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Analysis of Occurrence of Digit 3 in Prime Numbers till 1 Trillion

Neeraj Anant Pande
Associate Professor, Department of Mathematics & Statistics,
Yeshwant Mahavidyalaya (College), Nanded – 431602, Maharashtra, INDIA

Abstract: *Primes less than 10^{12} are analyzed for occurrence of digit 3 in them. Multiple occurrences of 3's are explored. The first and last occurrences of all multiple 3's in them are determined within blocks of powers of 10 till 1 trillion.*
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I. Introduction

It is interesting to see early understanding and exploration of numbers by humans [2]. Many simplest looking integer types pose great challenges even today; amongst them prime numbers are surely on forefront. Explorations of primes on theoretical front [1] and in initial vast ranges [4] have just reconfirmed their irregularities. This paper presents the analysis of occurrence of digit 3 within all primes in ranges of powers of 10 till 1 trillion, i.e., primes p such that $1 < p < 10^n$, $1 \leq n \leq 12$. Such analysis for primes is recently done for digit 0[8], [9], [10], digit 1 [14], [15], [16] and digit 2 [17], [18], [19]. Dividing digits in all natural numbers in two classes of zero [5], [6], [7] and non-zero [11], [12], [13], such study for them is also available.

II. Occurrence of Single Digit 3 in Prime Numbers

3 is first odd prime itself. The way in which digit 3 comes in all positive integers can be inferred from work of [11]. Now all prime numbers p in the range $1 < p < 10^{12}$ are considered for trends of occurrences of digit 3.

Table 1: Number of Prime Numbers in Various Ranges with Single 3 in Their Digits

Sr. No.	Range	Number of Primes with Single 3
1.	$1 – 10^1$	1
2.	$1 – 10^2$	9
3.	$1 – 10^3$	57
4.	$1 – 10^4$	457
5.	$1 – 10^5$	3,693
6.	$1 – 10^6$	30,928
7.	$1 – 10^7$	264,820
8.	$1 – 10^8$	2,296,417
9.	$1 – 10^9$	20,065,110
10.	$1 – 10^{10}$	176,290,694
11.	$1 – 10^{11}$	1,555,436,420
12.	$1 – 10^{12}$	13,767,790,131

III. Occurrence of Multiple Digit 3's in Prime Numbers

These results have required long executions on many electronic computers with the aid of efficient algorithms [3].Single, double, triple and multiple occurrences of digit 3 in all natural numbers in ranges of $1 – 10^n$ is available [11]. The same kind of exploration amongst prime numbers is done here.

Table 2: Number of Prime Numbers in Various Ranges with Multiple 3 in Their Digits

Sr. No.	Number Range <	Number of Prime Numbers with 2 3's	Number of Prime Numbers with 3 3's	Number of Prime Numbers with 4 3's
1.	10^3	9	0	0
2.	10^4	92	12	0
3.	10^5	1,023	154	7
4.	10^6	10,638	1,924	181
5.	10^7	107,948	23,518	3,023
6.	10^8	1,071,488	277,526	44,151
7.	10^9	10,539,313	3,153,036	596,113
8.	10^{10}	102,852,807	34,832,837	7,624,536
9.	10^{11}	997,245,608	376,822,954	93,623,978
0.	10^{12}	9,617,083,586	4,007,995,116	1,113,561,836

Table 2: Continued …

Sr. No.	Number Range <	Number of Prime Numbers with 5 3's	Number of Prime Numbers with 6 3's	Number of Prime Numbers with 7 3's
1.	10^6	14	0	0
2.	10^7	242	13	0
3.	10^8	4,440	270	11
4.	10^9	73,949	6,171	299
5.	10^{10}	1,128,161	114,918	7,956
6.	10^{11}	16,095,596	1,958,080	169,209
7.	10^{12}	217,784,557	30,791,807	3,175,699

Table 2: Continued …

Sr. No.	Number Range <	Number of Primes with 8 3's	Number of Primes with 9 3's	Number of Primes with 10 3's	Number of Primes with 11 3's
1.	10^9	8	0	0	0
2.	10^{10}	372	7	0	0
3.	10^{11}	10,056	388	9	0
4.	10^{12}	236,170	12,414	411	8

These values of multiple occurrences of digit 3's in primes in various ranges of 1 - 10^i is graphically plotted with vertical axis in on logarithmic scale.

