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Solving Fuzzy Travelling Salesman Problem Using Octagon Fuzzy Numbers with α-Cut and Ranking Technique

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Abstract: *The travelling salesman problem is to find the shortest route that the salesman visit all the places and return the source place with minimum cost. In this paper, we solve the assignment problem of maximize the profit and travelling salesman problem using octagonal fuzzy numbers . By ranking the octagonal fuzzy numbers it is possible to compare them and using this we convert the fuzzy valued number to a crisp value. It is proved that a better solution is obtained when it is solved using fuzzy octagonal number than when it is solved using trapezoidal fuzzy number. This method is easy to understand and apply to find optimal solution of fuzzy assignment problems and fuzzy travelling salesman problems occurring in real life situations.*

Keywords: *Octagonal Fuzzy Number, Fuzzy Assignment Problem, Fuzzy Travelling salesman problem, Fuzzy Ranking Method*
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I. Introduction

Fuzzy sets, introduced by Zadeh in 1965[12] .Provide us a new mathematical tool to deal with uncertainty of information. Since then, fuzzy set theory has been rapidly developed. The Assignment Problem (AP) is a special type of linear programming problem (LPP) in which our objective is to assign a number of origins to the equal number of destinations at a minimum cost (or maximum profit). We assume that one person can be assigned exactly one job, also each person can do at most one job. The problem is to find an optimal assignment so that the total cost of performing all jobs is minimize or maximize the total profit. Dubois and Fortemps[2] surveys refinements of the ordering of solutions supplied by the max-min formulation. They havegiven a general algorithm which computes all maximal solutions in the sense of these relations. Different kinds of fuzzy transportation problems are solved in the papers[4,5,8,9]. Dominance of fuzzy numbers can be explained by many ranking methods[1,3,6,7].

Here we investigate a more realistic problem, namely the assignment problem with fuzzy costs or times $\widetilde{C}_{ij}$. Since the objectives are to minimize the cost or to maximize the total profit, subject to some crisp constraints, the objective function is considered also as a fuzzy number. The method is to rank the fuzzy objective values of the objective function by some ranking method for fuzzy numbers to find the best alternative. On the basis of this idea a new ranking method is used to transform the fuzzy assignment problem to a crisp one so that the conventional solution methods may be applied to solve the assignment problem.

The idea is to transform a problem with fuzzy parameters to a crisp version in the LPP form and to solve it by ranking method. Other than the fuzzy assignment problem other applications of this method can be tried in project scheduling, Sequencing, Replacement problem, etc. A ranking using α-cut is introduced on octagonal fuzzy numbers. Using this ranking the fuzzy assignment problem or fuzzy travelling salesman problem is converted to a crisp valued problem, which can be solved using Hungarian method. The optimal solution can be got either as a fuzzy number or as a crisp number.

II. Preliminary

2.1. Fuzzy Set:A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse X to the unit interval [0, 1], i.e. $A = \{(x, \mu_A(x)) ; x \in X\}$. Here $\mu_A : X \to [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

2.2. Normal fuzzy set: A fuzzy set *A* of the universe of discourse X is called a normal fuzzy set implying that there exist at least one $x \in X$ such that $\mu_A(x) = 1$.

2.3. Definition: A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$, where $a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0\,, & x < a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ k\,, & a_2 \le x \le a_3 \\ k + (1-k)\left(\frac{x-a_3}{a_4-a_3}\right), & a_3 \le x \le a_4 \\ 1\,, & a_4 \le x \le a_5 \\ k + (1-k)\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ k\,, & a_6 \le x \le a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right), & a_7 \le x \le a_8 \\ 0\,, & x \ge a_8\,, \end{cases} \quad \text{Where } 0 < k < 1.$$

.4. Definition: Robust ranking technique which satisfy compensation, linearity and additive properties and provides results which are consistent with human intuition. If ã is a convex fuzzy number, the Robust ranking index is defined by $R(\tilde{a}) = \int_0^1 (0.5)(\alpha_\alpha^l, \alpha_\alpha^U)\,d\alpha$,
Where $(\alpha_\alpha^L\,, \alpha_\alpha^U) = [\{(b-a)\alpha + a, d-(d-c)\alpha\}, \{(f-e)\alpha + e, h-(h-g)\alpha\}]$ is a α-level cut of a fuzzy number ã.
2.5. Definition: Given a fuzzy set 'A' defined on 'X' and any number α ∈[0,1], the α –cut is denoted by A(α) and is defined by the crisp set $A(\alpha) = \{x: A(x) \ge \alpha\}$. i.e. A(α)={x: μ(x)≥α , α ∈ [0,1]}

III. Solving Fuzzy Travelling Salesman Problem

3.1. Example: Consider the assignment problem which maximize the total profit. Here the cost (C_{ij}) involved in executing a given job is considered as fuzzy quantifiers.

