

## **Modelling and Forecasting Exchange-rate Volatility with ARCH-type Models**

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**Abstract:** *This paper uses the Generalized Autoregressive Conditional Heteroskedasticity models to estimate volatility (conditional variance) in the monthly returns of the principal stock exchange of Sudan. over the period from January 1999 to December 2013. The models include both symmetric and asymmetric models that capture the most common stylized facts about index returns such as volatility clustering and leverage effect. The empirical results show that the conditional variance process is highly persistent (explosive process), and provide evidence on the existence of risk premium for the return series which support the positive correlation hypothesis between volatility and the expected stock returns. Our findings also show that the asymmetric models provide better fit than the symmetric models, which confirms the presence of leverage effect. These results, in general, explain that high volatility of return series is present in the Sudanese stock market over the period.*

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### **I. Introduction**

The economic crisis has had a differentiated impact on the world economies and on their trade, thereby changing trade patterns significantly in some cases. In the context of low employment related to recession, some policymakers are wanting to stimulate their exports, thereby hoping to improve their trade and current account balances. Policymakers interested in implementing such policies have taken a closer look at exchange rate movements. Simply stated, depreciation of a country's currency makes its exports cheaper and its imports more costly. In the reality of a globalised economy however, industries are vertically integrated, and exported products contain a large proportion of imported components. Imported components therefore become more costly for any given exporter and are not necessarily substitutable with domestically-produced products.

In addition, exchange rate levels have important implications for debt servicing and foreign investment flows. A depreciation in a country's currency implies that the nominal value of debt denominated in foreign currencies increases relative to the country's resources in local currency whereas its local-currency denominated debt decreases in value for foreign creditors. Capital investments become cheaper to foreign investors when the currency is depreciated, which is particularly important for large economies that attract capital investments like the United States and, to a lesser extent, the European Union. If depreciation is the result of a loss of confidence in the economy, however, foreign investors may be more hesitant to invest. Exchange rate changes affect firms within a given country differently.

Firms face number of risks when engaging in international trade, in particular economic and commercial risks that are determined by macroeconomic conditions over which they have little control, such as exchange rates and their volatility. Risk management tools are available to help firms mitigate the impact of such risks, especially in the short term. These techniques for securing exchange rate risk are sometimes complex, however, and do not cover all commercial and financial operations. Besides, such tools may not be available to all firms, and the cost of using them may be significant, especially for small firms and in situations of high volatility.

### **Exchange Rate Policy in Sudan: An Overview**

The exchange rate market in Sudan has undergone numerous policy interventions. Throughout the period 1956-1979, the exchange rate has been pegged at a fixed rate of approximately one Sudanese pound to US\$2.85. In 1979, the government shifted to a floating exchange rate system with the aim of boosting the economy since the country had witnessed many economic problems including, fiscal deficit, external disequilibrium, high inflation rates and mounting external debts during the 1970s (Ali 1985). Thus, the government launched the first version of the stabilization and liberalization programs, which focused on exchange rate devaluation as a key policy tool for economic recovery. As a result, the Sudanese pound underwent a significant devaluation to the rate of one US dollar to 0.35 Sudanese pounds. The main goal of this policy was to reduce the external imbalances through encouraging the volume of exports, and attracting private international capital, such as remittances of Sudanese nationals working abroad (SNWA). The monetary authority in that period had adopted a dual exchange markets, namely, the official and the parallel markets.

Although the parallel exchange market in the beginning was limited to foreign trade, the massive flows of migrants' remittances in the second half of that decade extended the parallel exchange activities and

increased the black market premium (Elbadawi 1994). Throughout the 1980s, the exchange rate in Sudan experienced a series of devaluations, owing to the economic and political instabilities. In the 1980s, the country experienced many factors affecting economic performance as drought and famines in 1984-1985 and the eruption of the second civil war in 1983. The country, therefore, suffered from a severe lack of foreign reserves and relied mainly on foreign aid in financing development projects. As such, the exchange rate was devalued in 1985 by 48 percent, with the official rate set at LS2.5/US\$ and the parallel at LS3.3/US\$. In early 1987, the exchange rate was devalued further to about LS4.00/US\$ and LS5.8/US\$, for the official and black market, respectively.