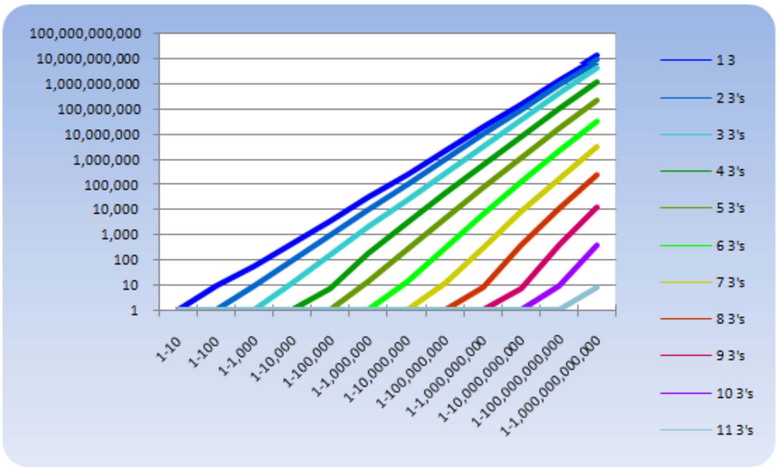


Figure 1: Number of Primes in Various Ranges with Multiple 3's in Their Digits

The percentage of primes containing multiple 3's calculated with respect to number of all such natural numbers with those many 3's in respective ranges fluctuates as follows.

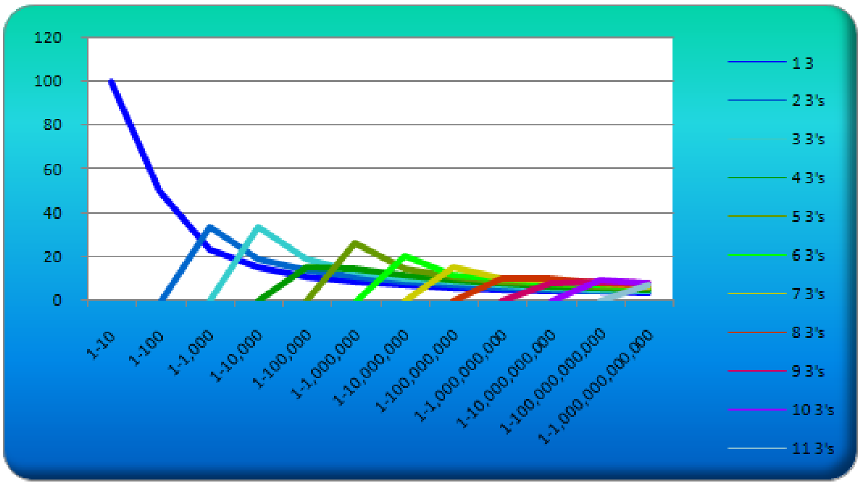


Figure2: Percentage of Primes in Various Ranges with Multiple 3's in Their Digits with Respect to All Such Integers in Respective Ranges.

The peak observed for single 3 in the range 1 – 10 is obvious; the only number 3 with one 3 in this range is itself a prime, so the percentage becomes 100. There will be no peak thereafter as in any range $1 - 10^n$ for occurrences of n 3's, as number having all n digits 3's will be greater than 3 and divisible by 3 and hence cannot be prime. We now compare differences of number of multiple occurrences of digits 1 and 2 in prime numbers with those of 3 in our ranges. Odd man out digit 0 is not considered in these comparisons as it doesn't occupy two places in any n digit prime number, viz., units place and leading n^{th} place.

