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Puzzling and a puzzling graphs on some graphs

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Abstract: *Let P be the partition of a graph G with chromatic number, P is puzzle on G, if there is a vertex coloring of G using 1,2,… χ (G) Such that the sums of the numbers assigned to the partition pieces are all same. P is a puzzle if there is unique vertex coloring such that the sums are all different. We investigate the concept of some graphs are puzzling, a puzzling and neither.*

Keywords: *Vertex coloring, Puzzles, Partition, Fan graph, Wheel graph, Ladder graph.*

I. Introduction

This work here is in a tradition of research into graphs with special colorings, a tradition that include the **"vertex colorable"** graphs[4], **"magic"** graphs[5-6] and more recently," **Anti magic"** graphs. In this paper we determine the puzzlity and a puzzlity of a number of common classes of graphs.

Many graphs are neither puzzling nor a puzzling. It is easy to see, for example, then for n>1, K_n, the complete graph on n vertices, is not puzzling. A puzzle must have a piece with at least two vertices. Any solution to the puzzle generates the another solution by switching the colors of the two vertices in the piece. A similar argument shows that K_n, is not puzzling.

By **"vertex coloring"**, we mean a minimal coloring of the vertices using the numbers 1,2,3,… χ (G).For this paper," partition" will mean a partition where the pieces are all connected.

In section I, the fan graphs are neither puzzling nor a puzzling. In section II, we show that the ladder graphs are puzzling and which are a puzzling. In section III, we determine wheels are puzzling and there conditions.

Definition 1: Let G be a connected graph with chromatic number χ (G).A **puzzle** on G such that there is exactly one vertex coloring with the property that the sums of the vertex labels of the partition pieces are all same.

A graph is **puzzling** if there is a puzzle on the graph.

Definition 2: An **a puzzle** on a graph G is a partition of G such that there is exactly one vertex coloring such that the sums of the vertex labels of the partition pieces are all different.

A graph is **a puzzling** if there is and a puzzle on the graph.

II. Fan Graphs

Theorem 1: For all n, F_n, the fan graphs on n vertices is not puzzling.

Proof: Suppose a fan graph partitioned into connected pieces. In any equal coloring, a piece of length 2k must have a vertex sum of 5k.A piece of length 2k+1 must have a vertex sum of 5k+2 or 5k+4.In equal coloring exists, all pieces must have the same length. If the length is even, both vertex coloring are equal coloring. Hence there is no equal coloring.

If the length of all pieces of the partition is odd, then the adjacent pieces must have different sums (one begins and ends with '1', the other begins and ends with '2') If the partition has more then one piece then here is no unique equal coloring. The partition has only one piece there is no unique equal coloring.

Theorem 2: For all n, F_n , the fan graph of n vertices , is not a puzzling.

Proof: A partition of a fan graph on unequal coloring if the pieces are all of different lengths. In that case, both vertex coloring are unequal. So there is no unique unequal coloring.

III. Ladder Graphs

Theorem 3: The ladder graph of n vertices are puzzling.

Proof: If ladder graph of n vertices are chromatic number of L_n is 2.In any equal coloring, a piece of length either 2k or 2k+1 must have a vertex sum of 3k.Any coloring of L_n must either color of all vertices in one or two and the reverse. Now all the partition pieces are same length, is puzzling. In that pieces are either odd or even the sums of the vertex labelling the partitions are same. One piece or more than pieces of the ladder graphs are all unique coloring.

Theorem 4: For n, L_n are a puzzling if and only if $2<n\leq5$, and if $n\geq5$, where n is odd.
Proof: Case 1: Suppose $n<5$, the minimum vertex coloring is two. In that the argument all the partitions are different length.
Case 2: If n is greater than 5 and n is odd, clearly the proof of the case 1, is a puzzling.
If n is even, there exist two coloring. The one of partition must have a repeated coloring. There is no unique unequal coloring.

IV. Wheel Graphs

Theorem 5: Wheels are apuzzling if $n\geq4$.
Proof: As with puzzles, if the number of vertices are either odd or even the proof of the theorem (2).So that wheels are a puzzling.

Theorem 6: Wheels are puzzling.
Claim 1: For all n, W_n, the wheel graph is puzzling if n not equal to either 5 or 9.
Proof: If $n\geq4$, W_n be the set of n vertices and the chromatic number 3 if n is odd and 4 if n is even such that the center vertex v_0 and every edge connected the all vertices in v_0. Suppose that W_n is puzzling. Let P be the puzzle on W_n and Let us say that in the solution, the center vertex v_0 is labeled one and all the other adjacent pieces labeled the other colors which the property that adjacent vertices does not the same color. Since all piece have the same sum. Hence, it is puzzling.
Claim 2: If n is 5 or 9.
Proof: For the last clause it is easy to check that W_5 and W_9 is not puzzling only the center vertex v_0 labeled one.
For the other direction, v_0 is either two or three, the proof of in this case, must break up into above claim (1).
Case 1: W_5 is not puzzling where the center vertex v_0 is one.
In this case, the partition must contain the following pieces (212) (312) (23) (231) (32) (21) (31).Thus the multiple solution can be obtain the different length. Thus the piece of partition cannot the same length.
Case 2: W_5 is puzzling where the center vertex v_0 is either two or three.
Now the center vertex is two, the possible pieces that sums five by (131) (23).Hence obtained by theorem (2).Then the same argument of the center vertex is three. Therefore, W_5 is puzzling.
Case 3: W_9 is not puzzling where the center v_0 is one.
Suppose the center vertex v_0 can be either two or three, there are possibilities the same length. Hence, proved.

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