In an attempt to attract the remittances of SNWA through the official channels, the monetary authority in 1987-1988, attempted to unify the two rates at LS4.5/US\$. However, by the end of 1980s the black market was active, and the speculation on foreign currency and non tradable goods were the dominant activities thereby causing the black market exchange rate to be set at more than LS20/US\$ in late 1989.

In the early 1990s, the economy witnessed several transformations, notably the transition from state control policies that characterized the period of 1970s and 1980s to free market policies. The Salvation Revolution government of 1989 launched many economic recovery programs, which aimed at encouraging the export through stabilizing the exchange rates. The Comprehensive National Strategy (CNS) of 1992-2002 was an ambitious one. The CNS focused on the liberalization of trade and exchange rate, the liberalization of the financial sector, the removal of agricultural subsidies, the reduction of trade tariffs and the privatization of inefficient public enterprises. Accordingly, the exchange rate policy received considerable attention from the government, because it was believed to be a core factor affecting the economic instability. Thus, at the beginning of the economic recovery program of 1990, the black market exchange was prohibited and considered an illegal practice and the government implemented strict punishment for illegitimate exchange dealers. Thus, all foreign exchange transactions were confined to the licensed commercial banks. Yet despite these measures, the exchange rate reported was higher in the early 1990s compared to the 1980s.

As a part of the economic liberalization policies in 1992, the government adopted a floating exchange rate as well as unifying the formal and parallel rate. However, due to the drastic depreciation of the local currency and the subsequent increase in inflation, the floating system was abandoned in October 1993 and replaced by the dual exchange system. The formal rate was set at LS215/US\$, while the parallel was set at LS300/US\$. Thereafter, the exchange rate underwent continuous devaluations as set by the Bank of Sudan at LS300/\$ and LS430/\$ in 1994 and 1995, respectively. Therefore, managing the exchange rate during 1990-1995 was a difficult task for the government owing to the scarcity of foreign exchange and economic distortions. Indeed, in the first half of 1990s, the economy saw many challenges, like the soaring inflation rate of more than 120% in 1995, as well as the decrease of foreign assistance due to political reasons in addition to a reduction in the agricultural exports.

In the second half of the 1990s, the exchange rate stabilized owing to the flow of FDI and the commercial exploitation of oil in 1999. Notably, the flow of oil revenues brought to the economy a huge amount of foreign reserves. As a result, the exchange rate saw substantial stability with a limit rate at LS2650-2600 per US dollar during 2000-2003. It is worth mentioning that oil exports in the early 2000-2007 became the major source of foreign exchange and accounted for around 85% of the total value of exports. Accordingly, during this period the Central Bank of Sudan adopted a managed floating exchange regime. Moreover, during the period that was accompanied by oil exportation, the economy witnessed a favorable economic performance. For example, the country reported a positive and high economic growth rate, leading Sudan to be one of the fastest growing countries in the region (World Bank 2008). The rate of inflation also fell into the one digit range during that period. Nevertheless, other sectors of the economy, like agriculture, suffered severely; this may have been because of the oil windfall, which appreciated the exchange rate and hence, reduced the sector's competitiveness. The appreciation of the exchange rate during that period was assumed to be a symptom of the Dutch disease. In fact, the share of the agricultural sector in GDP and total exports declined sharply after oil exploitation.

During the period 2008-2010, the exchange saw many fluctuations owing to the reduction in oil prices due to global economic crisis. The decline in the inflow of foreign currency that followed led to another split in exchange markets into official and black. Recently, in the aftermath of the secession of South Sudan in July 2011, Sudan has suffered from many economic challenges owing to the sudden stop of oil revenues. As a result, the exchange rate depreciated rapidly, leading to increase in the black market premium. In response to such a situation, in June 2012 the authorities adopted a new exchange rate measure, which devalued the currency to the rate of SDG4.42/US\$3. Overall, the exchange rate in Sudan had seen a continuous devaluation since 1979, particularly in the period which preceded the oil exploitation. Appendix V, reveals that the nominal exchange rate reported a positive trend with a slight increase during the period 1979-1991 but did not exceed LS500/US\$. After the economic liberalization policies of 1992 and up to 1996, the exchange rate depreciated dramatically to about LS2000/US\$ in 1997. However, during the period of the managed floating exchange rate regime and oil

exploitation (1997-2007), the exchange rate was stable at the rate of 2.5SDG/US\$ on average and then decreased subsequently to about SDG2/US\$ in 2008.