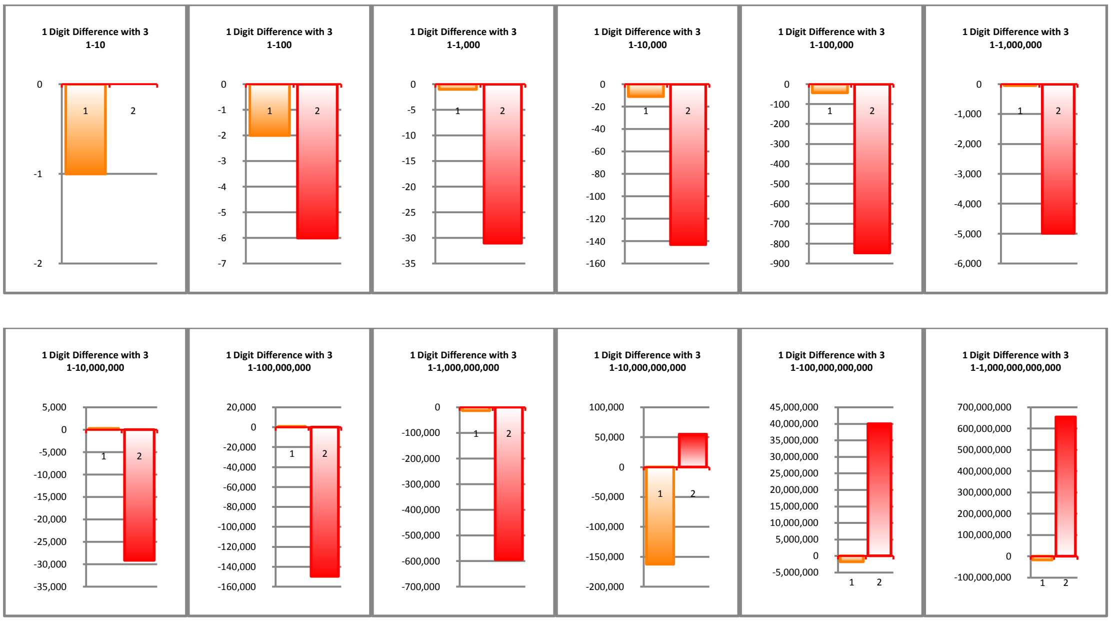


Figure3: Differences of Number of Primes having Single1 and Single2 in their Digits with those having Single3 in them in Ranges of $1 - 10^n$.

