

## Common Fixed Point Theorems for Two Pairs of Weakly Compatible Mappings in Fuzzy Metric Spaces Using $(CLR_{ST})$ Property

Pranjali Sharma<sup>1</sup>, Shailesh Dhar Diwan<sup>2</sup>

<sup>1</sup>Department of Mathematics, Shri Shankaracharya Institute of Prof. Mgmt. and Tech. Raipur (C.G.), India,

<sup>2</sup>Department of Mathematics, Sant Guru Ghasidas Govt. P. G. College, Kurud, Dhamtari (C.G.), India,

**Abstract:** In this paper, we establish some common fixed point theorems for two pairs of weakly compatible mappings in fuzzy metric spaces using the  $(CLR_{ST})$  property. Our results extend, generalize and improve several results of metric spaces to fuzzy metric spaces.

**Keywords:** Fuzzy metric space, t-norm, weakly compatible mappings,  $(CLR_{ST})$  property, (E. A) property, fixed point.

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### I. Introduction

The idea of fuzzy set was first introduced in 1965 by Iranian mathematician L.A. Zadeh [21]. Fuzzy set is defined by a membership function which assigns each object to a grade of membership between zero and one. Following the concept of fuzzy set, Kramosil and Michalek [11] established the concept of fuzzy metric space. George and Veeramani [5] modified the concept of fuzzy metric space by imposing some stronger conditions using continuous t-norm and defined the hausdorff topology of fuzzy metric spaces. Gregori and Sapena [7] defined the concepts of convergent sequence, cauchy sequence, completeness and compactness in sense of fuzzy metric space. Grabiec [6] introduced the fuzzy version of Banach contraction principle.

Generalizing the concept of commuting mapping Sessa [18] introduced the concept of weakly commuting mappings. In metric spaces, Pant [14] initiated the study of concept of  $R$ -weakly commuting mappings. Mishra [13] proved some common fixed point theorems for compatible mappings in fuzzy metric spaces. Generalization of compatible mappings is given by Jungck [10] and Pathak et al. [15]. Fang [3] proved some fixed point theorems in fuzzy metric spaces, which improve, generalize some results of Metric spaces. Vasuki [20] fuzzify weakly commuting maps.

Aamri and El Moutawakil [1] defined the (E.A) property for self mappings which contains the class of noncompatible as well as compatible mappings. It is observed that (E.A) property and common property (E.A) require the closedness of the subspaces for the existence of fixed point. Recently, Sintunavarat and Kuman [19] defined the notion of  $CLR_g$  property. It is important to note that  $CLR_g$  property never requires completeness of subspace.

In this paper, we prove some common fixed point theorems for two pairs of weakly compatible mappings in fuzzy metric space using  $(CLR_{ST})$  property. Our results generalize various known results of metric spaces to fuzzy metric spaces.

### II. Preliminaries

**Definition 2.1** [17] : A binary operation  $*$  :  $[0,1] \times [0,1] \rightarrow [0,1]$  is called continuous t-norm if it satisfies the following conditions:

- 1)  $*$  is commutative and associative;
- 2)  $*$  is continuous;
- 3)  $a * 1 = a$ , for all  $a \in [0, 1]$  ;
- 4)  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$ , for all  $a, b, c, d \in [0, 1]$

**Remark 2.2** [17]: The concept of t-norm can be considered as fuzzy union.

**Definition 2.3**[5] : The 3-tuple  $(X, M, *)$  is said to be a fuzzy metric space (FMS) if,  $X$  is a non empty set,  $*$  is a continuous t-norm,  $M$  is a fuzzy set on  $X \times X \times (0, \infty)$  satisfying the following conditions:

- 1)  $M(x, y, t) > 0$ ;
  - 2)  $M(x, y, t) = 1$  if and only if  $x=y$ ;
  - 3)  $M(x, y, t) = M(y, x, t)$ ;
  - 4)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ ;
- $M(x, y, \cdot) : (0, \infty) \rightarrow [0, 1]$  is continuous, for all  $x, y, z \in X$  and  $s, t > 0$ .

$M(x, y, t)$  is considered as the degree of nearness of  $x$  and  $y$  with respect to  $t$ .

**Example 2.4 [5]:**

Let  $(X, d)$  be a metric space,  $t$ -norm  $a * b = \min \{a, b\} \quad \forall x, y \in X$  and  $t > 0$ .

$$M_d(x, y, t) = \frac{t}{t + d(x, y)},$$

then  $(X, M, *)$  is a Fuzzy metric space.

