 5\} >$,
$A_{14} = < X,\{1, 2, 3, 4\}, \{5\} >$, $\tilde{X} = < X,X, \phi >$ and $\tilde{\phi} = < X, \phi, X >$. Then A_{12} is an IO set in $(X,\tau_{0,1})$ which implies that A_{12} is an ISO set in $(X,\tau_{0,1})$. Since there is no IO set A in (X,τ) such that $A \subset A_{12} \subset cl(A)$, A_{12} is not an ISO set in (X,τ).

The following Theorem 2.8. shows that every ISO set in $(X, \tau_{0,2})$ is an ISO set in (X,τ). Example 2.9 shows that the converse is not true.

Theorem 2.8. Let (X,τ) be an ITS. If A is an ISO set in $(X, \tau_{0,2})$, then A is an ISO set in (X,τ).
Proof. Suppose that A is ISO set in $(X, \tau_{0,2})$. Then there exists $G =< X, (G^2)^c,G^2 > \in \tau_{0,2}$ which implies that $< X,G^1,G^2 > \in \tau$ for some G^1 such that $G^1 \cap G^2 = \phi$. Now $< X, (G^2)^c,G^2 > \subset < X,A^1,A^2 >$ implies that $(G^2)^c \subset A^1,A^2 \subset G^2$. Since $G^1 \cap G^2 = \phi$ implies that $G^1 \subset (G^2)^c$ and so$< X,G^1,G^2 > \subset < X,A^1,A^2 >$. Now $< X,A^1,A^2 > \subset cl(< X,(G^2)^c,G^2 >) = \cap\{H_\alpha = < X, H^1_\alpha , H^2_\alpha >| H_\alpha$ is ICS in X and $< X, (G^2)^c,G^2 > \subset < X,H^1_\alpha ,H^2_\alpha >$ for every α } $=\{\cap H_\alpha =<X, \cap H^1_\alpha ,\cup H^2_\alpha >| H_\alpha$ is ICS in X and $(G^2)^c \subset \cap H^1_\alpha , \cup H^2_\alpha \subset G^2$ for every α }. $< X,A^1,A^2 > \subset \{< X,\cap H^1_\alpha , \cup H^2_\alpha >| H_\alpha$ is ICS in X and $(G^2)^c \subset \cap H^1_\alpha , \cup H^2_\alpha \subset G^2$ for every α . Now $G^1 \subset (G_2)^c$ which implies that $< X,A^1,A^2 > \subset \{\cap H_\alpha = < X, \cap H^1_\alpha , \cup H^2_\alpha >| H_\alpha$ is ICS in X and $G^1 \subset \cap H^1_\alpha , \cup H^2_\alpha \subset G^2$ for every $\alpha\}$ $=\{\cap H_\alpha =<X, \cap H^1_\alpha ,\cup H^2_\alpha >| H_\alpha$ is ICS in X and $G^1 \subset H^1_\alpha , H^2_{\alpha} \subset G^2\}$. Therefore, $< X,A^1,A^2 > \subset cl (< X,G^1,G^2 >)$. Finally, we have $< X,G^1,G^2 > \subset <X,A^1,A^2 > \subset cl(< X,G^1,G^2 >)$.
Thus $< X,A^1,A^2 >$ is an ISO set in (X, τ).