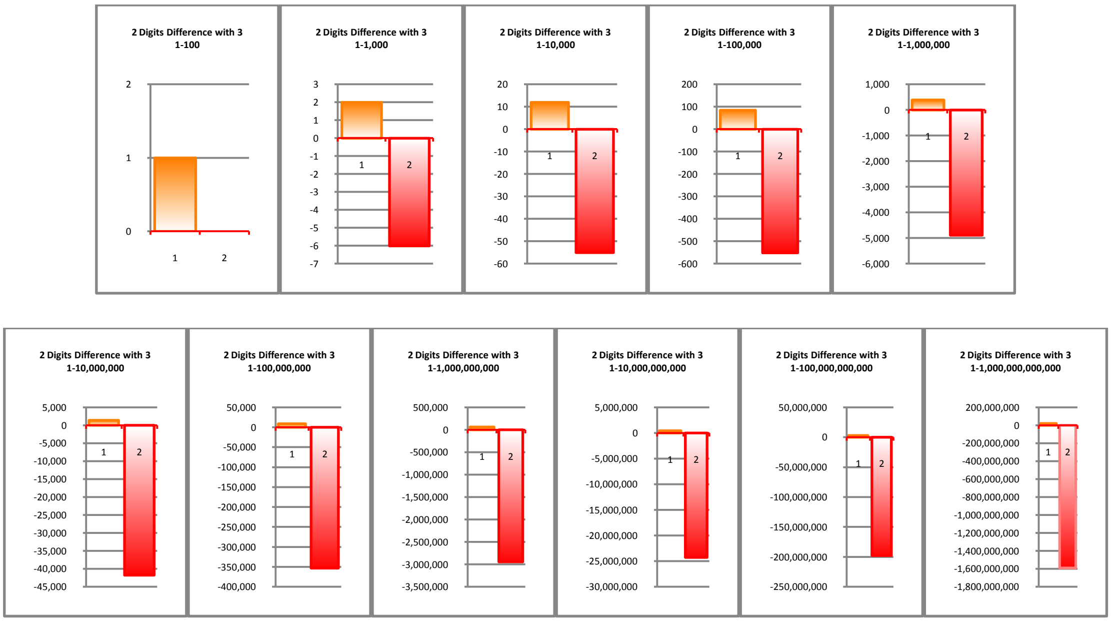


Figure4: Differences of Number of Primes having Two1's and Two2's in their Digits with those having Two 3's in them in Ranges of $1 - 10^n$.

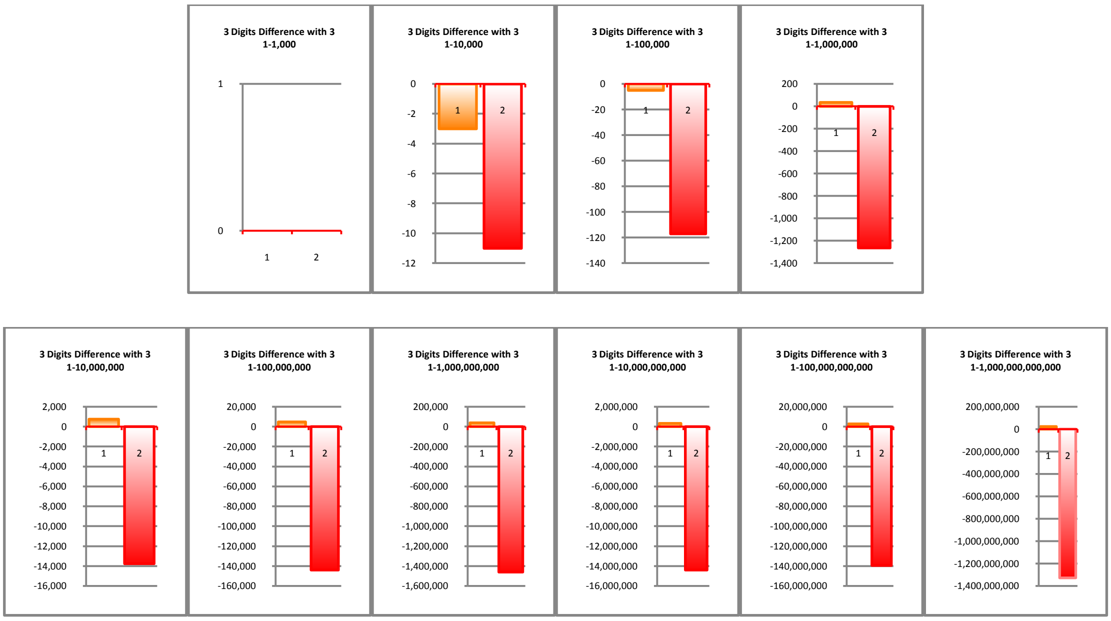


Figure5: Differences of Number of Primes having Three1's and Three2's in their Digits with those having Three3's in them in Ranges of $1 - 10^n$.