### **Empirical Literature Review**

To capture the volatility in financial time series, a comprehensive empirical analysis of the returns and conditional variance of the financial time series have been carried out using autoregressive conditional Heteroskedasticity models. Bellow a literature review of these studies:

Sharaf Obaid, Abdalla Suliman.(2013).Estimating Stock Returns Volatility of Khartoum Stock Exchange through GARCH Models this study modeled and estimated stock returns volatility of Khartoum Stock Exchange (KSE) index using symmetric and asymmetric GARCH family models namely :GARCH (1,1) GARCH-M (1,1) EGARCH (1,1) and GJR-GRACH (1,1) models, they carried out that based on daily closing prices over the period from Jan 2006 to Aug 2010 that high volatility processing present in KSE index return series the result also provided evidence on the existence of risk premium and indicate the presence of leverage effect in the KSE index returns series our findings indicate the student-t is the most favored distribution for all models estimated .

Ahmed E l sheikh M. Ahmed and Suliman Zakaria(2013) Modeling stock Market volatility using GARCH Models Evidence from Sudan, they used the Generalized Autoregressive conditional Heteroskedasticity Models to estimate volatility (conditional variance)

In the daily returns of the principal stock exchange of Sudan namely Khartoum stock Exchange (KSE) over the period from 2006 to 2010. daily Observations of 1326 for (KSE) The models include both symmetric and asymmetric models that capture the most common stylized facts about index returns such as volatility clustering and leverage effect the empirical result show that the conditional variance process is highly persistent (explosive process) and provide evidence on the existence of risk premium of the KSE index return series which support the positive correlation hypothesis between volatility and the expected stock returns was findings also show that the asymmetric models provide better fit than the symmetric models; which confirms . The presence of leverage effect.These results in general explains that high volatility of index return series is present in Sudanese stock market over the sample period.

AUWAL (2010) used 354 bi – weekly data points (from July 22, 2002 to March 27, 2006) of Retail Dutch Auction System (RDAS) sessions' period in Nigeria to show that RDAS as an institutional arrangement for foreign exchange does not bring about better stability in the exchange rate in the long – run. The short – run relationship is modeled by the GARCH while the long – run relationship is modeled through the ARDL. The study revealed that volatility is persistent in demand for foreign exchange and the exchange rate. There is evidence of long run relationship between demand for foreign exchange, marginal rate of exchange and the rate of success of bids under RDAS. Therefore, exchange rate and the effect of RDAS have significant influence towards the determination of demand for exchange rate both in the short run and in the long run. The study concluded that, RDAS, as an institutional arrangement for conducting a flexible exchange, cannot bring about better result in the long run than it can offers in the short run. The study suggests further studies on the recent phase Wholesale Dutch Auction System (WDAS).

## **II. Methodology**

Autoregressive conditional Heteroskedasticity (ARCH) and its generalization (GARCH) models represent the main methodologies that have been applied in modelling and forecasting stock market volatility in empirical finance. In this paper different GARCH specifications are employed to model stock returns volatility in Sudan these models are GARCH (1,1), GARCH-M (1,1), which will be used for testing symmetric volatility and EGARCH(1,1),TGARCH(1,1) and PGARCH (1,1) for modelling asymmetric volatility These models will be shortly discussed in the following subsections. For all these different models, there are two distinct equations, the first one for the conditional mean and the second one for the conditional variance. We are mainly interested in the second equation as it provides estimates and conditional forecast of volatility

### **ARCH Model**

ARCH models based on the variance of the error term at time t depends on the realized values of the squared error terms in previous time periods. The model is specified as:

$$y_t = u_t \tag{1}$$

$$u_t \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i u_{t-i}^2 \tag{2}$$

This model is referred to as ARCH(q), where q refers to the order of the lagged squared returns included in the model. If we use ARCH(1) model it becomes

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{3}$$

Since  $h_t$  is a conditional variance, its value must always be strictly positive; a negative variance at any point in time would be meaningless. To have positive conditional variance estimates, all of the coefficients in the conditional variance are usually required to be non-negative. Thus coefficients must satisfy  $\alpha_0 > 0$  and  $\alpha_1 \geq 0$ .