Example 2.9. Let X={1, 2, 3, 4, 5}. Consider the family
$\tau = \{ \phi, \tilde{X}, A_1, A_2, A_3, A_4\}$ where $A_1 =< X, \{1, 2,3\},\{4\} >$, $A_2 =< X, \{3, 4\}, \{5\} >$, $A_3 = < X, \{3\}, \{4, 5\} >$, $A_4 = < X, \{1, 2, 3, 4\}, \{ \phi \} >$, $\tilde{X} =< X,X, \phi >$,$\tilde{\phi} =< X, \phi,X >$. Then $\tau_{0,2}= \{\tilde{\phi} , \tilde{X}, A_{11},A_{12},A_{13},A_{14}\}$ where $A_{11}= < X, \{1, 2, 3, 5\}, \{4\} >$, $A_{12} = < X, \{1, 2, 3, 4\}, \{5\} >$, $A_{13} = < X, \{1, 2, 3\}, \{4, 5\} >$, $A_{14} = < X, \{1, 2, 3, 4, 5\}, \{ \phi \} >$,$\tilde{X}=< X,X, \phi >$,$\tilde{\phi} =< X, \phi ,X >$. Then A_3 is an IO set in (X,τ) which implies that A_3 is an ISO set in (X,τ). Since there is no IO set A in $(X,\tau_{0,2})$ such that $A \subset A_3 \subset cl(A)$ where cl(A) is the closure of A in $(X, \tau_{0,2})$, A_3 is not an ISO set in $(X, \tau_{0,2})$.

III. Intuitionistic Semiclosure and Intuitionistic Semiinterior Operator

Let (X,τ) be an ITS and A= $< X, A^{\mathbf{1}}, A^{\mathbf{2}} >$ be an IS in X. Then the semiinterior and the semiclosure of a subset A are denoted by Isint(A) and Iscl(A) and are defined as follows.

Iscl(A) = $\cap$ {K | K is an ISC in X and $A \subseteq K$} and
Isint(A) = $\cup$ {G | G is an ISO in X and $G \subseteq A$}.

It can be established that Iscl(A) is the smallest ISC set contained in all ISC sets containing A and Isint(A) is the largest ISO set contained in A, A is an ISC set in X if and only if Iscl(A) = A and A is an ISO set in X if and only if Isint(A) = A. We say that A is $I\text{−}dense$ if cl(A) = X. The following Theorem 3.1 gives some properties of the semiclosure and semiinterior operators.

Theorem 3.1. For any IS A in (X, τ), we have (i) $Iscl(\overline{A})$.= $\overline{Isint(A)}$ and (ii) $Isint(\overline{A}) = \overline{Iscl(A)}$.
Proof. (i) $Iscl(\overline{A}) = Iscl(X - A) = \cup$ {K | K is an ISC set in X and $X - A \subset K$}. Therefore,

$X - \mathrm{Iscl}(X - A) = \{X - K \mid X - K$ is an ISO set in X and $X - K \subset A\} = \cup\{G \mid G$ is an ISO set in X and $G \subset A\}$ = Isint(A). Therefore, $\mathrm{Iscl}(\overline{A}) = \overline{Isint(A)}$.

(ii) $\mathrm{Isint}(\overline{A}) = \mathrm{Isint}(X - A) = \cup\{G \mid G$ is an ISO set in X and $G \subset X - A\}$, $X - \mathrm{Isint}(X - A) = \cap\{X - G \mid X - G$ is an ISC set in X and $A \subset X - G\} = \cap\{B \mid B$ is an ISC set in X and $A \subset B\}$. Therefore, $X - \mathrm{Isint}(X - A)$ = Iscl(A). Hence $X - \mathrm{Iscl}(A) = \mathrm{Isint}(X - A)$. Therefore, $\mathrm{Isint}(\overline{A}) = \overline{Iscl(A)}$.

The following Theorem 3.2 gives characterizations of intuitionistic semiopen sets.

Theorem 3.2. Let A be a subset of X .Then the following are equivalent.
(a) A is an ISO set.
(b) $A \subset$ cl(int(A)).
(c) cl(A) = cl(int(A)).

Proof. (a) ⇒ (b). Suppose A is an intuitionistic semiopen set.Then there exists an IO set G such that $G \subset A \subset$ cl(G). Since G is an IO set, G = int(G) and so $A \subset$ cl(int(G)). Since $G \subset A$, $A \subset$ cl(int(A)).

(b) ⇒ (c). Also, cl(int(A)) $\subset$ cl(A). Thus cl(A)$\subset$ cl(cl(int(A))) = cl(int(A)).Therefore, cl(A) = cl(int(A)).