$[C_{ij}]_{5X5}$ =

$$\begin{pmatrix} (-2,-1,0,1,2,3,7,8) & (-4,-2,0,2,4,6,8,10) & (-3,-1,1,3,5,7,9,11) & (4,5,6,7,8,9,10,11) & (-3,-2,-1,0,1,2,5,6) \\ (1,3,5,7,9,11,12,13) & (9,10,11,12,13,14,15,16) & (2,4,5,7,9,10,12,13) & (6,7,8,9,10,11,12,13) & (-6,0,6,12,18,24,30,36) \\ (-3,-1,0,1,2,4,5,6) & (2,3,4,5,6,7,8,9) & (3,6,7,8,9,10,12,13) & (1,2,3,5,6,7,8,10) & (0,1,2,3,4,5,6,7) \\ (5,6,8,10,12,13,14,15) & (-1,0,1,3,5,7,9,10) & (9,10,11,12,13,14,15,16) & (-3,-1,1,2,3,4,7,10) & (-3,-1,1,3,5,7,9,11) \\ (5,6,7,10,12,14,15,17) & (-2,-1,0,1,2,3,4,5) & (-1,0,1,2,3,4,5,6) & (2,4,5,6,7,8,9,11) & (2,3,4,5,6,7,10,11) \end{pmatrix}$$

Solution: The problem then converted to Simplex method and using ranking technique we get single fuzzy number.

$$\begin{aligned} \text{Max } Z = {} & R(-2,-1,0,1,2,3,7,8)x_{11} + R(-4,-2,0,2,4,6,8,10)x_{12} + R(-3,-1,1,3,5,7,9,11)x_{13} + R(4,5,6,7,8,9,10,11)x_{14} \\ & + R(-3,-2,-1,0,1,2,5,6)x_{15} + R(1,3,5,7,9,11,12,13)x_{21} + R(9,10,11,12,13,14,15,16)x_{22} + R(2,4,5,7,9,10,12,13)x_{23} \\ & + R(6,7,8,9,10,11,12,13)x_{24} + R(-6,0,6,12,18,24,30,36)x_{25} + R(-3,-1,0,1,2,4,5,6)x_{31} + R(2,3,4,5,6,7,8,9)x_{32} \\ & + R(3,6,7,8,9,10,12,13)x_{33} + R(1,2,3,5,6,7,8,10)x_{34} + R(0,1,2,3,4,5,6,7)x_{35} + R(5,6,8,10,12,13,14,15)x_{41} \\ & + R(-1,0,1,3,5,7,9,10)x_{42} + R(9,10,11,12,13,14,15,16)x_{43} + R(-3,-1,1,2,3,4,7,10)x_{44} + R(-3,-1,1,3,5,7,9,11)x_{45} \\ & + R(5,6,7,10,12,14,15,17)x_{51} + R(-2,-1,0,1,2,3,4,5)x_{52} + R(-1,0,1,2,3,4,5,6)x_{53} + R(2,4,5,6,7,8,9,11)x_{54} \\ & + R(2,3,4,5,6,7,10,11)x_{55}, \quad \text{and} \quad R(\tilde{a}) = \int_0^1 (0.5)(\alpha_\alpha^L, \alpha_\alpha^U)\,d\alpha \end{aligned}$$

Where $(\alpha_\alpha^L, \alpha_\alpha^U) = [\{(b-a)\alpha + a, d-(d-c)\alpha\}, \{(f-e)\alpha + e, h-(h-g)\alpha\}]$.