**GARCH Model**

Bollerslev (1986) and Taylor (1986) developed the GARCH(p,q) model. The model allows the conditional variance of variable to be dependent upon previous lags; first lag of the squared residual from the mean equation and present news about the volatility from the previous period which is as follows:

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i u_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i} \tag{4}$$

In the literature most used and simple model is the GARCH(1,1) process, for which the conditional variance can be written as follows:

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1} \tag{5}$$

Under the hypothesis of covariance stationarity, the unconditional variance  $h_t$  can be found by taking the unconditional expectation of equation 5.

We find that

$$h = \alpha_0 + \alpha_1 h + \beta_1 h \tag{6}$$

Solving the equation 5 we have

$$h = \frac{\alpha_0}{1 - \alpha_1 - \beta_1} \tag{7}$$

For this unconditional variance to exist, it must be the case that  $\alpha_1 + \beta_1 < 1$  and for it to be positive, we require that  $\alpha_0 > 0$

**GARCH-in-Mean (GARCH-M) Model**

In finance, the return of a security may depend on its volatility. To model such a phenomenon one may consider the GARCH-M model developed by Engle, Lilien, and Robins (1987), where "M" stands for GARCH in the mean (Tsay 2010). This model is an extension of the basic GARCH framework which allows the conditional mean of a sequence to depend on its conditional variance or standard deviation. A simple GARCH-M(1,1) model can be written as :

Mean equation  $r_t = \mu + \lambda h_t^2 + \varepsilon_t$  (8)

Variance equation  $h_t^2 = \delta + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}^2$ , (9)

The parameter  $\lambda$  in the mean equation is called the risk premium parameter. A positive  $\lambda$  indicates that the return is positively related to its volatility. In other words, a rise in mean return is caused by an increase in conditional variance as a proxy of increased risk. Engle, Lilien, and Robins assume that the risk premium is an increasing function of the conditional variance of  $\varepsilon_t$  in other words, the greater the conditional variance of returns, the greater the compensation necessary to induce the agent to hold the asset (Enders 2004).

**Exponential GARCH**

Exponential GARCH (EGARCH) proposed by Nelson (1991) which has form of leverage effects in its equation. In the EGARCH model the specification for the conditional covariance is given by the following form:

$$\log(h_t) = \alpha_0 + \sum_{j=1}^q \beta_j \log(h_{t-j}) + \sum_{i=1}^p \alpha_i \left| \frac{u_{t-i}}{\sqrt{h_{t-i}}} \right| + \sum_{k=1}^r \gamma_k \frac{u_{t-k}}{\sqrt{h_{t-k}}} \tag{10}$$

Two advantages stated in Brooks (2008) for the pure GARCH specification; by using  $\log(h_t)$  even if the parameters are negative, will be positive and asymmetries are allowed for under the EGARCH formulation.

In the equation  $\gamma_k$  represent leverage effects which accounts for the asymmetry of the model. While the basic GARCH model requires the restrictions the EGARCH model allows unrestricted estimation of the variance

If  $\gamma_k < 0$  it indicates leverage effect exist and if  $\gamma_k \neq 0$  impact is asymmetric. The meaning of leverage effect bad news increase volatility.

Applying process of GARCH models to return series, it is often found that GARCH residuals still tend to be heavy tailed. To accommodate this, rather than to use normal distribution the Student's t and GED distribution used to employ ARCH/GARCH type models

### **The Threshold GARCH (TGARCH) Model**

Another volatility model with common use to handle leverage effects is the Threshold GARCH (TGARCH) model. In the (TGARCH) model the conditional variance of the model can be expressed as:

$$h_t = \delta + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \gamma_i d_{t-i} \varepsilon_{t-i}^2 + \sum_{i=1}^q \beta_i h_{t-j} \quad (11)$$

Where

$d_i = 1$  if  $\varepsilon_t < 0$  and  $d_i = 0$  otherwise.

