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On $\delta-\alpha$ - Open Sets

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Abstracts: *In this paper, we introduce a new class of open sets called $\delta-\alpha$-open sets together with its corresponding operators $\delta-\alpha$-interior and $\delta-\alpha$-closure. This class of sets is strictly contained in the class of $\delta-preopen$ and $\delta-semiopen$ sets. We also study some of the fundamental properties of this set.*

Keywords: *open set, closed set, $\delta-open$, $\delta-closed$, $\delta-\alpha-open$, $\delta-\alpha-closed$.*

I. Introduction

Njastad [2] introduced a new class of open sets in topological space, called $\alpha-open$ sets. This class of sets is strictly contained in the class of preopen [6] and semiopen sets [5] and strictly contains open sets. Velico [1] introduced the concept of $\delta-open$ sets. Since then authors like Noiri [3] and Chakraborty [4] generalized the concept and introduced $\delta-preopen$, $\delta-semiopen$ and $\delta-b-open$ sets. In this paper, we have introduced the notion of $\delta-\alpha-open$ sets. The class of $\delta-\alpha-open$ sets is contained in the class of $\delta-preopen$ sets and $\delta-semiopen$ sets and contains $\delta-open$ sets. We also obtain some of the properties of this set.

II. Preliminaries

In this paper, (X,τ) or X always represents a topological spaces in which no separation axioms are assumed unless explicitly stated. For a subset A of X, the interior and the closure of A in X with respect to τ are denoted by $\text{int}(A)$ and $cl(A)$ respectively. The complement of a set A of X is denoted by $X-A$. A subset A of X is called regular open [7] if $A=\text{int}(cl(A))$ and regular closed if $A=cl(\text{int}(A))$. In [7] it is shown that the class of regular open sets of X is a base for a topology τ_s on X coarser than τ and this space (X,τ_s) is called the semi-regularization space of (X,τ). The $\delta-\text{int}erior$ [1] of a subset A of X is denoted by $\text{int}_\delta(A)$ and is defined as the union of all regular open sets contained in A. A set A of X is called $\delta-open$ [1] if $A=\text{int}_\delta(A)$. The above mentioned topology τ_s consists of all $\delta-open$ sets in X. The $\delta-closure$ [1] of a set A is denoted by $cl_\delta(A)$ and is defined as the intersection of all regular closed sets containing A. A set A of X is called a $\delta-closed$ set [1] if $A=cl_\delta(A)$. The complement of a $\delta-open$ set is a $\delta-closed$ set.

Definition 3.1. A subset A of X is called:
(a) $\alpha-open$ [2] if $A\subset\text{int}(cl(\text{int}(A)))$,
(b) $semiopen$ [5] if $A\subset cl(\text{int}(A))$,
(c) $preopen$ [6] if $A\subset\text{int}(cl(A))$,
(d) $b-open$ [9] if $\text{int}(cl(A))\cup cl(\text{int}(A))$,
(e) $\beta-open$ [8] if $A\subset cl(\text{int}(cl(A)))$,
(f) $\delta-semiopen$ [3] if $A\subset cl(\text{int}_\delta(A))$,
(g) $\delta-preopen$ [3] if $A\subset\text{int}(cl_\delta(A))$,
(h) $\delta-b-open$ [4] if $A\subset\text{int}(cl_\delta(A))\cup cl(\text{int}_\delta(A))$,
(i) $\delta-\beta-open$ [10] if $A\subset cl(\text{int}(cl_\delta(A)))$,
(j) $\delta-\alpha-open$ if $A\subset\text{int}(cl(\text{int}_\delta(A)))$.

The family of all $\delta-\alpha-open$ (resp. $\alpha-open, semiopen, preopen, b-open, \beta-open, \delta-semiopen, \delta-preopen, \delta-b-open, \delta-\beta-open$) sets of X is denoted by $\delta\alpha O(X)$ (resp. $\alpha O(X), SO(X), PO(X), BO(X), \beta O(X), \delta SO(X), \delta PO(X), \delta BO(X), \delta\beta O(X)$).

Definition 3.2. The complement of a $\delta-\alpha-open$ (resp. $\alpha-open, semiopen, preopen, b-open, \beta-open, \delta-semiopen, \delta-preopen, \delta-b-open, \delta-\beta-open$) set is called $\delta-\alpha-closed$ (resp. $\alpha-closed$ [2], $semiclosed$ [5], $preclosed$ [6], $b-closed$ [9], $\beta-closed$ [8], $\delta-semiclosed$ [3], $\delta-preclosed$ [3], $\delta-b-closed$ [4], $\delta-\beta-closed$[10]) set.