**Definition 2.5** ([20], [14]): Two self mappings  $f$  and  $g$  of a fuzzy metric space  $(X, M, *)$  are said to be weakly commuting if,

$$M(fgx, gfx, t) \geq M(fx, gx, t), \text{ for all } x \in X, t > 0.$$

**Definition 2.6** ([9], [13]): Two self mappings  $f$  and  $g$  of a fuzzy metric space  $(X, M, *)$  are said to be compatible if,

$$\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t) = 1,$$

whenever  $\{x_n\}$  is a sequence in  $X$  such that,

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = x,$$

for some  $x \in X, t > 0$ .

**Definition 2.7** [10]: Two self mappings  $f$  and  $g$  of a fuzzy metric space  $(X, M, *)$  are said to be weakly compatible if they commute at their coincidence point that is, if  $fx = gx$  for some  $x \in X$ , then

$$M(fgx, gfx, t) = 1$$

It is obvious that if two mappings are compatible then they are weakly compatible, but converse is not true.

**Definition 2.8** ([1], [12]): Two self mappings  $f$  and  $g$  of a fuzzy metric space  $(X, M, *)$  are said to satisfy the (E.A) property if there exist a sequence  $\{x_n\}$  in  $X$  such that for all  $t > 0$ ,

$$\lim_{n \rightarrow \infty} M(fx_n, gx_n, t) = 1.$$

**Definition 2.9** [19]: Two self mappings  $f$  and  $g$  of a fuzzy metric space  $(X, M, *)$  are said to satisfy the common limit in the range of  $g$  (CLR<sub>g</sub>) property if there exist a sequence  $\{x_n\}$  in  $X$  such that,

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = gu, \text{ for some } u \in X.$$

**Definition 2.10** [2]: Two pairs  $(A, S)$  and  $(B, T)$  of self mappings of a fuzzy metric space  $(X, M, *)$  are said to satisfy the (E.A) property if there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that for all  $t > 0$ ,

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = z, \text{ for some } z \text{ in } X.$$

**Definition 2.12** [8]: Two pairs  $(A, S)$  and  $(B, T)$  of self mappings of a fuzzy metric space  $(X, M, *)$  are said to satisfy the (CLR<sub>ST</sub>) property if there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that for all  $t > 0$ ,

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = z,$$

where  $z \in S(X) \cap T(X)$ .

### III. Main Result

**Theorem 3.1** Let  $A, B, S$  and  $T$  be self mappings a fuzzy metric space  $(X, M, *)$  such that,

I. The pairs  $(A, S)$  and  $(B, T)$  shares the (CLR<sub>ST</sub>) property,

II. Both the pairs  $(A, S)$  and  $(B, T)$  are weakly compatible.

III. There exist a constant  $k \in (0, 1)$  such that,

$$(M(Ax, By, kt))^2 \geq \min\{M(Sx, Ty, t)\}^2,$$

$$M(Sx, Ax, t)M(Ty, By, t), M(Sx, By, 2t)M(Ty, Ax, t), M(Ty, Ax, t), M(Sx, By, 2t)M(Ty, By, t)\}$$

for all  $x, y \in X$  and  $t > 0$ , then  $A, B, S$  and  $T$  have a unique common fixed point in  $X$ .

**Proof:** Since the pairs  $(A, S)$  and  $(B, T)$  share the (CLR<sub>ST</sub>) property, then there exists two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that,

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ty_n = \lim_{n \rightarrow \infty} By_n = z,$$

where,  $z \in S(X) \cap T(X)$ . Since  $z \in S(X)$ , there exists a point  $u \in X$  such that,  $Su = z$ . We assert that  $Au = Su$ . On using inequality III with  $x = u, y = y_n$ , we get,

$$(M(Au, By_n, kt))^2 \geq \min \{ (M(Su, Ty_n, t))^2, M(Su, Au, t) M(Ty_n, By_n, t), M(Su, By_n, 2t) M(Ty_n, Au, t), M(Ty_n, Au, t), M(Su, By_n, 2t) M(Ty_n, By_n, t) \}$$

Taking as limit  $n \rightarrow \infty$

$$\begin{aligned} (M(Au, z, kt))^2 &\geq \min \{ (M(z, z, t))^2, M(z, Au, t) M(z, z, t), M(z, z, 2t) M(Au, z, t), M(Au, z, t), M(z, z, 2t) \\ &M(z, z, t) \} \\ &= \min \{ 1, M(z, Au, t), 1, 1, M(Au, z, t), M(Au, z, t), 1, 1 \} \\ &= M(z, Au, t) \end{aligned}$$

It is possible only when  $Au = z$ .

