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# Some New Approaches of Fuzzy Graph

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***Abstract**: It is observed that fuzzy graph is defined in different way in available literature .different approaches like fuzzy line graph, linear fuzzy graph and product fuzzy graphs were introduced with their properties.*
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***Keywords:** Fuzzy graph, Linear fuzzy graph, fuzzy line graph, product Fuzzy Graphs.*

## I. Introduction

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and the relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a fuzzy graph model.Fuzzy graph is also a symmetric binary fuzzy relation on a fuzzy subset. The concept of fuzzy sets and fuzzy relations was introduced by L.A. Zadeh in 1965After Zadeh many authors have extensively developed the theory of fuzzy sets and it's application.In 1975 Rosendfeld [4] and Yeh and Beng [10] independently developed the theory of fuzzy graph. A fuzzy graph is a pair G:( σ,μ) , where σ is a fuzzy subset of V and μ is fuzzy relation on V such that, μ (uv) ≤ σ (u) Λ σ (v) for all u,v∈V.

The fuzzy line graph,L(G) of a graph G is graph of the set of lines of G.Hence the vertices of L(G)are the lines of G with two vertices of L(G).In linear fuzzy graph the field of the values of μ is extended to the interval [-1,+1] and in product fuzzy graph "minimum" in the definition of fuzzy graph is replaced by "product" and call the resulting structure product fuzzy graph.The concept of fuzzy line graph, linear fuzzy graph and product fuzzy graph s were introduced and their basic properties are developed.

## II. Preliminary

**Definition 2.1** When a vertex $\sigma\,(u_i)$ is an end vertex of some edges $\mu\,(u_i, v_j)$ of any fuzzy graph G:(σ, μ) . Then $\sigma\,(u_i)$ and $\mu\,(u_i, v_j)$ are said to be incident to each other.

**Definition 2.2** Let G:(σ, μ) is a fuzzy graph of the graph $G:(V,E)$, then the distance $d[\sigma\,(v_i), \sigma\,(v_j)]$ between two of its vertices $\sigma\,(v_i)$ and $\sigma\,(v_j)$ is the length of shortest path between them , i.e . $d[\sigma\,(v_i), \sigma\,(v_j)] = Min[\sum_{i,j\in\wedge} \mu\,(u_i, v_j)$

**Definition 2.3** Fuzzy intersection graph: Let G = (V,E) be a graph where V = {v1,v2,…….,vn}. Let Si = {vi, ei1,ei2,…..,eij,….,eiqi} where eij∈E and eij has vi as a vertex, j = 1,2…,qi; i = 1,2,….,n. Let S = (S1,S2,….Sn). Let T = {SiSj:Si,Sj∈S,Si∩Sj ≠ ϕ, i≠j}. Then þ(S) = (S,T) is an intersection graph and G = þ(S). Any fuzzy subgraph (ι,γ) of þ(S) with γ* = T is called a fuzzy intersection graph

**Proposition 2.4**[6] Let (σ,μ) and $(\sigma', \mu')$ be fuzzy sub graphs of graphs G and G′, respectively. If $f$ is a weak isomorphism of (σ,μ) onto$(\sigma', \mu')$, then $f$ is an isomorphism of (σ*,μ*) onto$(\sigma'^*, \mu'^*)$.

**Proposition 2.5** [5,6] If (λ,ω) is the fuzzy line graph of (σ,μ), then (λ*,ω*) is the fuzzy line graph of cycle (σ*,μ*).

## III. Main Results

**3.1 Fuzzy Line Graphs:** Let L(G)=(Z,W) the line subgraph of G where and $W = \{S_e S_x : S_e \cap S_x \neq \emptyset, e, x \in E, e \neq x\}$ and where $S_e = \{(e) \cup (u_e, v_e), e \in E$. Let (σ,μ) be a fuzzy subgraph of G. Define the fuzzy subset λ, ω of Z,W, respectively, as follows: $\forall S_e \in Z$, $\lambda(S_e) = \mu_e$;$\forall S_e S_x \in W$, ω(SeSx))= min{μ(e),μ(x)}.Then the pair (λ, ω) is called fuzzy line Graph.

**Lemma 3.1.1** if ( λ, ω) is the fuzzy line graph of (σ,μ), then ( λ*, ω*) is the line graph of (σ*,μ*).

**Theorem 3.2** Let ( λ, ω) be the fuzzy line graph corresponding to (σ,μ). Suppose that (σ*,μ*) is connected. Then ∃ a weak isomorphism of (σ,μ) onto ( λ, ω) if and only if (σ*,μ*) is a cycle an ∀v∈ σ*,∀e∈μ*, σ(v) = μ(e), i.e. σ and μ are constant function on σ* andμ*, respectively, taking on the same value. And if $f$ is weak isomorphism of (σ,μ) onto (λ,ω) then $f$ is an isomorphism.