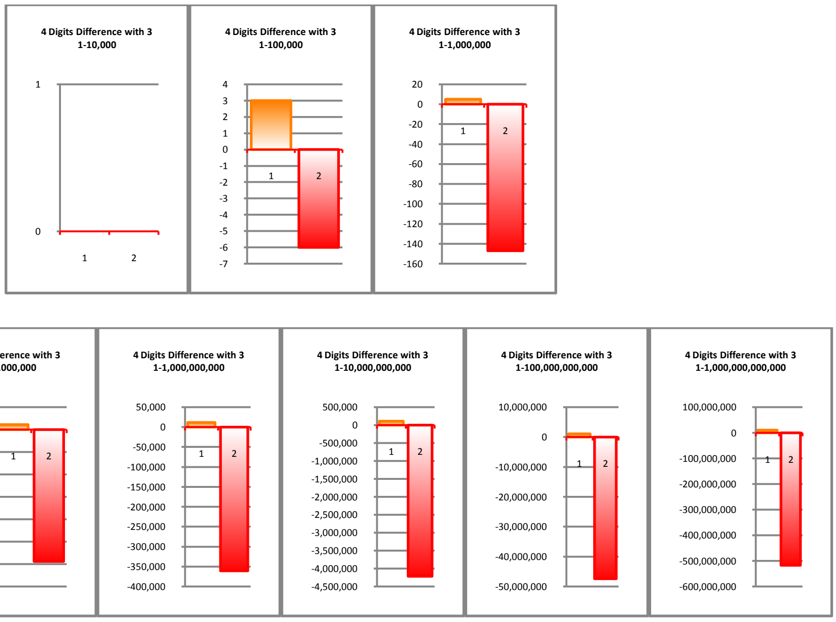
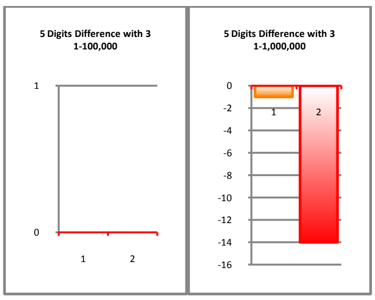


Figure6: Differences of Number of Primes having Four1's and Four2's in their Digits with those having Four 3's in them in Ranges of $1 - 10^n$.



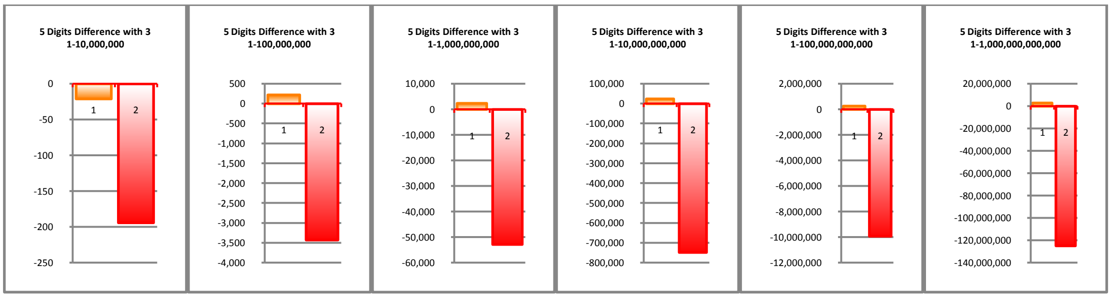


Figure7: Differences of Number of Primes having Five1's and Five2's in their Digits with those having Five 3's in them in Ranges of $1 - 10^n$.