Therefore  $Au = Su = z$ .

Also  $z \in T(X)$ , there exists a point  $v \in X$  such that  $Tv = z$ . We show that  $Bv = Tv$ .

On using inequality (III) with  $x = u, y = v$ , we get,

$$\begin{aligned} (M(z, Bv, kt))^2 &\geq \min \{ (M(z, z, t))^2, M(z, z, t) M(z, Bv, t), M(z, Bv, 2t) M(z, z, t), M(z, z, t), M(z, Bv, 2t) \\ &M(z, Bv, t) \} \\ (M(z, Bv, kt))^2 &\geq \min \{ 1, 1, M(z, Bv, t), M(z, Bv, 2t), 1, 1, M(z, Bv, 2t) M(z, Bv, t) \} \end{aligned}$$

$$(M(z, Bv, kt))^2 \geq (M(z, Bv, t))^2$$

It is possible when  $Bv = z$ , therefore  $Bv = Tv = z$ .

Since the pair  $(A, S)$  is weakly compatible, therefore  $Az = ASu = SAu = Sz$ . Putting  $x = z$  and  $y = v$  in inequality (III), we have

$$(M(Az, Bv, kt))^2 \geq \min \{ (M(Sz, Tv, t))^2, M(Sz, Az, t) M(Tv, Bv, t), M(Sz, Bv, 2t) M(Tv, Az, t), M(Tv, Az, t), M(Sz, Bv, 2t) M(Tv, Bv, t) \}$$

$$(M(Az, z, kt))^2 \geq \min \{ (M(Az, z, t))^2, M(Az, Az, t) M(z, z, t), M(Az, z, 2t) M(z, Az, t), M(z, Az, t), M(Az, z, 2t) M(z, z, t) \}$$

$$(M(Az, z, kt))^2 \geq \min \{ (M(Az, z, t))^2, 1, 1, M(Az, z, 2t) M(z, Az, t), M(z, Az, t), M(Az, z, 2t), 1 \}$$

$$(M(Az, z, kt))^2 \geq (\min M(Az, z, t))^2$$

It is possible when  $Az = z$ , therefore  $Az = Sz = z$ .

Which shows that  $z$  is a common fixed point of the pair  $(A, S)$ .

Also the pair  $(B, T)$  is weakly compatible, therefore  $Bz = BTv = TBv = Tz$ . On using inequality (III) with  $x = u, y = z$ , we have

$$(M(Au, Bz, kt))^2 \geq \min \{ (M(Su, Tz, t))^2, M(Su, Au, t) M(Tz, Bz, t), M(Su, Bz, 2t) M(Tz, Au, t), M(Tz, Au, t), M(Su, Bz, 2t) M(Tz, Bz, t) \}$$

$$(M(z, Bz, kt))^2 \geq \min \{ (M(z, Bz, t))^2, M(z, z, t) M(Bz, Bz, t), M(z, Bz, 2t) M(Bz, z, t), M(Bz, z, t), M(z, Bz, 2t) M(Bz, Bz, t) \}$$

$$(M(z, Bz, kt))^2 \geq \min \{ (M(z, Bz, t))^2, 1, 1, M(z, Bz, 2t) M(Bz, z, t), M(Bz, z, t), M(z, Bz, 2t), 1 \}$$

$$(M(z, Bz, kt))^2 \geq (M(z, Bz, t))^2$$

It is possible when  $Bz = z$ , therefore  $Bz = Tz = z$ , which shows that  $z$  is a common fixed point of the pair  $(B, T)$ .

Therefore  $z$  is a common fixed point of both the pairs  $(A, S)$  and  $(B, T)$ .