Adverse market conditions and bad news ( $\varepsilon_{t-1}^2 < 0$ ) such as frost, drought, or political instability has an impact of  $(\alpha+\gamma)$ . Good news about the demand and supply conditions in the commodity market ( $\varepsilon_{t-1}^2 > 0$ ) has an impact of  $\alpha$ .

### **The APARCH Model**

Ding, Granger and Engle (1993) also introduced the Power GARCH (PGARCH) specification to deal with asymmetry. Unlike other GARCH models, in this model, the standard deviation is modeled rather than the variance as in most of the GARCH-family. In Power GARCH an optional parameter  $\gamma$  can be added to account for asymmetry (Floros, 2008). The model also affords one the opportunity to estimate the power parameter  $\delta$  instead of imposing it on the model (Ocran and Biekts, 2007)The general asymmetric Power GARCH model specifies  $h_t^\delta$  as of the following form:

$$. h_t^\delta = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^\delta + \sum_{j=1}^q \beta_j h_{t-j}^\delta + \gamma \varepsilon_{t-i}^\delta I_{t-i} \quad (12)$$

Where  $\omega > 0$ ,  $\delta \geq 0$ ,  $\alpha_i \geq 0$ ,  $\beta_j \geq 0$ ,  $-1 < \gamma_i < 1$ ,  $i=1,2,\dots,p$ ,  $j=1,2,\dots,q$ .

The model is couples the flexibility of varying exponent with the asymmetry coefficient, moreover The APARCH includes other ARCH extensions as special cases.

In the model, effect of good news shows their impact by  $\alpha_i$ , while bad news shows their impact by  $\alpha + \gamma$ . In addition if  $\gamma \neq 0$  news impact is asymmetric and  $\gamma > 0$  leverage effect exists. To satisfy non-negativity condition coefficients would be  $\alpha_0 > 0$ ,  $\alpha_i > 0$ ,  $\beta \geq 0$  and  $\alpha_i + \gamma_i \geq 0$ . That is the model is still acceptable, even if  $\gamma_i < 0$ , provided that  $\alpha_i + \gamma_i \geq 0$  (Brooks,2008).

#### **Basic concepts:**

##### **Leverage effect:**

Changes in stock prices tend to be negatively correlated with changes in volatility, i.e., volatility is higher after negative chocks than after positive chocks of same magnitude.

##### **Volatility clustering:**

Large changes tend to be followed by large changes -of either sign- and small changes tend to be followed by small changes.

##### **Leptokurtic:**

Distribution has a kurtosis value greater than that of a standard, normal distribution which gives the distribution a high peak, a thin midrange, and fat (heavy) tails

##### **Data:**

The data will be use in the analysis of this paper are monthly readings of Exchange Rate in the Sudan covered the period from 01/01/1999 to 31/12/2013 obtained from Central Bureau of Statistics, Bank of Sudan and Khartoum Stock market. and then transformed into logarithmic return series. The corresponding transform price series into monthly logarithmic return are calculated by using. the formula:  $r_t = \ln(x_t/x_{t-1})$  where  $x_t$  is the exchange rate and  $r_t$  denotes the returns



**Descriptive Statistics**

**Table(1):** Summary Statistics of Exchange rate Returns (SDG/ USA (\$))

Sample size	Mean	Median	S.Dev	Min.	Max.	Skew.	Kurt.	JarqueBera	P-value
178	0.0051	0.000	0.036	-0.076	0.373	6.717	62.241	27521.52	0.000

The summary statistics of this study is presented in table(1).This indicates that the returns series have monthly positive mean of (0.0051) while the monthly volatility is (0.013),without loss of generality the mean grows at a linear rate while the volatility grows approximately at a square root rate. The lowest monthly returns correspond to (-0.076) and the best monthly exchange rate returns is (0.373). The returns series of the exchange rate shows positive skewness. This implies that the series is flatter to the right. The kurtosis value is higher than the normal value of perfectly normal distribution in which value for skewness is ‘zero’ and kurtosis ‘three’ and this suggest that the kurtosis curve of the exchange rate return series is leptokurtic. The results of this study reveal that, the series is not normally distributed. Our empirical result is consistent with the Jarque-Bera (JB) tests obtain above which is used to assess whether the given series is normally distributed or not. Here, the null hypothesis is that the series is normally distributed. Results of JB test find that the null hypothesis is rejected for the return series and suggest that the observed series are not normally distributed

**Testing for Stationarity**

To investigate whether the daily price index and its returns are stationary series, the Augmented Dickey–Fuller(ADF) test (Dickey and Fuller, 1981) has been applied. Thereby, the lag length has been selected automatically based on the Schwarz information criterion with a preset maximum lag length of 13. The results are reported in Table(2).