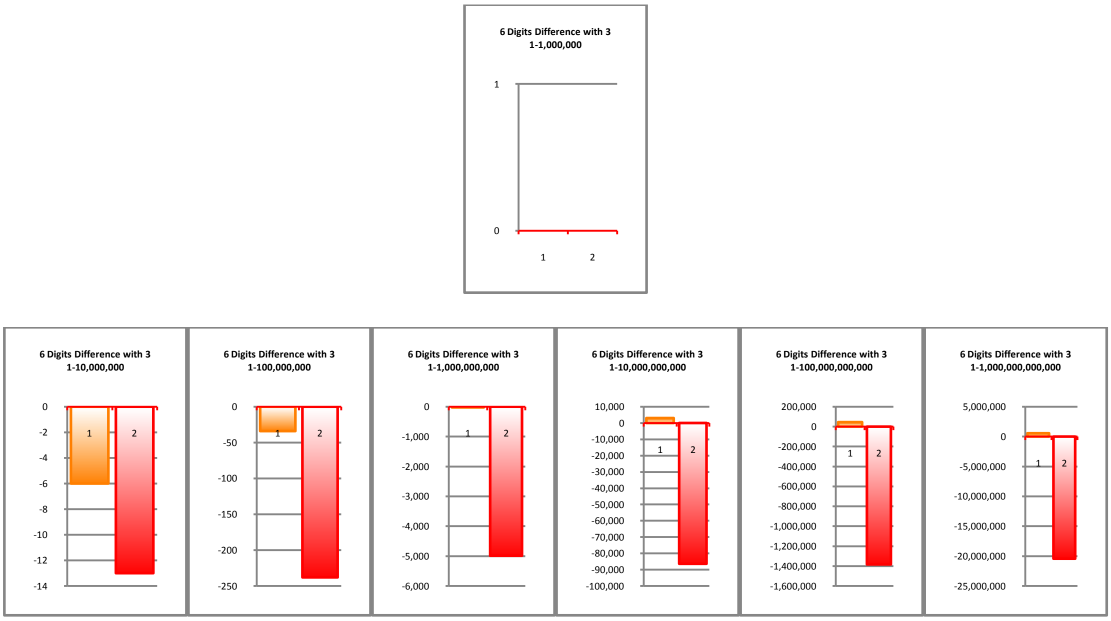


Figure8: Differences of Number of Primes having Six1's and Six2's in their Digits with those having Six 3's in them in Ranges of $1 - 10^n$.

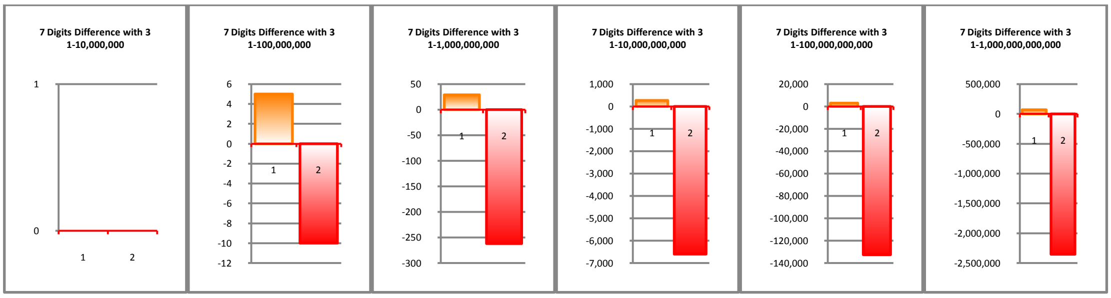


Figure9: Differences of Number of Primes having Seven1's and Seven2's in their Digits with those having Seven 3's in them in Ranges of $1 - 10^n$.

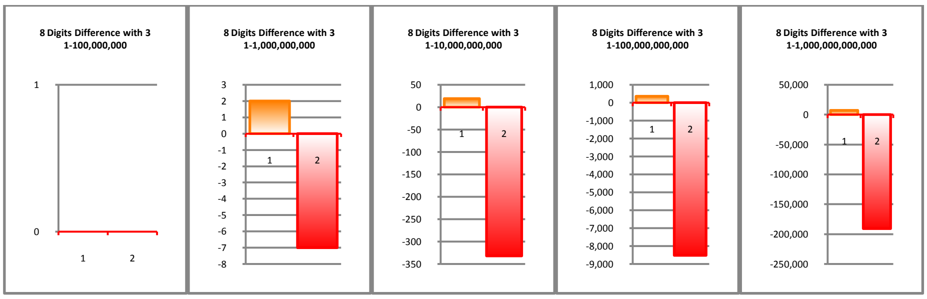


Figure10: Differences of Number of Primes having Eight1's and Eight2's in their Digits with those having Eight 3's in them in Ranges of $1 - 10^n$.

