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Topology optimization of 3D structures using ANSYS and MATLAB

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Abstract: *This work presents a study of three-dimensional topology optimization of some academic structures as Messerschmitt Bolkow Bolhm beam (MBB beam) and cantilever beam using the ANSYS APDL (ANSYS Parametric Design Language) based Optimality Criteria approach and the MATLAB numerical results. The basic concept solves minimum compliance problem subject to volume constraints using the Solid Isotropic Material with Penalization (SIMP) method. We compare different parameters like-stresses, displacement, and von mises stress and compliance values. In order to give a faster and better code for implementation of domain decomposition method applying to 3d structures.*
Keywords: *Topology optimization, SIMP method, optimality criteria, minimum compliance, material density.*

I. Introduction

Optimization is a mathematical field for finding an alternative with the most cost or highest achievable performance under the given constraints. The topology optimization is the very important field in structural optimization that searches the more suitable density of material to minimize compliance under volume constraints. This discipline has attracted the interest of applied mathematicians and engineering designers. From the work of Bendsøe and Kikuchi **[1]** which handles optimal topologies using a homogenization method, then Bendsøe and Sigmund explain in detail this theory with various examples **[2]**. In this paper we present a comparative study of 3D topology optimization of MBB beam treated numerically via Matlab and the same results by ANSYS. Many authors have interest by this item like; Sigmund **[3]** which introduced the 99-line program for two-dimensional topology optimization using the SIMP approach (Solid Isotropic Material with Penalization). His program uses stiffness matrix assembly and optimality criteria (OC) methods and presents the optimal topology via filtering strategies. Also for MATLAB, but in three-dimensional case, Kai Liu and Andrés Tovar **[4]** introduced the 169 lines to solve three-dimensional topology optimization problems. This MATLAB code includes finite element analysis, sensitivity analysis, density filter and optimality criterion.

In this paper, we explains the use of ANSYS in minimum compliance, compliant mechanism, and optimality criteria (OC) methods in 3D topology optimization of MBB beam and cantilever beam; we compare the results with "top3d.m" **[4]** and also “top3dfmincon.m”. In section 2 a reviews on some theoretical approaches in topology optimization with focus on the SIMP method applying to continuous and discrete case. Section 3 introduces 3D finite element analysis and its numerical implementation via Ansys and Matlab; we compare different numerical results subject to 50% volume constraints.

II. Theoretical Background

We can define the topology optimization problem as a mathematical programming problem in which the aim is to search the distribution of material or density of the area or volume. A classical formulation is to find the "black and white" layout (i.e., solids and voids) that minimizes the work leads by external forces (compliance) subject to a volume constraint.

The Solid Isotropic Material with Penalization (SIMP) method has been presented by Bendsøe **[2]**, known that the material properties can be expressed in terms of the design variable material density using a simple factor means to suppress intermediate values of the density. The common choice of design parameterization is to take ρ as the design variable by convention, $\rho = 1$ at a point signifies a material region else, $\rho = 0$ represents void.

We search an optimal density which solves the problem:

$$\min_{\rho} l\big(u(\rho)\big)$$
$$a_\rho(u, v) = (f, v)_\Omega \quad \forall v \in H_0^1(\Omega)^3 \qquad (1)$$
$$E_{ijkl}(x) = \rho^p E_{ijkl}^0 \quad with \;\; 1 < p < 3$$

$$\int_{\Omega} \rho(x)d\Omega - \bar{V} \le 0 \ , \qquad 0 < \rho_{min} \le \rho(x) \le 1$$

Where the bilinear form can be written as:

$$a_{\rho}(u,v) = \int_{\Omega} \rho^{p}(x)E^{0}_{ijkl}\,\varepsilon_{ij}(u)\varepsilon_{kl}(v)d\Omega, \qquad and \qquad 1 < p < 3 \qquad (\mathbf{2})$$

With $\varepsilon_{ij}(u) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ represent the linearized tensor of deformations, and E^{0}_{ijkl} is a rigidity tensor of isotropic material. The Sobolev space $H^1(\Omega)$ is defined:

$$H^1(\Omega) = \left\{ v \in L^2(\Omega);\ \forall i = 1..n\ ;\ \frac{\partial v}{\partial x_i} \in L^2(\Omega) \right\} \qquad (\mathbf{3})$$

$\frac{\partial v}{\partial x_i}$ is the weak derivative of function v.