Now we prove the uniqueness of common fixed, let  $z_1$  and  $z_2$  be two common fixed points of  $f$  and  $g$  then from condition III,

$$(M(z_1, z_2, kt))^2 = (M(Az_1, Bz_2, kt))^2 \geq \min \{ (M(Sz_1, Tz_2, t))^2, M(Sz_1, Az_1, t) M(Tz_2, Bz_2, t), M(Sz_1, Bz_2, 2t) M(Tz_2, Az_1, t), M(Tz_2, Az_1, t), M(Sz_1, Bz_2, 2t) M(Tz_2, Bz_2, t) \}$$

$$\begin{aligned} &= \min \{ (M(z_1, z_2, t))^2, M(z_1, z_1, t) M(z_2, z_2, t), \\ &M(z_1, z_2, 2t) M(z_2, z_1, t), M(z_2, z_1, t), M(z_1, z_2, 2t) M(z_2, z_2, t) \} \end{aligned}$$

$$= (M(z_1, z_2, t))^2$$

Therefore,

$$(M(z_1, z_2, kt))^2 \geq (M(z_1, z_2, t))^2$$

It is possible only when  $z_1 = z_2$ .

Therefore  $z$  is unique common fixed point of both the pairs  $(A, S)$  and  $(B, T)$ .

**Example 3.5:** Let  $(X, M, *)$  be a fuzzy metric space, with  $a * b = \min\{a, b\}$  where  $X = [1, 15)$ , and  $M(x, y, t) = \frac{t}{t + |x - y|}$ , for all  $x, y \in X$  and  $t > 0$ . Define self maps  $A, B, S, T$  as,

$$A(x) = \begin{cases} 1, & x \in \{1\} \cup (2, 15) \\ 14, & x \in (1, 2] \end{cases}$$

$$B(x) = \begin{cases} 1, & x \in \{1\} \cup (2, 15) \\ 4, & x \in (1, 2] \end{cases}$$

$$S(x) = \begin{cases} \frac{x+2}{4}, & x \in (2, 15) \\ 1, & x = 1 \\ 14, & x \in (1, 2] \end{cases}$$

$$T(x) = \begin{cases} x-1, & x \in (2, 15) \\ 1, & x = 1 \\ 14, & x \in (1, 2] \end{cases}$$

Let  $\{x_n\} = \{1 + \frac{2}{n}\}$  and  $\{y_n\} = \{1 + \frac{1}{n}\}$

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = 1$$

And  $1 \in S(X) \cap T(X)$

all the conditions of Theorem 3.1 are satisfied for some fixed  $k \in (0, 1)$  and  $1$  is the unique common fixed point of the pairs  $(A, S)$  and  $(B, T)$ .

**Corollary 3.3:** Let  $A, B, S$  and  $T$  be self mappings of a fuzzy metric space  $(X, M, *)$  such that

- I. The pair  $(A, S)$  satisfies  $CLR_s$  property.
- II.  $A(X) \subset T(X)$
- III.  $T(X)$  is a closed subset of  $S(X)$
- IV.  $B(y_n)$  converges for every sequence  $\{y_n\}$  in  $X$ , whenever  $T(y_n)$  converges.
- V. Both the pairs  $(A, S)$  and  $(B, T)$  are weakly compatible.
- VI. There exist a constant  $k \in (0, 1)$  such that,

$$(M(Ax, By, kt))^2 \geq \min \{ (M(Sx, Ty, t))^2, M(Sx, Ax, t)M(Ty, By, t), M(Sx, By, 2t)M(Ty, Ax, t), M(Ty, Ax, t), M(Sx, By, 2t)M(Ty, By, t) \}$$

Then  $A, B, S$  and  $T$  have a unique common fixed point.

**Proof:** Let the pair  $(A, S)$  satisfy the  $CLR_s$  property, then there exist a sequence  $x_n$  in  $X$  such that,

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z, \text{ where } z \in S(X)$$

As  $A(X) \subset T(X)$  therefore  $Ax_n = Ty_n$

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ty_n = z, \text{ where } z \in S(X) \cap T(X).$$

Now we show that  $\lim_{n \rightarrow \infty} By_n = z$

In inequality (VI) put  $x=x_n$  and  $y=y_n$  we get,

$$(M(Ax_n, By_n, kt))^2 \geq \min \{ (M(Sx_n, Ty_n, t))^2, M(Sx_n, Ay_n, t)M(Ty_n, By_n, t), M(Sx_n, By_n, 2t)M(Ty_n, Ax_n, t), M(Ty_n, Ax_n, t), M(Sx_n, By_n, 2t)M(Ty_n, By_n, t) \}$$

Taking limit as  $n \rightarrow \infty$

$$(M(z, l, kt))^2 \geq \min \{ (M(z, z, t))^2, M(z, z, t)M(z, l, t), M(z, l, 2t)M(z, z, t), M(z, z, t), M(z, l, 2t)M(z, l, t) \}$$

$$= (M(z, l, t))^2$$

It is possible only when  $z=l$ .