(c) ⇒ (a). Since int(A) $\subset A \subset$ cl(A) = cl(int(A)), A is an intuitionistic semiopen set.

The following Theorem 3.3, gives a property of intuitionistic semiopen sets.

Theorem 3.3. If $A \subset B \subset$ cl(A) and A is an ISO in (X,τ), then B is an ISO. In particular, the intuitionistic closure of every intuitionistic semiopen set is an intuitionistic semiopen set.
Proof. Since A is an intuitionistic semiopen set, by Theorem 3.2(c), cl(A) = cl(int(A)) and so cl(A) $\subset$ cl(int(B)). Since $B \subset$ cl(A), $B \subset$ cl(int(B)).Therefore, B is an ISO set.

The following Theorem 3.4 gives characterizations of intuitionistic semiclosed sets.

Theorem 3.4. Let A be an IS in (X,τ). Then the following are equivalent.
(a) A is an intuitionistic semiclosed set.
(b) int(cl(A)) $\subset$ A.
(c) int(cl(A)) = int(A).
(d) There exist an Intuitionistic closed set F such that int(F) $\subset A \subset F$.

Proof. (a) ⇒ (b). Since A is an intuitionistic semiclosed set, $X - A$ is an intuitionistic semiopen set and so $X - A \subset$ cl(int($X - A$)) by Theorem 3.2(b). By Theorem 3.1, cl(int($X - A$)) = $X -$ int(cl(A)) and so int(cl(A)) $\subset$ A.

(b) ⇒ (c) int(cl(A)) $\subset$ A implies that int(cl(A)) $\subset$ int(A) and so it follows that int(cl(A)) = int(A).

(c) ⇒ (d) If F = cl(A), then F is an intuitionistic closed set such that int(F) = int(cl(A)) = int(A) $\subset A \subset F$.

(d) ⇒ (a) If there exists an Intuitionistic closed set F such that int(F) $\subset A \subset F$, then $X - F \subset X - A \subset X -$ int(F) = cl($X - F$) Since $X - F$ is an Intuitionistic semiopen set, $X - A$ is an intuitionistic semiopen set by Theorem 3.3 and so A is intuitionistic semiclosed.
The following Lemma 3.5 is essential to prove the remaining results of the section.
Lemma 3.5. Let (X,τ) be an ITS and A be an IS. Then $x \in$ Iscl(A) if and only if every ISO set U containing x, intersects A.
Proof. The proof follows from [7], Lemma 2.3
The following Theorem 3.6, gives characterizations of $I -$ dense sets.
Theorem 3.6. If (X,τ) is any ITS and A is an IS, then the following are equivalent.
(a) A is $I -$ dense.
(b) Iscl(A) = X.
(c) If B is any intuitionistic semiclosed subset of X such that $A \subset B$, then B = X.
(d) Every nonempty intuitionistic semiopen set has a nonempty intersection with A.
(e) Isint($X - A$) = ϕ.

Proof. (a) ⇒ (b). Suppose $x \notin$ Iscl(A).Then there exists an intuitionistic semiopen set G containing x such that $G \cap A = \phi$. Since G is a nonempty intuitionistic semiopen set, there is a nonempty intuitionistic open set H such that $H \subset G$ and so $H \cap A = \phi$, a contradiction. Therefore, Iscl(A) = X.

(b) ⇒ (c). If B is any intuitionistic semiclosed set such that $A \subset B$, then $X = Iscl(A) \subset Iscl(B) = B$ which implies that $B = X$.

(c) ⇒ (d). If G is any nonempty intuitionistic semiopen set such that $G \cap A = \phi$, then $A \subset X - G$ and $X - G$ is intuitionistic semiclosed. By (c), it folllows that $G = \phi$, a contradiction.