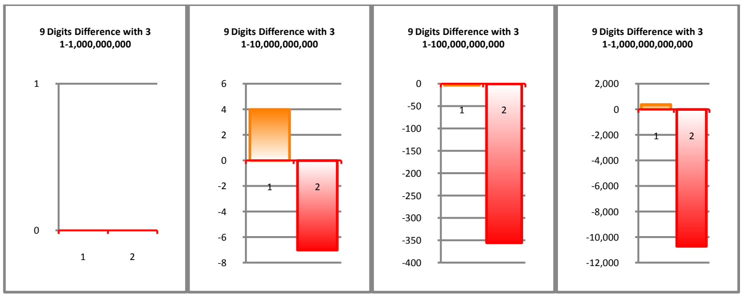


Figure11: Differences of Number of Primes having Nine1's and Nine2's in their Digits with those having Nine 3's in them in Ranges of $1 - 10^n$.

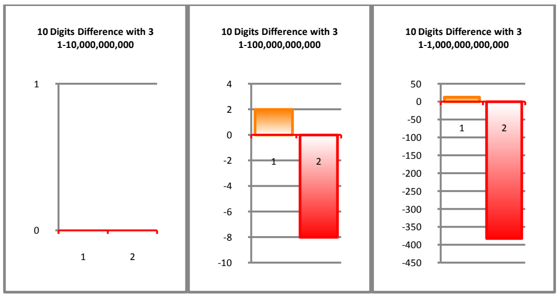


Figure12: Differences of Number of Primes having Ten1's and Ten2's in their Digits with those having Ten 3's in them in Ranges of $1 - 10^n$.

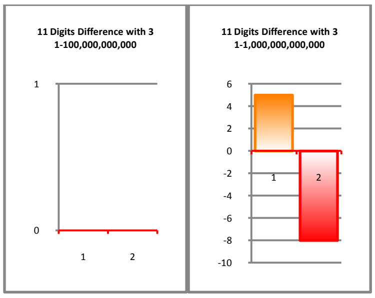


Figure13: Differences of Number of Primes having Eleven1's and Eleven2's in their Digits with those having Eleven 3's in them in Ranges of $1 - 10^n$.

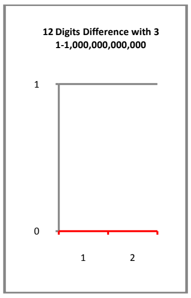


Figure14: Differences of Number of Primes having Twelve1's and Twelve2's in their Digits with those having Twelve 3's in them in Ranges of $1 - 10^n$.

IV. First Occurrence of Digit 3 in Prime Numbers

The first positive integer with single digit 3 is 3 itself! For sufficiently large ranges, first positive integer containing 2 3's 33, 3 3's is 333 and so on. Simple formulation for this is generalized in [11].

Formula 1 [11] : If n and r are natural numbers, then the first occurrence of r number of 3's in numbers in range $1 \le m < 10^n$ is

$$f = \begin{cases} - , & \text{if } r > n \\ \sum_{j=0}^{r-1} \left(3 \times 10^j\right), & \text{if } r \le n \end{cases}.$$

This was for all natural numbers. The first occurrences of r number of 3's in prime numbers in range $1 \le m < 10^n$ doesn't have a common formula yet. Owing to this they have actually been determined.

Table 3: First Prime Numbers in Various Ranges with Multiple 3's in Their Digits

Sr. No.	Range	First Prime Number in Range with						
		1 3	2 3's	3 3's	4 3's	5 3's	6 3's	7 3's
1.	$1 - 10^1$	3	-	-	-	-	-	-
2.	$1 - 10^2$	3	-	-	-	-	-	-
3.	$1 - 10^3$	3	233	-	-	-	-	-
4.	$1 - 10^4$	3	233	2,333	-	-	-	-
5.	$1 - 10^5$	3	233	2,333	23,333	-	-	-
6.	$1 - 10^6$	3	233	2,333	23,333	313,333	-	-
7.	$1 - 10^7$	3	233	2,333	23,333	313,333	3,233,333	-
8.	$1 - 10^8$	3	233	2,333	23,333	313,333	3,233,333	31,333,333
9.	$1 - 10^9$	3	233	2,333	23,333	313,333	3,233,333	31,333,333
0.	$1 - 10^{10}$	3	233	2,333	23,333	313,333	3,233,333	31,333,333
1.	$1 - 10^{11}$	3	233	2,333	23,333	313,333	3,233,333	31,333,333
2.	$1 - 10^{12}$	3	233	2,333	23,333	313,333	3,233,333	31,333,333