Therefore, $R(-2,-1,0,1,2,3,7,8) = \int_0^1 (0.5)(\alpha - 2 + 1 - \alpha, \alpha + 2 + 8 - \alpha)\,d\alpha = \int_0^1 (0.5)(9)\,d\alpha = 4.5$

Similarly, $R(-4,-2,0,2,4,6,8,10) = 6$, $R(-3,-1,1,3,5,7,9,11)=4$, $R(4,5,6,7,8,9,10,11)=15$, $R(-3,-2,-1,0,1,2,5,6)=2$, $R(1,3,5,7,9,11,12,13)=15$, $R(9,10,11,12,13,14,15,16)=25$, $R(2,4,5,7,9,10,12,13)=15.5$, $R(6,7,8,9,10,11,12,13)=19$, $R(-6,0,6,12,18,24,30,36)=30$, $R(-3,-1,0,1,2,4,5,6)=3.5$, $R(2,3,4,5,6,7,8,9)=11$, $R(3,6,7,8,9,10,12,13)=17$, $R(1,2,3,5,6,7,8,10)=10.5$, $R(0,1,2,3,4,5,6,7)=7$, $R(5,6,8,10,12,13,14,15)=21$, $R(-1,0,1,3,5,7,9,10)=8.5$, $(9,10,11,12,13,14,15,16)=25$, $R(-3,-1,1,2,3,4,7,10)=6$, $R(-3,-1,1,3,5,7,9,11)=4$, $R(5,6,7,10,12,14,15,17)= 21.5$, $R(-2,-1,0,1,2,3,4,5)=3$, $R(-1,0,1,2,3,4,5,6)=5$, $R(2,4,5,6,7,8,9,11)=13$, $R(2,3,4,5,6,7,10,11) = 12$.

We get the table after the ranking

4.5	**6**	**4**	**15**	**2**
15	**25**	**15.5**	**19**	**30**
3.5	**11**	**17**	**10.5**	**7**
21	**8.5**	**25**	**6**	**4**
21.5	**3**	**5**	**13**	**12**

After using Hungarian method we get,

			15	
				30
	11			
		25		
21.5				

Now using the allotment rules, the solution of the problem can be obtained in the form of octagon fuzzy numbers

			(-10,-5,0,5,10,15,20,25)	
				(-6,0,6,12,18,24,30,36)
	(2,3,4,5,6,7,8,9)			
		(9,10,11,12,13,14,15,16)		
(5,6,7,10,12,14,15,17)				

C_{14}=(-10,-5,0,5,10,15,20,25), C_{25}=(-6,0,6,12,18,24,30,36), C_{32}=(2,3,4,5,6,7,8,9), C_{43}=(9,10,11,12,13,14,15,16), C_{51}=(5,6,7,10,12,14,15,17)
And the optimal (Maximum)assignment cost is = 15+30+11+25+21.5 = 102.5

3.2. Example:
Consider the fuzzy travelling salesman problem so as to minimise the cost for unit quantity of the C_{ij} where

$$[C_{ij}]_{5X5} = \begin{pmatrix} \infty & (0,1,2,3,4,5,6,7) & (8,9,10,11,12,13,14,15) & (4,5,6,7,8,9,10,11) & (1,3,5,6,7,8,10,12) \\ (-2,-1,0,1,2,3,4,5) & \infty & (2,4,5,6,7,8,9,11) & (-3,-1,0,1,2,4,5,6) & (-2,-1,0,1,2,3,4,5) \\ (2,3,4,5,6,7,8,9) & (3,6,7,8,9,10,12,13) & \infty & (5,6,8,9,10,11,12,15) & (8,9,10,11,12,13,14,15) \\ (4,5,6,7,8,9,11,12) & (1,2,3,5,6,7,8,10) & (0,1,2,3,4,5,6,7) & \infty & (-1,0,1,2,3,4,5,6) \\ (5,6,7,10,12,14,15,17) & (0,1,2,3,4,5,6,7) & (9,10,11,12,13,14,15,16) & (5,6,7,8,9,10,12,13) & \infty \end{pmatrix}$$