Definition 3.3. The union of all $\delta-\alpha-open$ (resp. $\alpha-open, semiopen, preopen, b-open, \beta-open, \delta-semiopen, \delta-preopen, \delta-b-open, \delta-\beta-open$) sets contained in A is called the $\delta-\alpha-interior$ (resp. $\alpha-interior$[2], $semi-interior$[5], $preinterior$ [6], $b-interior$ [9], $\beta-interior$ [8], $\delta-semi-interior$ [3], $\delta-preinterior$ [3], $\delta-b-interior$ [4], $\delta-\beta-interior$ [10]) of A and is denoted by $\alpha\operatorname{int}_\delta(A)$ (resp. $\alpha\operatorname{int}(A), \sin t(A), p\operatorname{int}(A), b\operatorname{int}(A), \beta\operatorname{int}(A), \sin t_\delta(A), p\operatorname{int}_\delta(A), b\operatorname{int}_\delta(A), \beta\operatorname{int}_\delta(A)$).

Definition 3.4. The intersection of all $\delta-\alpha-closed$ (resp. $\alpha-closed, semiclosed, preclosed, b-closed$ $\beta-closed, \delta-semiclosed$ $\delta-preclosed, \delta-b-closed$ $\delta-\beta-closed$) sets containing A is called the $\delta-\alpha-closure$ (resp. $\alpha-closure$ [2], $semiclosure$ [5], $preclosure$ [6], $b-closure$ [9], $\beta-closure$ [8], $\delta-semiclosure$ [3], $\delta-preclosure$ [3], $\delta-b-closure$ [4], $\delta-\beta-closure$ [10]) of A and is denoted by $\alpha cl_\delta(A)$ (resp. $\alpha cl(A), s\,cl(A), pcl(A), bcl(A), \beta cl(A), s\,cl_\delta(A), pcl_\delta(A), bcl_\delta(A), \beta cl_\delta(A)$).

III. Basic Properties of $\delta-\alpha-open$ Sets

Theorem 4.1. Every $\delta-\alpha-open$ set is a $\delta-preopen$ set.

Proof: Let A be $\delta-\alpha-open$ set. Then $A\subset \operatorname{int}(cl(\operatorname{int}_\delta(A)))$. Since $\operatorname{int}_\delta(A)\subset A$ and $cl(A)\subset cl_\delta(A)$, so $cl(\operatorname{int}_\delta(A))\subset cl(A)\subset cl_\delta(A)$, and hence $A\subset \operatorname{int}(cl(\operatorname{int}_\delta(A)))\subset \operatorname{int}(cl_\delta(A))$.

The converse of the above theorem need not be true. This is shown by the following example.

Example 4.1. In the topological space (X,τ), where $X=\{a,b,c\}$ and $\tau=\{\phi, X,\{a\},\{a,b\}\}$, the set $\{c\}$ is $\delta-preopen$ set but not a $\delta-\alpha-open$ set.

Theorem 4.2. Every $\delta-\alpha-open$ set is a $\delta-semiopen$ set.

Proof: Let A be $\delta-\alpha-open$ set. Then $A\subset \operatorname{int}(cl(\operatorname{int}_\delta(A)))\subset cl(\operatorname{int}_\delta(A))$.

The converse of the above theorem need not be true. This is shown by the following example.

Example 4.2. In the topological space (X,τ), where $X=\{a,b,c\}$ and $\tau=\{\phi, X,\{a\},\{b\}\{a,b\}\}$, the set $\{b,c\}$ is $\delta-semiopen$ set but not a $\delta-\alpha-open$ set.

Theorem 4.3. Every $\delta-\alpha-open$ set is a $\alpha-open$ set.

Proof: Let A be $\delta-\alpha-open$ set. Then $A\subset \operatorname{int}(cl(\operatorname{int}_\delta(A)))$. Since $\operatorname{int}_\delta(A)\subset \operatorname{int}(A)$, so $\operatorname{int}(cl(\operatorname{int}_\delta(A)))\subset \operatorname{int}(cl(\operatorname{int}(A)))$.Hence $A\subset \operatorname{int}(cl(\operatorname{int}_\delta(A)))\subset \operatorname{int}(cl(\operatorname{int}(A)))$.