Null Hypothesis: RT has a unit root  
 Exogenous: Constant  
 Lag Length: 0 (Automatic - based on SIC, maxlag=13)

	t-Statistic	Prob.*
<b>Augmented Dickey-Fuller test statistic</b>	<b>-10.60353</b>	<b>0.0000</b>
Test critical values:		
1% level	-3.467205	
5% level	-2.877636	
10% level	-2.575430	

Table(2) reports the results of the ADF test were applied to the exchange return series the result illustrate that the absolute value of the ADF test (10.60353)is greater than the 1%, 5% and 10% critical values in absolute terms (3.467205 ,2.877636 and 2.575430) respectively, this result conclude that the exchange rate series is stationary

**Testing for Heteroskedasticity**

One of the most important issues before applying the generalized autoregressive conditional Heteroskedasticity (GARCH) methodology is to examine the residuals for evidence of heteroskedasticity. To test for the presence of heteroskedasticity in the residuals of the daily return of stock and exchange rate series, the Lagrange Multiplier (LM) test proposed by Engle (1982) and the Ljung-Box Statistics Q(m) test proposed by McLeod and Li (1983) are carried out. The null hypothesis is that the first *m* lags of autocorrelation of the squared residuals series are equal to zero. The first step of both tests is to entail the estimation of the mean equation by simple ordinary least square (OLS). The residuals used for Lagrange Multiplier and Ljung-Box test are obtained from the mean equation to run an auxiliary regression of the squared residuals upon the lagged squared terms.

In summary, according to Enders (2004) and Tsay (2002), the test procedure is performed by the following steps:

**Step1:** Obtain the residuals  $e_t$  using ordinary least squares (OLS) on the Conditional mean equation (which might be regression equation or ARIMA model) and calculate the squares of the fitted errors ( $e_t^2$ ).

**Step2:** For the Ljung-Box test which has been proposed by McLeod and Li (1983), the methodology involves the following:

First: Calculate the sample autocorrelations of the squared residuals ( $e_t^2$ ) as:

$$Pi = \frac{\sum_{t=1}^T (e_t^2 - \sigma^2)(e_{t-1}^2 - \sigma^2)}{\sum_{t=1}^T (e_t^2 - \sigma^2)^2}$$

Where  $\sigma^2$  is the sample variance of the residuals,  $T$  is the number of residuals

Second: use Ljung-Box Q-statistics to test for groups of significant coefficients. The test statistic is given by :

$$Q_{LB} = T(T + 2) \sum_{k=1}^m \frac{\hat{\rho}^2(k)}{(T-m)}$$

It has an asymptotic chi-squares ( $\chi^2$ ) distribution with  $n$  degrees of freedom if the ( $e_t^2$ ) sequence is serially uncorrelated. The Q-statistics are significant at all lags, indicating significant serial correlation in the residuals. Rejecting the null hypothesis that the  $e_t^2$  are serially uncorrelated is equivalent to rejecting the null hypothesis of no ARCH or GARCH errors.

Step3: For the Lagrange multiplier test which has been proposed by Engle (1982), the methodology also involves the following:

First: Regress the squared residuals on a constant and  $q$  lags as in the following equation:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \dots + \alpha_q e_{t-q}^2 + v_t$$