And $H_0{}^1(\Omega) = \{ v \in H^1(\Omega);\ v_{|\partial\Omega} = 0 \}$ $(\mathbf{4})$.

The problem of minimum compliance **(1)** is unbounded and, therefore, ill-posed. One alternative to make it well posed, by imposing an additional constraint on the gradient of the artificial function of local density ρ by taking:

$$\|\rho\|_{H^1(\Omega)} = \left[\int_{\Omega} (\rho^2 + (\nabla\rho)^2)d\Omega \right]^{\frac{1}{2}} \le M \ , \quad \forall x \in \Omega \qquad (\mathbf{5})$$

or a filtering techniques **[2]**.

Optimality criteria (OC) are necessary conditions to minimize the objective function (compliance); it was a classical approach to structural optimization problems. The discrete topology optimization problem is a large scale mathematical programming problem **[5]**, at each iteration, the design variables are updated using this scheme **[2]**:

$$\rho_e^{new} = \begin{cases} \max\{\rho_e - m, 0\} & if\ \ \rho_e B_e^{\eta} \le \max\{\rho_e - m, 0\} \\ \min\{\rho_e + m, 1\} & if\ \rho_e B_e^{\eta} \ge \min\{\rho_e + m, 1\} \\ \rho_e B_e^{\eta} & otherwise \end{cases} \qquad (\mathbf{6})$$

The parameter ρ_e denotes the value of the density variable at the older iteration, and η is a tuning parameter and m a move limit. B_e^{η} is given by the expression:

$$B_e = \Lambda_e^{-1}.\mathrm{p}.\rho_e^{p-1}(x).E^{0}_{ijkl}\,\varepsilon_{ij}(u_e)\varepsilon_{kl}(u_e)$$

where u_e is the displacement field at the older iteration.

The minimum compliance problem in the SIMP approach is given by:

$$\begin{gathered} \min_{\boldsymbol{\rho}} \boldsymbol{\ell}(\boldsymbol{\rho}) = \ \boldsymbol{u^t}.\boldsymbol{K}(\boldsymbol{\rho}).\boldsymbol{u} \\ \boldsymbol{v}(\boldsymbol{\rho}) = \ \boldsymbol{\rho^t}.\boldsymbol{v} \ \le \bar{\boldsymbol{v}} \\ \boldsymbol{\rho_{min}} \le \boldsymbol{\rho_i} \le \mathbf{1} \\ \boldsymbol{\rho} \in \{\boldsymbol{x} \in \mathbb{R}^{\mathbf{3}} \ ; \ \ \mathbf{0} \le \boldsymbol{x} \le \mathbf{1}\} \end{gathered} \qquad (\mathbf{7})$$

The Lagrangian function is defined as

$$\boldsymbol{\mathcal{L}}(\boldsymbol{\rho}) = \boldsymbol{u^tKu} + \boldsymbol{\Lambda}(\boldsymbol{\rho^t}.\boldsymbol{v} - \bar{\boldsymbol{v}}) + \boldsymbol{\lambda}(\boldsymbol{Ku} - \boldsymbol{F}) + \sum_{i=1}^{n} \boldsymbol{\nu_i}(\boldsymbol{\rho_{min}} - \boldsymbol{\rho_i}) \ + \sum_{i=1}^{n} \boldsymbol{\gamma_i}(\boldsymbol{\rho_i} - \mathbf{1}) \qquad (\mathbf{8})$$

Where Λ, λ, ν_i and γ_i are Lagrange multipliers for the different constraints. The optimality condition is given by:

$\frac{\partial \mathcal{L}}{\partial \rho_i} = 0\ ;\ i = 1..n$ $(\mathbf{9})$

III. Numerical Implementation

We study here two academic structures, the one is a Messerschmitt Bolkow Bolhm beam (MBB beam) in sub section III.1 and the second is a Cantilever Beam treated in sub section III.2.