**Proof.** Suppose that $f$ is a weak isomorphism of (σ,μ) onto (λ,ω). By Proposition 1.1 $f$ is an isomorphism of (σ*,μ*) onto ( λ*, ω*). By proposition 1.2 (σ*,μ*) is cycle. Let

$\sigma^* = \{v_1,v_2,\ldots..v_n\}$ and $\mu^* = \{v_1v_2.v_2v_3\ldots..,v_nv_i\}$ where $v_1v_2,\ldots\ldots v_nv_i$ is a cycle.
Let $\sigma(v_i) = S_i$ and $\mu(v_i,v_{i+1}) = r_i$, i=1,2,….n where $v_{n+1} = v_1$.
Then for $S_{n+1} = s_1$

$$r_i \le \min\{S_i,S_{i+1}\},\ i = 1,2,\ldots..,n \quad \ldots\ldots(1)$$

Now $\lambda^* = \{S_{v,vi+1}:\ i = 1,2,\ldots.,n\}$
$\omega^* = \{S_{v,vi+1}S_{vi+1vi+2}: i=1,2,\ldots..,n-1.$
Also for $r_{n+1} = r_1$,
$\lambda(Svi,vi+1) = \mu(vivi+1) = ri$ and
$\omega(S_{v,vi+1},S_{vi+1,vi+2}) = \min\{\mu(v_iv_{i+1}),\mu(v_{i+1}v_{i+2})\}$
$=\min\{r_i,r_{i+1}\}$.
i = 1,2,….n, where $v_{n+2} = v_2$. Since $f$ is an isomorphism of $(\sigma^*,\mu^*)$ onto $(\lambda^*, \omega^*)$,$f$ maps $\sigma^*$ one to one onto $\lambda^* = \{S_{v1v2},\ldots.S_{vnvi}\}$.Also $f$ preserves adjancency . Hence $f$ induces a permutation $\pi$ of {1,2,….,n} such that $f(v_i)=S_{v\pi(i)v\pi(i)+1}$
and $V_iv_{i+1}\rightarrow f(v_i)\, f(v_{i+1}) = S_{v\pi(i)v\pi(i)+1}S_{v\pi(i)+1v\pi(i+1)+1}$, i= 1,2,……n-1.Now

$$S_i = \sigma(v_i) \le \lambda(f(v_i)) = \lambda(Sv\pi iv\pi i+1) = r_{\pi(i)}$$

and

$$r_i = \mu(v_iv_{i+1}) \le \omega(\ f(v_i)\ f(v_{i+1}))$$
$$= \omega(\ S_{v\pi(i)v\pi(i)+1}S_{v\pi(i)+1v\pi(i+1)+1})$$
$$= \min\{\mu(v_{\pi(i)}v_{\pi(i)+1}),\mu(v_{\pi(i)+1}v_{\pi(i+1)+1})\} = \min\{r_{\pi(i)},r_{\pi(i)+1}\},\quad i= 1,2,\ldots..,n.$$

that is $S_i \le r_{\pi(i)}$ and $r_i \le \min\{r_{\pi i},r_{\pi(i)+1}\}$, i = 1,2,…,n ……………………….(2)
Now we have $r_i \le r_{\pi(i)}$, i= 1,2,…,n, and so $r_{\pi I} \le r_{\pi(\pi i)}$, i= 1,2,…,n. Continuing we have that $r_i \le r_{\pi(i)} \le\ldots\ldots\le r_{\pi j(i)} \le r_i$ and so $r_i=r_{\pi i}$, i= 1,2,….n, where $\pi^{j+1}$ is the identity map. By (2) again we have $r_i \le r_{\pi i+1} = r_{i+1}$, i= 1,2,…,n where $rn+1 = r_1$.Hence By (1)and (2) $r_1=,\ldots\ldots=r_n=S_1=\ldots..=S_n$.thus we have proved that σ and μ are constant function and $f$ is an isomorphism. Conversely, Suppose that $(\sigma^*,\mu^*)$ is a cycle and $\forall\ v\in \sigma^*,\forall\ e\in\mu^*$, $\sigma(v) = \mu(e)$.
By proposition 1.2 $(\lambda^*, \omega^*)$ is the line graph of $(\sigma^*,\mu^*)$.Since $(\sigma^*,\mu^*)$is a cycle, $(\sigma^*,\mu^*)\cong(\lambda^*, \omega^*)$by This isomorphism induces an isomorphism of $(\sigma,\mu)$ onto $(\lambda,\omega)$ Since $\sigma(v)=\mu(e)\ \forall v\in V,\ \forall e\in E$ and so $\sigma = \mu = \lambda$ on their respective domains.