Table 3: Continued

Sr. No.	Range	First Prime Number in Range with			
		8 3's	9 3's	10 3's	11 3's
1.	$1 - 10^1$	-	-	-	-
2.	$1 - 10^2$	-	-	-	-
3.	$1 - 10^3$	-	-	-	-
4.	$1 - 10^4$	-	-	-	-
5.	$1 - 10^5$	-	-	-	-
6.	$1 - 10^6$	-	-	-	-
7.	$1 - 10^7$	-	-	-	-
8.	$1 - 10^8$	-	-	-	-
9.	$1 - 10^9$	333,233,333	-	-	-
0.	$1 - 10^{10}$	333,233,333	3,233,333,333	-	-
1.	$1 - 10^{11}$	333,233,333	3,233,333,333	23,333,333,333	-
2.	$1 - 10^{12}$	333,233,333	3,233,333,333	23,333,333,333	333,313,333,333

V. Last Occurrence of Digit 3 in Prime Numbers

The largest natural number with r number of 3's in its digits in ranges $1 - 10^n$, $1 \le n \le 12$, fits in a formula.

Formula 2 [11] : If n and r are natural numbers, then the last occurrence of r number of 3's in numbers in range $1 \le m < 10^n$ is

$$l = \begin{cases} - & \text{, if } r > n \\ \sum_{j=0}^{r-1}\left(3\times10^j\right) + \begin{cases} 0 & \text{, if } r = n \\ \sum_{j=r}^{n-1}\left(9\times10^j\right), & \text{if } r < n \end{cases} \end{cases}.$$

Since such primes don't fit in any formula, the last prime numbers with r number of 3's in them in ranges $1 - 10^n$, $1 \le n \le 12$, have been computationally determined.

Table 4: Last Prime Numbers in Various Ranges with Multiple 3's in Their Digits

Sr. No.	Number of 3's	Last Prime Number in Range 1 –							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
1.	1	3	83	983	9,973	99,923	999,983	9,999,973	99,999,931
2.	2	-	-	733	9,833	99,833	999,433	9,999,533	99,999,373
3.	3	-	-	-	7,333	93,383	997,333	9,998,333	99,993,833
4.	4	-	-	-	-	38,333	973,333	9,943,333	99,983,333
5.	5	-	-	-	-	-	733,333	9,533,333	99,338,333
6.	6	-	-	-	-	-	-	3,733,333	93,733,333
7.	7	-	-	-	-	-	-	-	83,333,333
8.	8	-	-	-	-	-	-	-	-
9.	9	-	-	-	-	-	-	-	-
0.	10	-	-	-	-	-	-	-	-
1.	11	-	-	-	-	-	-	-	-

Table 4: Continued …

Sr. No.	Number of 3's	Last Prime Number in Range 1 –		
		10^9	10^{10}	10^{11}
1.	1	999,999,937	9,999,999,943	99,999,999,943
2.	2	999,999,733	9,999,999,833	99,999,999,833
3.	3	999,993,833	9,999,993,833	99,999,993,833
4.	4	999,935,333	9,999,937,333	99,999,933,433
5.	5	999,343,333	9,999,533,333	99,999,335,333
6.	6	995,333,333	9,995,333,333	99,994,333,333
7.	7	983,333,333	9,943,333,333	99,934,333,333
8.	8	373,333,333	9,433,333,333	99,433,333,333
9.	9	-	3,334,333,333	97,333,333,333
0.	10	-	-	38,333,333,333
1.	11	-	-	-

Remark : The maximum number of 3's in any prime in the range $1 – 10^n$ is at most $n - 1$, except $n = 1$.
The numbers coming in all sections of this work give new integer sequences and which are important enough to merit independent analysis.

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