3.1 MBB beam

Let $\boldsymbol{\Omega} \subset \mathbb{R}^{\mathbf{3}}$ be a bounded region with regular boundary $\partial\boldsymbol{\Omega}$ we define**:**

$$\boldsymbol{\Omega} = \{(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \in \mathbb{R}^3 \ : \ \mathbf{0} \le \boldsymbol{x} \le \mathbf{3}, \mathbf{0} \le \boldsymbol{y} \le \mathbf{1}, \mathbf{0} \le \boldsymbol{z} \le \mathbf{1}\} \qquad (\mathbf{10})$$

with the boundary conditions as in **Fig -1**

We discretize the volume using the hexagonal cubic elements with the size 0.05 then we have:
$nelx = 120e, nely = 20e, nelz = 20e$, so the volume $V = nelx \times nely \times nelz = 48000e$

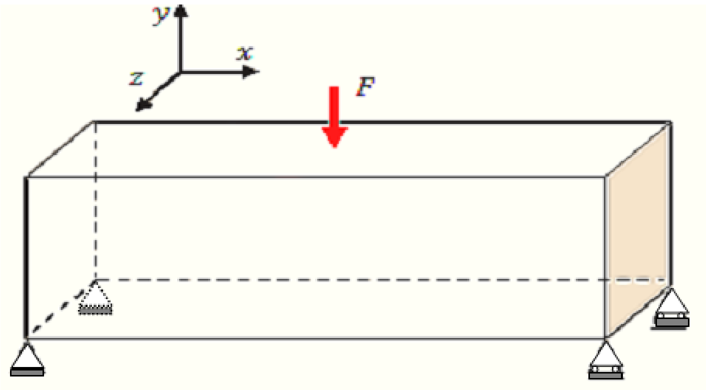

Fig -1: MBB Beam with boundary conditions and loads

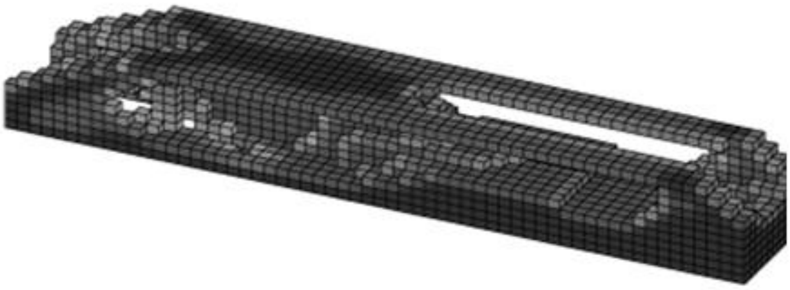

Fig -2: Topology optimization of MBB-Beam (Matlab)

And by executing the ANSYS code, we have the initial Structure **(Fig -3)** and The final structure **(Fig -4)**

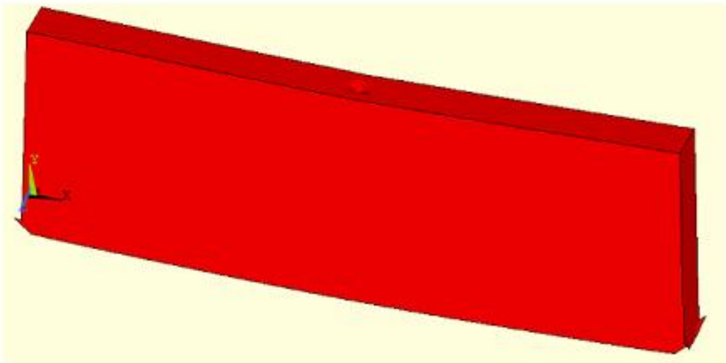

Fig -3: Initial structure (MBB-Beam)

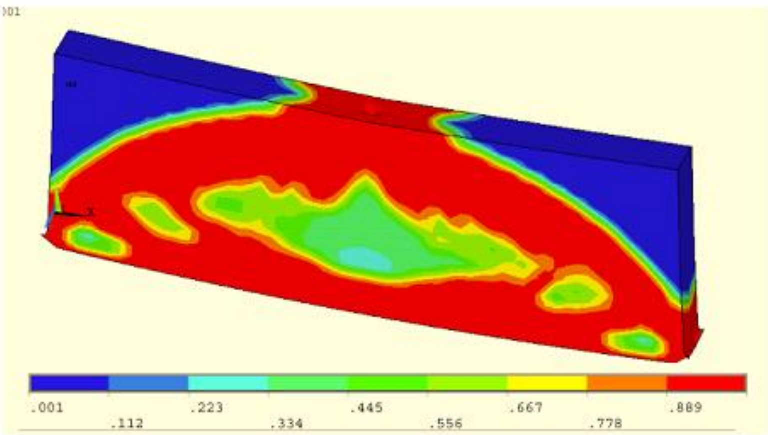

Fig -4: Topology optimization of MBB-Beam

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We mention here the minimum values of displacements, Von Mises strains $\mathcal{E}_{vmtot}$ and Von Mises stresses σ_{vm}