**3.3 Linear Fuzzy Graph:** A fuzzy graph is a pair (σ,μ) where σ stands for a finite set and μ is a mapping from V×V to the interval [0,1]. We consider here fuzzy antisymmetrical graph, i.e. such that $\forall\ a,b\in V:\mu(a,b) >0 \Rightarrow\mu(b,a) = 0$.In order to define the concept of linear fuzzy graph the field of the value of μ is extended to the interval [-1,+1], setting $\mu(b,a) = -\mu(a,b)\ \forall a,b$ such that $\mu(a,b)>0$
Thus we obtain a fuzzy graph (σ,μ) where μ is a mapping from V×V to the interval[-1,+1] such that $\forall\ a,b\in V$: $\mu(a,b)+ \mu(b.a) = 0$.
This fuzzy graph is called linear if there are no four elements a,b,c,d of V such that:
$\mu(a,c)<\mu(b,c)$ and $\mu(b,d)<\mu(a,d)$
**Remark 3.3.1** Every linear fuzzy graph is transitive.

**3.4 Matrix representation of linear fuzzy graph**
Let r be an enumeration of the elements of V.A fuzzy graph (σ,μ) can be represented by a matrix $M^r$ such that $M^r_{ij}= \mu(r^{-1}(i),r^{-1}(j))$.i.e. the space(i,j)of the matrix $M^r$ contains the value of the arc which joins the numbered vertex i to the numbered vertex.
**Theorem 3.5** : if (σ,μ) is a linear fuzzy graph, then matrix Mr has the following property:
i) $i < i' \Rightarrow M^r_{ij} \ge M^r_{i'j}\quad \forall j$ and
ii) $j > j' \Rightarrow M^r_{ij} \ge M^r_{ij'}\quad \forall\ i$
**Proof:** Let r be an enumeration of V i.e. a bijection from V to {1,2,…..|V|}where |V| is the cardinality of V such that $r(a) < r(b) \Leftrightarrow aTb$. and let R be the total preorder associated with (σ,μ) such that $aTb \Rightarrow aRb$ Where T is total order consistent with R.
Since $r(a) < r(b) \Leftrightarrow aTb$
If $i<i'$, then

$$r^{-1}(i)Tr^{-1}(i')$$

Thus $r^{-1}(i)Sr^{-1}(i')$, i.e. $\mu(r^{-1}(i),c) \ge \mu(r^{-1}(i'),c), \forall c$,
i.e. $M^r_{ij} \ge M^r_{i'j}\ ,\forall j$
similarly if $j > j'$, then $r^{-1}(j')Tr^{-1}(j)$,
hence $r^{-1}(j')Sr^{-1}(j)$, i.e.$\mu\left(c,r^{-1}(j)\right) \ge \mu\left(c,r^{-1}(j')\right), \forall\ c$
or $M^r_{ij} \ge M^r_{ij'}\ ,\forall i$.

**3.6 Product Fuzzy Graph:** Let G be a graph whose vertex set is V,$\sigma$ be a fuzzy sub set of V and $\mu$ be a fuzzy sub set of V×V then the pair $(\sigma,\mu)$ is called product fuzzy graph if

$\mu(u,v) \le \sigma(u) \times \sigma(v) \quad ,\forall\, u,v \in V$

**Remark 3.6.1** If $(\sigma,\mu)$ is a product fuzzy graph and since $\sigma(u)$ and $\sigma(v)$ are less than or equal to 1, it follows that

$$\boldsymbol{\mu(u,v) \le \sigma(u) \times \sigma(v) \le \sigma(u) \wedge \sigma(v) \quad ,\forall\, u,v \in V}$$

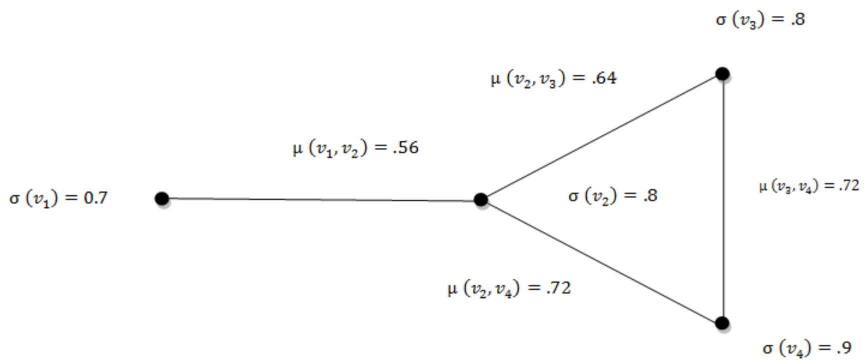
Hence $(\sigma,\mu)$ is a fuzzy graph thus every product fuzzy graph is a fuzzy graph but converse is need not true we illustrate these by example and counter example.