**Second:** The null hypothesis that there is no ARCH effect up to order  $q$  can be formulated as:

$$H_0 : \alpha_0 = \alpha_1 = \alpha_2 = \dots = \alpha_q = 0$$

Against the alternative:

$$H_1 : \alpha_i > 0 \text{ for at least one } i = 1, 2, 3, \dots, q$$

Hence, this regression will have little explanatory power so that the coefficient of determination ( $R^2$ ) will be quite low. Using a sample of  $T$  residuals, under the null hypothesis of no ARCH errors, the test statistic for the joint significance of the  $q$  lagged squared residuals is  $TR^2$  which under  $H_0$  converges to a chi-square distribution with  $q$  degrees of freedom,  $\chi^2(q)$ . If  $TR^2$  is sufficiently large, rejection of the null hypothesis that  $\alpha_0$  through  $\alpha_q$  are jointly equal to zero is equivalent to rejection of the null hypothesis of no ARCH errors. On the other hand, if  $TR^2$  is sufficiently low, it is possible to conclude that there are no ARCH effects

**Table (3).** ARCH-LM Test for residuals of Exchange rate Returns (SDG/ USA (\$))

**Heteroskedasticity Test: ARCH**

F-statistic	6.180302	Prob. F(1,175)	0.0139
Obs*R-squared	6.037707	Prob. Chi-Square(1)	0.0140

Note:  $H_0$ : There are no ARCH effects in the residual series

The ARCH-LM test results in Table 3 provide strong evidence for rejecting the null hypothesis. Rejecting  $H_0$  is an indication of the existence of ARCH effects in the residuals series of the mean equation and therefore the variance of the returns series indicates are non-constant

**Table (4)** Estimation results of different GARCH models Exchange rate Returns (SDG/ USA (\$))

Coefficients	GARCH (1,1)	GARCH-M (1,1)	EGARCH (1,1)	TGARCH (1,1)	PGARCH (1,1)
Mu( $\mu$ )	0.038	14.53	-0.0007	0.002	0.003
	0.019	0.999	0.209	0.504	0.36
Arl( $\phi$ )	0.997	1.000	0.599	0.506	0.451
	0.000	0.000	0.000	0.124	0.149
Ma( $\theta_1$ )	-0.852	-0.679	-0.605	-0.069	-0.032
	0.000	0.000	0.000	0.839	0.897
Ma( $\theta_2$ )	-0.139	-0.271	0.022	0.004	0.002
	0.005	0.000	0.319	0.982	0.99
Omega ( $\omega$ )	0.00001	0.00003	-15.107	0.0002	0.0006
	0.906	-	0.000	0.000	0.863
Alpha ( $\alpha$ )	5.452	0.117	0.689	1.442	0.918
	0.000	0.000	0.000	0.006	0.0007
Beta ( $\beta_1$ )	0.008	0.882	-0.075	-0.841	-0.197
	0.325	0.000	0.042	0.136	0.094
Gamma( $\gamma$ )			-0.806	-0.027	-0.043
			0.000	0.557	0.739
Delta ( $\delta$ )					1.804
					0.198
□□□□	5.46	0.999	0.614	0.601	0.721
Log likelihood	506.08	507.5	468.83	467.15	464.77
ARCH-LM test	0.012	25.66	5.42	1.22	1.766
	0.912	0.000	0.0198	0.269	0.183

In the results for the variance equation reported in Tables 4 provides the estimates of the GARCH (1,1) model for return series of exchange rate in Sudan. The estimation result shown that the coefficients in the conditional variance equation the  $\alpha$  significant and  $\beta$  not significant at 5% significant level. The sum of ARCH and GARCH coefficients ( $\alpha+\beta$ ) = 5.46 (persistence coefficients) in the GARCH (1,1) model is the greater than one, suggesting that the conditional variance process is explosive. the ARCH-LM for lagged conditional variance and squared disturbance is 0.012, under  $\chi^2_{(1)}$  the null hypothesis is accepted since the p- value is 0.912 where it has greater than 5% of significance level. Means that the Accept the null hypothesis at the same condition. Therefore the ARCH-LM test on the residuals of this model indicates that the conditional heteroskedasticity is not present.

The GARCH-M (1,1) model is estimated by allowing the mean equation of the return series to depend on a function of the conditional variance. From estimation results in Table 4 the estimated coefficient (risk premium) of  $R_t$  in the mean equation is positive for the markets, which indicates that the mean of the return sequence depends on past innovations and the past conditional variance.