	U_X	U_Y	U_Z	$\mathcal{E}_{vmtot}$	σ_{vm}
nodes	1	3570	1	2341	4681
minimum	-0.48. 10^{-3}	-0.8. 10^{-3}	-0.85. 10^{-5}	0.1. 10^{-2}	0.42. 10^{-5}

And also the maximum values:

	U_X	U_Y	U_Z	$\mathcal{E}_{vmtot}$	σ_{vm}
nodes	60	60	3601	1	1
maximum	-0.16. 10^{-3}	0.22. 10^{-4}	0.22. 10^{-4}	0.25. 10^{-2}	0.25. 10^{7}

We see that values on one hand between the minimal and maximal displacements and on the other hand deformations are very similar. Approximately the gap $\in [3.10^{-5},\ 1.5.10^{-3}]$, contrary to the Von Mises constraints which have a very important gap approximately $2.5.10^6$.

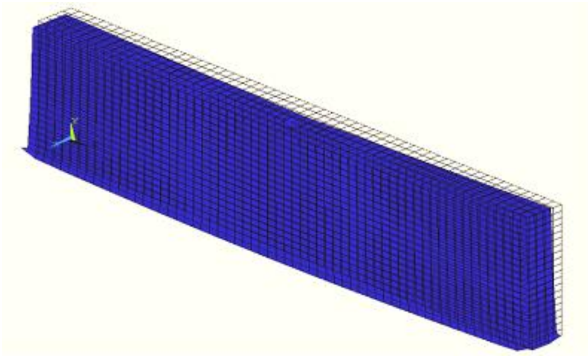


Fig -5: Displacement plot with deformed structure

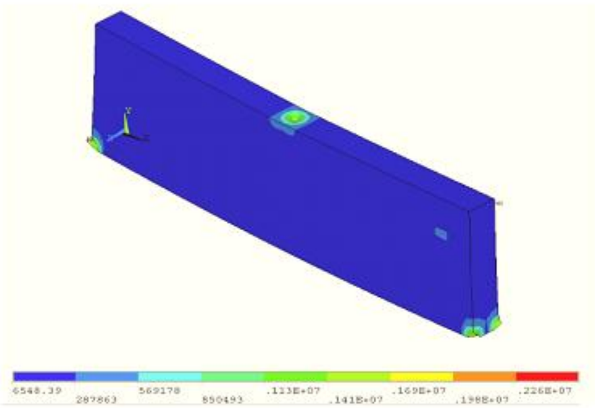


Fig -6: Von Mises stress plot

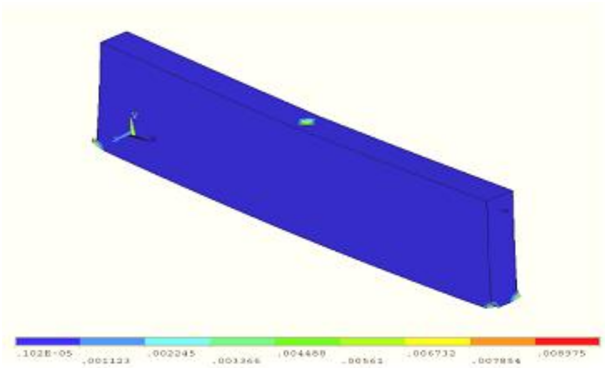


Fig -7: Von Mises elastic strain plot

Cantilever beam

In this sub section we use the domain $\boldsymbol{\Omega} \subset \mathbb{R}^{\mathbf{3}}$ to be a bounded region with regular boundary $\boldsymbol{\partial\Omega}$ we define**:**

$$\boldsymbol{\Omega} = \{(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in \mathbb{R}^3 : \mathbf{0} \leq \boldsymbol{x} \leq \mathbf{1.2}, \mathbf{0} \leq \boldsymbol{y} \leq \mathbf{0.4}, \mathbf{0} \leq \boldsymbol{z} \leq \mathbf{0.3}\} \qquad (\mathbf{11})$$

with the boundary conditions:

$$\{\boldsymbol{U_x} = \boldsymbol{U_y} = \boldsymbol{U_z} = \mathbf{0} \ \ \boldsymbol{if} \ \ \boldsymbol{x} = \mathbf{0}, \boldsymbol{y} \in [\mathbf{0}, \mathbf{0.4}], \boldsymbol{z} \in [\mathbf{0}, \mathbf{0.3}]\} \qquad (\mathbf{12})$$