Then the problem becomes

	A	B	C	D	E
A	∞	(0,1,2,3,4,5,6,7)	(8,9,10,11,12,13,14,15)	(4,5,6,7,8,9,10,11)	(1,3,5,6,7,8,10,12)
B	(−2, −1,0,1,2,3,4,5)	∞	(2,4,5,6,7,8,9,11)	(−3, −1,0,1,2,4,5,6)	(−2, −1,0,1,2,3,4,5)
C	(2,3,4,5,6,7,8,9)	(3,6,7,8,9,10,12,13)	∞	(5,6,8,9,10,11,12,15)	(8,9,10,11,12,13,14,15)
D	(4,5,6,7,8,9,11,12)	(1,2,3,5,6,7,8,10)	(0,1,2,3,4,5,6,7)	∞	(−1,0,1,2,3,4,5,6)
E	(5,6,7,10,12,14,15,17)	(0,1,2,3,4,5,6,7)	(9,10,11,12,13,14,15,16)	(5,6,7,8,9,10,12,13)	∞

Solution:
The fuzzy travelling salesman problem can be formulated in the following mathematical programming form
Min Z = 0 + R(0,1,2,3,4,5,6,7)x_{12}+R(8,9,10,11,12,13,14,15)x_{13}+R(4,5,6,7,8,9,10,11)x_{14}+R(1,3,5,6,7,8,10,12)x_{15}
+R(-2,-1,0,1,2,3,4,5)x_{21}+0+R(2,4,5,6,7,8,9,11)x_{23}+R(-3,-1,0,1,2,4,5,6)x_{24} +R(-2,-1,0,1,2,3,4,5)x_{25}
+ R(2,3,4,5,6,7,8,9)x_{31}+ R(3,6,7,8,9,10,12,13)x_{32}+0 +R(5,6,8,9,10,11,12,15)x_{34}+
R(5,6,7,10,12,14,15,17)x_{35}+ R(4,5,6,7,8,9,11,12)x_{41}+R(1,2,3,5,6,7,8,10)x_{42} +R(0,1,2,3,4,5,6,7)x_{43}
+ 0 + R(-1,0,1,2,3,4,5,6)x_{45}+R(5,6,7,10,12,14,15,17)x_{51}+ R(0,1,2,3,4,5,6,7))x_{52}+
R(9,10,11,12,13,14,14,15,16)x_{53}+ R(5,6,7,8,9,10,12,13)x_{54}+0

And, R(ã) = $\int_0^1 (0.5)(\alpha_\alpha^L, \alpha_\alpha^U)d\alpha$

Where $(\alpha_\alpha^L, \alpha_\alpha^U)$ = $[\{(b-a)\alpha + a, d-(d-c)\alpha\}, \{(f-e)\alpha + e, h-(h-g)\alpha\}]$

We get, R(0,1,2,3,4,5,6,7) = $\int_0^1 (0.5)(\alpha + 3 - \alpha, \alpha + 4 + 7 - \alpha)d\alpha$ = $\int_0^1 (0.5)(14)d\alpha$ = 7

Similarly

R(8,9,10,11,12,13,14,15)=23, R(4,5,6,7,8,9,10,11)=15, R(1,3,5,6,7,8,10,12)=13, R(-2,-1,0,1,2,3,4,5) =3 , R(2,4,5,6,7,8,9,11)=13, R(-3,-1,0,1,2,4,5,6)=3.5, R(-2,-1,0,1,2,3,4,5)=3, R(2,3,4,5,6,7,8,9)=11, R(3,6,7,8,9,10,12,13)=17,R(5,6,8,9,10,11,12,15)=19, R(8,9,10,11,12,13,14,15)=23,R(4,5,6,7,8,9,11,12)=15.5, R(1,2,3,5,6,7,8,10)=10.5, R(0,1,2,3,4,5,6,7)=7, R(-1,0,1,2,3,4,5,6)=5, R(5,6,7,10,12,14,15,17)=21.5, R(0,1,2,3,4,5,6,7) =7 ,R(9,10,11,12,13,14,15,16)=25, R(5,6,7,8,9,10,12,13)=17.5,

The corresponding table after ranking

	A	B	C	D	E
A	∞	7	23	15	13
B	3	∞	13	3.5	3
C	11	17	∞	19	23
D	15.5	10.5	7	∞	5
E	21.5	7	25	17.5	∞

Applying the Hungarian method the solution is

	A	**B**	**C**	**D**	**E**
A					**13**
B				**3.5**	
C	**11**				
D			**7**		
E		**7**			

Now using the allotment rules, the solution of the problem can be obtained can be in the form of octagon fuzzy number