Therefore,  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ty_n = \lim_{n \rightarrow \infty} By_n = z$ ,  
 where  $z \in S(X) \cap T(X)$ , hence the pairs (A, S) and (B, T) share the  $(CLR_{ST})$  property.  
 Therefore by theorem 3.1 A, B, S and T have a unique common fixed point.

**Theorem 3.4:** Let A and S be self mappings a fuzzy metric space  $(X, M, *)$  such that,

- I. The pairs (A, S) satisfies the  $(CLR_S)$  property,
- II. The pair (A, S) is weakly compatible.
- III. There exist a constant  $k \in (0, 1)$  such that,

$$(M(Ax, Ay, kt))^2 \geq \min\{(M(Sx, Sy, t))^2, M(Sx, Ax, t)M(Sy, Ay, t), M(Sx, Ay, 2t)M(Sy, Ax, t), \\ M(Sy, Ax, t), M(Sx, Ay, 2t)M(Sy, Ay, t)\}$$

for all  $x, y \in X$  and  $t > 0$ . Then A and S have a unique common fixed point in X.

**Proof:** Since the pairs (A, S) satisfies the  $(CLR_{ST})$  property, then there exists a sequence  $\{x_n\}$  in X such that,  
 $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$ ,

where  $z \in S(X)$ . Therefore, there exists a point  $u \in X$  such that  $Su = z$ . We assert that  $Au = Su$ . On using inequality ( III ) with  $x = u, y = x_n$ , we get,

$$(M(Au, Ax_n, kt))^2 \geq \min\{(M(Su, Sx_n, t))^2, M(Su, Au, t)M(Sx_n, Ax_n, t), M(Su, Ax_n, 2t)M(Sx_n, Au, t), \\ M(Sx_n, Au, t), M(Su, Ax_n, 2t)M(Sx_n, Ax_n, t)\}$$

Taking limit as  $n \rightarrow \infty$

$$(M(Au, z, kt))^2 \geq \min\{(M(z, z, t))^2, M(z, Au, t)M(z, z, t), M(z, z, 2t)M(z, Au, t), \\ M(z, Au, t), M(z, z, 2t)M(z, z, t)\}$$

It is possible only when  $Au = z$ . Therefore  $Au = Su = z$  and hence u is a coincidence point of (A, S). Since the pair (A, S) is weakly compatible therefore,

$$Az = ASu = SAu = Sz.$$

Putting  $x = z, y = x_n$  in inequality (III), we get

$$(M(Az, Ax_n, t))^2 \geq \min\{(M(Sz, Sx_n, t))^2, M(Sz, Az, t)M(Sx_n, Ax_n, t), M(Sz, Ax_n, 2t)M(Sz, Az, t), \\ M(Sx_n, Az, t), M(Sz, Ax_n, 2t)M(Sx_n, Ax_n, t)\}$$

Taking limit as  $n \rightarrow \infty$

$$(M(Az, z, kt))^2 \geq \min\{(M(Az, z, t))^2, M(Az, Az, t)M(z, z, t), M(Az, z, 2t)M(z, Az, t), \\ M(z, Az, t), M(Az, z, 2t)M(z, z, t)\} \\ = \min\{(M(Az, z, t))^2, 1.1, M(Az, z, 2t)M(z, Az, t), \\ M(z, Az, t), M(Az, z, 2t).1\} \\ = (M(Az, z, t))^2$$

It is possible only when  $Az = z$ , therefore  $Az = Sz = z$ , therefore z is a common fixed point of A and S.