The converse of the above theorem need not be true. This is shown by the following example.

Example 4.3. In the topological space (X,τ), where $X=\{a,b,c\}$ and $\tau=\{\phi, X,\{a\},\{a,b\}\}$, the set $\{a,b\}$ is $\alpha-open$ set but not a $\delta-\alpha-open$ set.

Theorem 4.4. Every $\delta-open$ set is a $\delta-\alpha-open$ set.

Proof: Let A be $\delta-open$ set. Then $A=\operatorname{int}_\delta(A)$. Now $\operatorname{int}(cl(\operatorname{int}_\delta(A)))=\operatorname{int}(cl(A))$. Since $A\subset cl(A)$, so, $A=\operatorname{int}_\delta(A)\subset \operatorname{int}(A)\subset \operatorname{int}(cl(A))=\operatorname{int}(cl(\operatorname{int}_\delta(A)))$.

Remark 1.

$$regular\ open \Rightarrow \delta-open \Rightarrow \begin{cases} open \Rightarrow \alpha-open \Rightarrow \begin{cases} preopen \Rightarrow b-open \Rightarrow \beta-open \\ semiopen \Rightarrow b-open \end{cases} \\ \delta-\alpha-open \Rightarrow \begin{cases} \alpha-open \\ \delta-preopen \Rightarrow \delta-b-open \Rightarrow \delta-\beta-open \\ \delta-semiopen \Rightarrow \delta-b-open \end{cases} \end{cases}.$$

Theorem 4.5. Arbitrary union (resp. intersection) of $\delta-\alpha-open$ (resp. $\delta-\alpha-closed$) sets is a $\delta-\alpha-open$ (resp. $\delta-\alpha-closed$) set.

Proof: let $\{A_\alpha\}_{\alpha\in\Delta}$ be a family of $\delta-\alpha-open$ sets in X. Then $A_\alpha \subset \operatorname{int}(cl(\operatorname{int}_\delta(A_\alpha))),\ \forall\alpha\in\Delta$.
Now,

$$\cup A_\alpha \subset \cup\{\operatorname{int}(cl(\operatorname{int}_\delta(A_\alpha)))\} \subset \operatorname{int}[\cup\{cl(\operatorname{int}_\delta(A_\alpha))\}] = \operatorname{int}[cl\{\cup(\operatorname{int}_\delta(A_\alpha))\}] \subset \operatorname{int}(cl(\operatorname{int}_\delta(\cup A_\alpha))).$$

Remark 2. The $\alpha\operatorname{int}_\delta(A)$ is $\delta-\alpha-open$ set and $\alpha cl_\delta(A)$ is a $\delta-\alpha-closed$ set.

Theorem 4.6. A subset A of X is closed if and only if $cl(\operatorname{int}(cl_\delta(A))) \subset A$.

Theorem 4.7. The following properties hold for $\delta-\alpha-\operatorname{int}erior$ operator.

(a) A set A of X is $\delta-\alpha-open$ if and only if $A=\alpha\operatorname{int}_\delta(A)$.

(b) $\alpha\operatorname{int}_\delta(A)\subset\alpha\operatorname{int}_\delta(B)$, if $A\subset B\subset X$.

(c) $\alpha\operatorname{int}_\delta(A)$ is the largest $\delta-\alpha-open$ set contained in A.

(d) $\alpha\operatorname{int}_\delta(\alpha\operatorname{int}_\delta(A))=\alpha\operatorname{int}_\delta(A)$.

Theorem 4.8. The following properties hold for $\delta-\alpha-closure$ operator.

(a) A set A of X is $\delta-\alpha-closed$ if and only if $A=\alpha cl_\delta(A)$.

(b) $\alpha cl_\delta(A)\subset\alpha cl_\delta(B)$, if $A\subset B\subset X$.

(c) $\alpha cl_\delta(A)$ is the smallest $\delta-\alpha-closed$ set containing A.

(d) $\alpha cl_\delta(\alpha cl_\delta(A))=\alpha cl_\delta(A)$.

IV. Conclusion

The author in this paper studies a new type of set called $\delta-\alpha-open$ set. Open sets and its generalizations are very important in many branches of Mathematics. Properties of this type of set is investigated. Also the relationship between this set and many other sets are introduced.

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