From Table 4 the estimates of the EGARCH (1,1) model for return series of exchange rate, the estimation results shows that; the estimates  $\gamma$  is negative and significant, meaning that returns series have asymmetry and has greater impact of negative shocks on the return series of exchange rate volatility. Moreover, the estimates  $\beta = -0.075$  is significantly at 5% significant level which is an indication of not persistence of volatility. In addition the estimates  $\alpha$  is statistically significant while  $\gamma$  is statistically significant,  $\alpha > 0$  indicating that the conditional variance has leverage effect. Furthermore  $\gamma \neq 0$ ; meaning that an asymmetry of negative shocks on the conditional variance is present. the ARCH-LM for lagged conditional variance and squared disturbance is 5.42, under  $\chi^2_{(1)}$  the null hypothesis is rejected since the p- value is 0.019 where it has less than 5% of significance level the null hypothesis is rejected This result indicates that the ARCH effect occur in the residuals of EGARCH (1,1) model this model is adequate to presents return series of exchange rate.

From above table demonstrate the estimation result of the TGARCH (1,1) for return series of exchange rate model. It can be seen that the ARCH is statistical significant and GARCH term is not statistical significant. The sum of ARCH and GARCH is equal to 0.601 less than one indicating that volatility shocks is quite persistent. Moreover, the symmetry term in the TGARCH (1,1) model  $\gamma = -0.027$  it is not statistically significant at 5% significant level, means that the conditional variance has not a leverage effect , Adverse market conditions and bad news such as, the effect of risk management policies and political instabilities has an impact of  $\alpha + \gamma = 1.415$  . Good news about demand and supply conditions in the price of exchange rate market has an impact of  $\alpha = 1.442$  the ARCH-LM for one lag difference of residuals squared is 1.22, under  $\chi^2_{(1)}$  the null hypothesis is not rejected since the p- value is 0.269 where it has greater than 5% of significance level. accept the null hypothesis Therefore the ARCH-LM test on the residuals of this model this results confirm that the model is appropriate

From table 4 demonstrate the estimates of the APARCH (1,1) model for returns exchange rate series, the estimated power term  $\delta = 1.8$  is not statistically significant at 5% significant level. Moreover the a symmetry term  $\gamma$  in the APARCH (1,1) model is not statistically significant at 5% significant level. In addition  $\gamma = -0.043$  is less than zero, this means that the conditional variance has not a symmetric term on the price volatility. The output on the ARCH test as shown in table 4 signifies that the null hypothesis did not rejected, that there is no ARCH effect in the residuals, because of the insignificant squared residual term (p-value of 0.183 is more than 0.05 level of significance). This result confirms that the APGARCH (1,1) model for returns exchange rate series model is adequate.

### **III. Conclusion**

Modelling and forecasting the volatility of returns series in stock markets has become fertile field of empirical research in financial markets. This is simply because volatility is considered as an important concept in many economic and financial applications like asset pricing; risk management and portfolio allocation This paper attempts to explore the comparative ability of different statistical and econometric volatility forecasting models in the context of Sudanese stock market. A total of five different models were considered in this study. The volatility of the exchange rate returns in sudan have been modeled by using a univariate Generalized Autoregressive Conditional heteroskedasticity (GARCH) models including both symmetric and asymmetric models that captures most common stylized facts about index returns such as volatility clustering and leverage effect, these models are GARCH(1,1), GARCH-M(1,1), exponential GARCH(1,1), threshold GARCH(1,1) and power GARCH(1,1). Based on the empirical results presented, the following can be concluded: The paper finds strong evidence that monthly returns could be characterised by the above mentioned models exchange data showed a significant departure from normality and existence of conditional heteroskedasticity in the residuals series. For all periods specified, the empirical analysis was supportive to the symmetric volatility hypothesis, which means returns are volatile and that positive and negative shocks (good and bad news) of the same magnitude have the same impact and effect on the future volatility level. The parameter estimates of the



GARCH (1,1) models ( $\alpha$  and  $\beta$ ) indicates a high degree of persistent in the conditional volatility of stock returns Exchange which means an explosive volatility. The parameter describing the conditional variance in the mean equation, measuring the risk premium effect for GARCH-M (1,1), is statistically significant in all periods, and the sign of the risk premium parameter is positive. The implication is that increase in volatility would increase returns, which is an expected result.

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