Now we prove the uniqueness of common fixed, let  $z_1$  and  $z_2$  be two common fixed points of f and g then from condition 3,

$$(M(z_1, z_2, kt))^2 = (M(Az_1, Bz_2, kt))^2 \geq \min\{(M(Sz_1, Tz_2, t))^2, M(Sz_1, Az_1, t)M(Tz_2, Bz_2, t), \\ M(Sz_1, Bz_2, 2t)M(Tz_2, Az_1, t), M(Tz_2, Az_1, t), M(Sz_1, Bz_2, 2t)M(Tz_2, Bz_2, t)\} \\ = \min\{(M(z_1, z_2, t))^2, M(z_1, z_1, t)M(z_2, z_2, t), \\ M(z_1, z_2, 2t)M(z_2, z_1, t), M(z_2, z_1, t), M(z_1, z_2, 2t)M(z_2, z_2, t)\} \\ = (M(z_1, z_2, t))^2$$

Therefore,

$$(M(z_1, z_2, kt))^2 \geq (M(z_1, z_2, t))^2$$

It is possible only when  $z_1 = z_2$ , therefore z is unique common fixed point of A and S.

#### IV. Conclusion

Our results improve several known results in the following ways:

- (i) Our results fuzzify, generalize and improve several results of metric space.
- (ii) The completeness of space is not required.
- (iii) Closedness of space is not required in case of  $CLR_{ST}$  property.
- (iv) Continuity of mappings is not required.
- (v) Weakly compatible mappings are used which are more general among all existing weak commutativity concepts.

#### References

- [1]. Aamri M. and Moutawakil D. E., "Some new common fixed point theorems under strict contractive conditions", Journal of Mathematical Analysis and Applications, vol. 270, no.1, (2002) pp. 181–188.
- [2]. Ali J., Imdad M. and Bahuguna D., "Common fixed point theorems in Menger spaces with common property (E.A)", Comput. Math. Appl., 60(12) (2010) 3152-3159.
- [3]. Fang J.X., "On fixed point theorems in fuzzy metric spaces", Fuzzy sets and systems, 46(1992), 107-113
- [4]. Gregus Jr. M., "A fixed point theorem in Banach space", Bull. Un. Math. Ital. 17-A (5) (1980) 193-198.
- [5]. George A., Veeramani P., "On some results in Fuzzy Metric Spaces", Fuzzy Sets and System, 64 (1994), 395-399.
- [6]. Grabiec M., "Fixed points in fuzzy metric spaces", Fuzzy Sets and Systems, 27(1989), 385-389.
- [7]. Gregori V. and Sapena A., "On Fixed Point Theorem in Fuzzy Metric Spaces". Fuzzy Sets and Systems, vol. 125, (2002), pp. 245–252.
- [8]. Imdad M., Pant B. D. and Chauhan S., "Fixed point theorems in Menger spaces using the CLRST property and applications", J. Nonlinear Anal. Optim., 3 (2) (2012), 225 - 237.
- [9]. Jungck G. "Compatible mappings and common fixed points.", Internet. J. Math. Math. Sci.9 (1986), 771-779.
- [10]. Jungck G. and Rhoades B.E., "Fixed point Theorems for occasionally weakly compatible mappings", Fixed point theory, 7(2006), 286-296.
- [11]. Kramosil O. and Michalek J., "Fuzzy Metric and statistical metric spaces", Kybernetika, 11 (1975), 326-334.
- [12]. Mihet D., "Fixed point theorems in fuzzy metric spaces using property (E.A)", Nonlinear Anal. 73 (2010) 2184–2188.
- [13]. Mishra S.N., "Common fixed points of compatible mappings in PM spaces", Math. Japon. 36 (1991), 283-289.
- [14]. Pant R.P. "Common fixed points of non-commuting mappings", J. Math. Anal. Appl. 188 (1994), 436-440.
- [15]. Pathak H.K., Khan M.S., "Compatible mappings of type (B) and common fixed point theorems of Gregus type", Czechoslovak Math. J. 45 (120) (1995) 685-698.
- [16]. Singh B. and Chauhan M. S., "Common fixed points of compatible maps in fuzzy metric spaces", Fuzzy Sets and Systems, 115(2000), 471-475.
- [17]. Schweizer B. and sclar A. "Statistical Metric space" pacific j. Math, (1960) 314-334.
- [18]. Sessa S., "On a weak commutativity condition in a fixed point considation." Publication of Inst. Mathematics, (1986) 32(46).149-153.
- [19]. Sintunavarat W., Kuman P., "Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces", Journal of Applied Mathematics, Vol. 2011 (2011), Artcal ID 637958, 14 pages.
- [20]. Vasuki R., "Common fixed points for R-weakly commuting maps in fuzzy metric spaces", Indian J. Pure Appl. Math. 30 (1999), 419-423.
- [21]. Zadeh L.A., "Fuzzy sets", Inform and Control 8 (1965), 338-353.