	A	B	C	D	E
A	∞	(0,1,2,3,4,5,6,7)	(8,9,10,11,12,13,14,15)	(4,5,6,7,8,9,10,11)	**(1,3,5,6,7,8,10,12)**
B	(−2,−1,0,1,2,3,4,5)	∞	(2,4,5,6,7,8,9,11)	**(−3,−1,0,1,2,4,5,6)**	(−2,−1,0,1,2,3,4,5)
C	**(2,3,4,5,6,7,8,9)**	(3,6,7,8,9,10,12,13)	∞	(5,6,8,9,10,11,12,15)	(8,9,10,11,12,13,14,15)
D	(4,5,6,7,8,9,11,12)	(1,2,3,5,6,7,8,10)	**(0,1,2,3,4,5,6,7)**	∞	(−1,0,1,2,3,4,5,6)
E	(5,6,7,10,12,14,15,17)	**(0,1,2,3,4,5,6,7)**	(9,10,11,12,13,14,15,16)	(5,6,7,8,9,10,12,13)	∞

The optimum travelling schedule is : A$\rightarrow E \rightarrow B \rightarrow D \rightarrow C \rightarrow A$

C_{15} =(1,3,5,6,7,8,10,12),C_{24}=(-3,-1,0,1,2,4,5,6),C_{31}=(2,3,4,5,6,7,8,9),C_{43}=(0,1,2,3,4,5,6,7),C_{52}=(0,1,2,3,4,5,6,7)

Total distance travelled = (1,3,5,6,7,8,10,12)+(1,0,1,2,4,5,6)+(2,3,4,5,6,7,8,9)+(0,1,2,3,4,5,6,7)+(0,1,2,3,4,5,6,7)

=(0,7,13,18,23,29,35,41) = 41.5

IV. Conclusion

In this paper, we derived the method of fuzzy assignment problem with maximize the profit and fuzzy travelling salesman problem using octagon fuzzy numbers with α-cut and ranking technique. Also some numerical examples are discussed and also observed the answers are optimal than other existing methods. In future, we tried to use method of octagon fuzzy numbers in fuzzy sequencing problem and fuzzy replacement problem.

References

[1]. C.B. Chen and C.M. Klein, A simple approach to ranking a group of aggregated uzzy utilities, *IEEE Trans. Syst., Man, Cybern. B, SMC*-27 (1997), 26-35.
[2]. D. Dubois and P. Fortemps, Computing improved optimal solutions to max-min flexible constraint satisfaction problems, *European*
[3]. *J. Operations Research*, 118 (1999), 95-126.
[4]. F. Choobineh and H. Li, An index for ordering fuzzy numbers, *Fuzzy Sets and Systems*, 54 (1993), 287-294.
[5]. G.J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*,Prentice-Hall, International Inc.,1995.
[6]. M. Tada and H. Ishii, An integer fuzzy transportation problem, *Comput. Math. Appl.*31 (1996), 71-87.
[7]. P. Fortemps and M. Roubens, Ranking and defuzzification methods based on area compensation, *Fuzzy Sets and Systems*, 82 (1996), 319-330.
[8]. R.R.Yager, A procedure for ordering fuzzy subsets of the unit interval, *Information Sciences*, 24 (1981), 143-161.
[9]. S. Chanas and D. Kuchta, A concept of the optimal solution of the transportation problem with fuzzy cost coefficients, *Fuzzy Sets and Systems*, 82 (1996), 299-305.
[10]. S. Chanas, W. Kolodziejczyk and A. Machaj, A fuzzy approach to the transportation problem, *Fuzzy Sets and Systems*, 13 (1984),
[11]. 211-221.
[12]. S.H. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, *Fuzzy Sets and Systems*, 17 (1985), 113-129.
[13]. S.U.Malini, C.kennedy Felbin, An Approach for solving fuzzy transportation problem Using octagonal Fuzzy numbers, Applied mathematical Sciences 7(54) (2013) 2661-2673.
[14]. L.A. Zadeh, Fuzzy Sets, Information & Control, 8 (1965), 338 - 353. http://dx.doi.org/10.1016/s0019-9958(65)90241-x.