(d) ⇒ (e). Suppose that $Isint(X - A) \neq \phi$. Then $Isint(X - A)$ is a nonempty intuitionistic semiopen set such that $Isint(X - A) \cap A \neq \phi$, a contradiction to the hypothesis.

(e) ⇒ (a). $Isint(X - A) = \phi$ implies that $X - Iscl(X - (X - A)) = \phi$ and so $Iscl(A) = X$. Hence $cl(A) = X$ which shows that A is $I -$ dense.

The following Theorem 3.7. gives some properties of the operators Isint and Iscl.

Theorem 3.7. If X is any nonempty set and A is a subset of X, then the following hold.

(a) $Isint(A) = A \cap cl(int(A))$.
(b) $Iscl(A) = A \cup int(cl(A))$.
(c) $Isint(Iscl(A)) = Iscl(A) \cap cl(int(cl(A)))$.
(d) $Iscl(Isint(Iscl(A))) = Isint(Iscl(A))$.
(e) $A \cup Isint(Iscl(A)) = Iscl(A)$.
(f) $Iscl(Isint(A)) = Isint(A) \cup int(cl(int(A)))$.
(g) $Isint(Iscl(Isint(A))) = Iscl(Isint(A))$.
(h) $A \cap Iscl(Isint(A)) = Isint(A)$.

Proof.

(a) Since $int(int(A)) = int(A)$ and $int(A) \subset cl(int(A))$ for every subset A of X, by Theorem 1.3 of [6], it follows that $Isint(A) = A \cap cl(int(A))$.

(b) Proof follows from (a).

(c) $Isint(Iscl(A)) = Iscl(A) \cap cl(int(Iscl(A)))$ by (a) and so $Isint(Iscl(A)) \subset Iscl(A) \cap cl(int(cl(A)))$, since $Isint(A) = A \cap cl(Isint(A))$.
Again, $Isint(Iscl(A)) = Iscl(A) \cap cl(int(Iscl(A))) = Iscl(A) \cap cl(int(A \cup int(cl(A)))) \supset Iscl(A) \cap cl(int(int(cl(A)))) = Iscl(A) \cap cl(int(cl(A)))$. This proves (c).

(d) Since every intuitionistic closed set is intuitionistic semiclosed, from(c), $Iscl(A) \cap cl(int(cl(A)))$ is intuitionistic semiclosed and so $Isint(Iscl(A))$ is intuitionistic semiclosed. Hence (d) follows.

(e) $A \cup Isint(Iscl(A)) = A \cup (Iscl(A) \cap Iscl(Isint(Iscl(A)))) = (A \cup Iscl(A)) \cap A \cup Iscl(Isint(Iscl(A)))) = Iscl(A) \cap Iscl(A) = Iscl(A)$.Therefore $A \cup (Isint(Iscl(A))) \supset Iscl(A)$. The reverse direction is clear.

(f), (g) and (h) are similarly proved.

Example 2.2. Shows that the intersection of two intuitionistic semiopen set is not an intuitionistic semiopen set. The following Theorem 3.8 shows the intersection is an intuitionistic semiopen set, if one of the set is an intuitionistic open set.

Theorem 3.8. Let (X,τ) be any space. If A is an intuitionistic open set and B is an intuitionistic semiopen set, then $A \cap B$ is an intuitionistic semiopen set.

Proof. Since B is an intuitionistic semiopen, there exist an intuitionistic open set G such that $G \subset B \subset Icl(G)$ and so $A \cap G \subset A \cap B \subset A \cap Icl(G)$.

Since $A \cap G$ is an intuitionistic open set, $A \cap G = Iint(A \cap G)$ and by Proposition 2.1 of [5], $A \cap Icl(G) \subset Icl(A \cap G)$.

Therefore , $A \cap B \subset A \cap Icl(G) \subset Icl(A \cap G) = Icl(Iint(A \cap G)) \subset Icl(Iint(A \cap B))$. Hence $A \cap B$ is an intuitionistic semiopen.

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