We discretize the volume using the hexagonal cubic elements with the size 0.05 then we have: $nelx = 60e, nely = 20e, \ nelz = 4e$ so the volume $V = nelx \times nely \times \ nelz = 4800e$

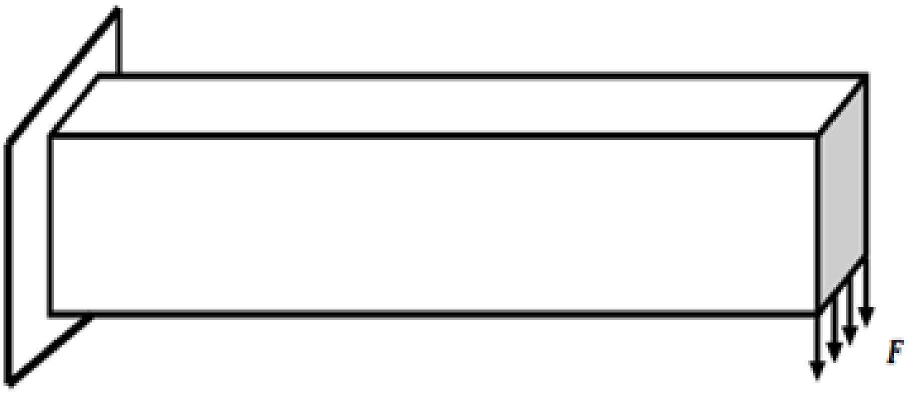


Fig -8: Cantilever Beam with boundary conditions and loads

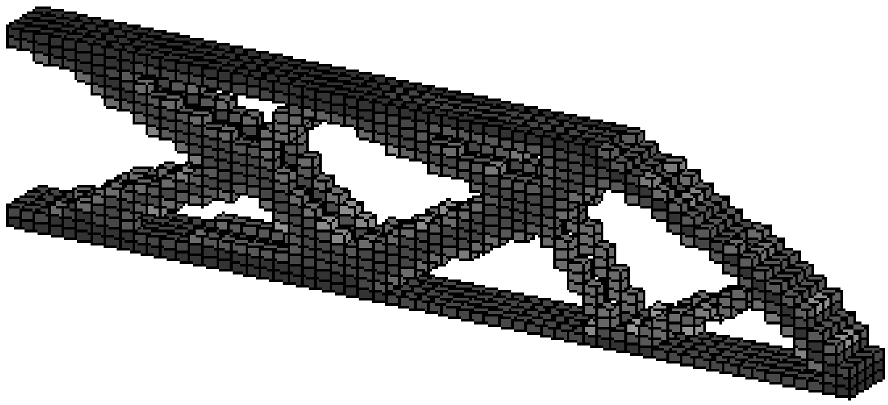
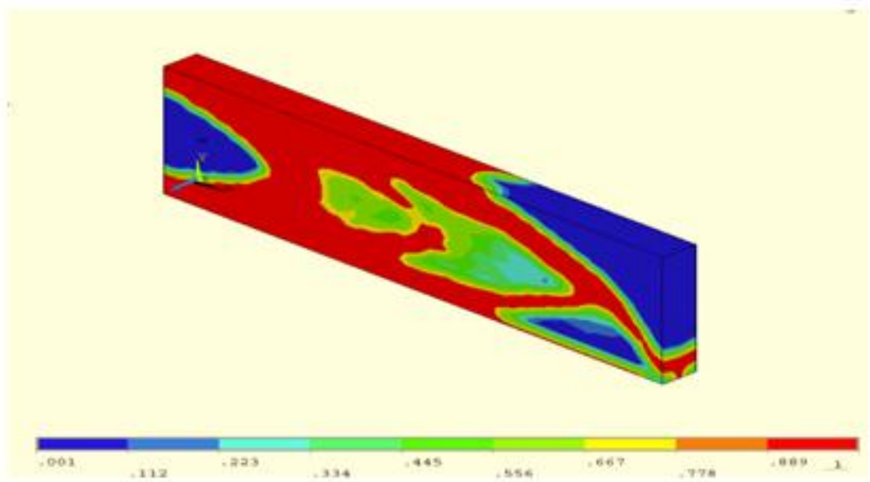
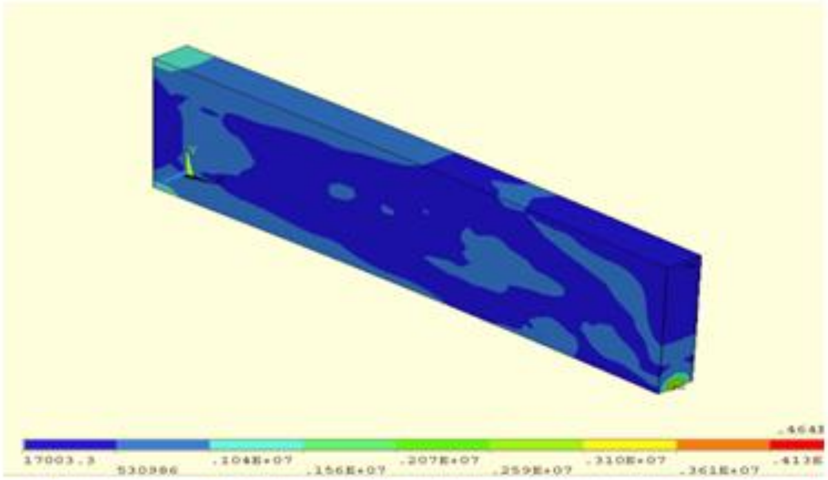