**Example** Let V = { u,v,w}, $\sigma$ be the fuzzy subset of V defined as $\sigma(u) = \frac{1}{4}, \sigma(v) = \frac{1}{2} and\ \sigma(w) = \frac{3}{4}$. Let $\mu$ be the fuzzy subset of VxV defined as $\mu(u,v) = \frac{1}{10}, \mu(v,w) = \frac{2}{8} and\ \mu(u,w) = \frac{2}{16}$. It is easy to see that G: $(\sigma,\mu)$is a product fuzzy graph and hence a fuzzy graph.

**Example** Let V = { u,v,w}, $\sigma$ be the fuzzy subset of V defined as $\sigma(u) = \frac{1}{4}, \sigma(v) = \frac{1}{2} and\ \sigma(w) = \frac{3}{4}$. Let $\mu$ be the fuzzy subset of VxV defined as $\mu(u,v) = 0.2, \mu(v,w) = 0.4 and\ \mu(u,w) = 0.2$. It is easy to see that G: $(\sigma,\mu)$is fuzzy graph and hence a fuzzy graph but it is not a product fuzzy graph. Since $\sigma(u) \times \sigma(v) = \frac{1}{4} \times \frac{1}{2} =$ $1/8 < \ \mu(u,v)$

**Definition 3.7** Let G:(σ, μ) is a product fuzzy graph of the graph $G:(V,E)$, then the distance $d[\sigma(v_i), \sigma(v_j)]$ between two of its vertices $\sigma(v_i)$ and $\sigma(v_j)$ is the length of shortest path between them , i.e . $d[\sigma(v_i), \sigma(v_j)] = Min[\sum_{i,j\in\wedge} \mu(u_i, v_j)$

**Example**




In this product fuzzy graph , there is two path between $\sigma(v_1)$ and $\sigma(v_4)$

i) $\{\mu(v_1, v_2), \mu(v_2, v_4)$

ii) $\{\mu(v_1, v_2), \mu(v_2, v_3), \mu(v_3, v_4)\}$

But i) is the shortest path between $\sigma(v_1)$ and $\sigma(v_4)$.

Hence the distance between the vertices $\sigma(v_1)$ and $\sigma(v_4)$ is given as

$$d[\sigma(v_1), \sigma(v_4)] = \{\mu(v_1, v_2) + \mu(v_2, v_4) = 0.56 + 0.72$$

i.e $d[\sigma(v_1), \sigma(v_4)] = 1.28$

**Definition 3.8** Let G:$(\sigma,\mu)$ be a product fuzzy graph with G:(V,E),a function $d: I^V \times I^V \to [0,1]$, is said to be a metric in product fuzzy graph if it satisfies the following conditions

i) $d(\sigma(u), \sigma(v)) > 0 \quad$ if $\sigma(u) \ne \sigma(v)$

ii) $d(\sigma(u), \sigma(v)) = 0 \quad$ if $\sigma(u) = \sigma(v)$

iii) $d(\sigma(u), \sigma(v)) = d(\sigma(v), \sigma(u)) = d(\bar{\sigma}(u), \bar{\sigma}(v))$

i.e. $d(\sigma(u), \sigma(v)) = d(\sigma(v), \sigma(u))$

iv) $d(\sigma(u), \sigma(z)) \le d(\sigma(u), \sigma(v)) + d(\sigma(v), \sigma(z))$ where $\sigma(v)$ lie between $\sigma(u)$ and $\sigma(z)$.

**Theorem 3.9** Let $G: (\sigma,\mu)$ be a connected product fuzzy graph with G:(V,E), then the distance between vertices of these fuzzy graph is metric.

**Proof:-** Given that $G: (\sigma,\mu)$ be a connected fuzzy graph , and we have to show that the distance between vertices of these fuzzy graph is metric.

For this, let $d: I^V \times I^V \to [0,1]$ then the condition (i) and (ii) of definition of metric on fuzzy graph is clear from definition of distance on fuzzy graph.

Since distance $d[\sigma(v_i), \sigma(v_j)]$ is the length of the shortest path between $\sigma(v_i)$ and $\sigma(v_j)$ this path cannot be longer than a path between $\sigma(v_i)$ and $\sigma(v_j)$ which goes through another vertex $\sigma(v_k)$.Hence

$d[\sigma(v_i), \sigma(v_j)] \leq d[\sigma(v_i), \sigma(v_k)] + d[\sigma(v_k), \sigma(v_j)]$.

Thus $d[\sigma(v_i), \sigma(v_j)]$is metric in product fuzzy graph.

This completes the proof of the theorem.

## IV. Conclusion

Different approaches like fuzzy line graph, linear fuzzy graph and product fuzzy graphs were introduced with their properties. Distance in product fuzzy graph and Metric in Product fuzzy graph were defined and given a proof of an independent theorem which is fuzzy version of classical graph.

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