Fig -9: Topology optimization of Cantilever-Beam (MATLAB)

The plots results by executing Ansys code



Optimal topology



Cantilever Von Mises stress

	U_X	U_Y	U_Z	$\mathcal{E}_{vmtot}$	σ_{vm}
nodes	1	3570	1	2341	4681
minimum	-0.19. 10^{-2}	-0.84. 10^{-2}	-0.25. 10^{-4}	0.21. 10^{-4}	1.52. 10^{4}

	U_X	U_Y	U_Z	$\mathcal{E}_{vmtot}$	σ_{vm}
nodes	60	60	3601	1	1
maximum	0.16. 10^{-2}	-0.48. 10^{-2}	0.25. 10^{-4}	0.11· 10^{-1}	0.46· 10^{7}

We are choosing the top3dfmincon.m programs because the following reasons cited below (table 2 and table 3).

Table -2 the MBB-Beam matlab results.

	Mesh size	Volume $\bar{v}$	Max. it	$\boldsymbol{\ell}_{min}$	Time elapsed
top3d.m	48,8,8	0.5	48	5.3626	45,27 s
	60,10,10	0.5	40	5.1427	62,91 s
Top3d.m (fmincon)	48,8,8	0.5	36	5.4699	56,20 s
	60,10,10	0.5	50	5.3104	69,33 s
Ansys	48,8,8	0.5	30	4.4356	63,1 s
	60,10,10	0.5	30	4.1902	87,96 s

Table -3 the cantilever matlab results.

	Mesh size	Volume $\bar{v}$	Max. it	$\boldsymbol{\ell}_{min}$	Time elapsed
top3d.m	48,16,12	0.5	67	24,909	45,27 s
	60,20,4	0.5	62	20,082	62,91 s
Top3d.m (fmincon)	48,16,12	0.5	48	21,921	72 s
	60,20,4	0.5	38	20,362	79,6 s
Ansys	48,16,12	0.5	30	22.764	54,12 s
	60,20,4	0.5	30	18.397	63,05 s

IV. Conclusions

In this paper, we compare the results obtained by Ansys and Matlab, and as we have already seen that the results are almost similar especially in the last iterations. These results allow us to use this study in the next work for a new implementation for domain decomposition method **[6]**, applying to industrial and real world structures. Our second future work is an application of finite element analysis for a structure subject to the variable and dynamic loads. So we study randomly the structure. Finally, we calculate some parameters as the rate of failure, reliability of the structure.

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