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$\left[\frac{p}{2}\right]$- Cordiallabeling in graphs

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Abstract:
A $\left[\frac{p}{2}\right]$*- **cordiallabeling** of a graph G with p verticesisabijection*

f: V(G) $\rightarrow$*{1, 2,3,..., p}definedby* $f(e = uv) = \begin{cases} 1 & if\ |f(u) - f(v)| \leq \left[\frac{p}{2}\right] \\ 0 & otherwise \end{cases}$ *and* $|e_f(0) - e_f(1)| \leq 1$

.If a graphhas $\left[\frac{p}{2}\right]$*- cordiallabeling then it is called* $\left[\frac{p}{2}\right]$ *- cordial where* $\left[\frac{p}{2}\right]$

representsthenearestintegerlessthanor equalto $\frac{p}{2}$*.In this paper we prove* $K_{m,n}$*if m, n are of different parity, paths, star graphs are* $\left[\frac{p}{2}\right]$ *-cordial.And* $K_{m,n}$*with m,n are of same pairity are not* $\left[\frac{p}{2}\right]$*- cordial.*

By a graph we mean a finite, undirected graph without multiple edges or loops. For graph theoretic terminology, we refer to Harary [2] and Bondy.

Keywords: $\left[\frac{p}{2}\right]$ **-** cordial, path, complete bipartite, star.

I. Introduction

Definition1.1: LetG=(V(G),E(G))beagraph.Amapping
f:V(G) $\rightarrow$ {0,1}iscalled a **binary vertex labeling**ofGwhere f(v)isthe labelofthevertexvofG underf.
Foranedgee=uv,theinducededgelabeling
f*:E(G) $\rightarrow$ {0,1}is givenbyf*(e)= $|f(u) - f(v)|$. Let $v_f(0), v_f(1)$ bethenum berof verticesofG havinglabel 0 and1respecti vely underfand $e_f(0), e_f(1)$ bethe

numberofedgeshavinglabel0and1respectivelyunderf*.

Definition 1.2: Abinary vertex labelingo fagraph Giscalleda **cordiallabeling**if $|v_f(0) - v_f(1)| \leq 1$and $|e_f(0) - e_f(1)| \leq 1$.Agraph G issaid to be cordialifitadmitscordiallabeling.

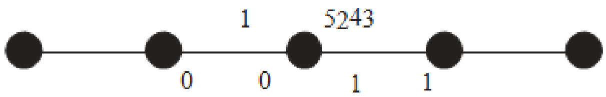
Theorem 2.1:$P_n$is $\left[\frac{p}{2}\right]$ - cordialforeveryn.

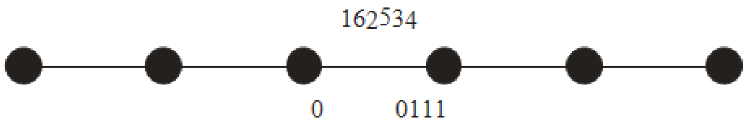
Proof: LettheverticesofP$_n$bev$_1$,v$_2$,…,v$_n$respectively.
Definef:V(P$_n$) $\longrightarrow$ {1,2,3,…,n},weconsiderthefollowingtwo cases

Define $f(v_i)=\begin{cases}\frac{i+1}{2} & if \quad i \quad is \quad odd\\ n-\frac{i-2}{2} & if \quad i \quad is \quad even\end{cases}$

Case(i):nis odd.
Thenf(v$_1$),f(v$_2$),f(v$_3$), …, f(v$_{n-1}$),f(v$_n$)arerespectively,
1,n,2,n-1,3, n-2,… $\frac{n-1}{2},\frac{n+1}{2}$
Thereforethedifferencesofthevertexlabelsarerespectively
n-1,n-2,n-3,…, 3,2,1.
Thuse$_f$(0)=e$_f$(1) = $\frac{n-1}{2}$

Case(ii):niseven.
Thenf(v$_1$),f(v$_2$),f(v$_3$),…,f(v$_{n-1}$),f(v$_n$)are respectively
1,n,2,n-1,3, n-2,…, $\frac{n}{2},\frac{n+1}{2}$
Thereforethedifferencesofthevertexlabelsarerespectively
n-1,n-2,n-3,…, 3,2,1.
Thuse$_f$(0)= $\frac{n-1}{2}$ and e$_f$(1) = $\frac{n}{2}$





$\left[\frac{p}{2}\right]$ - cordial labeling of P$_5$andP$_6$

Theorem 2.1:K$_{m,n}$is $\left[\frac{p}{2}\right]$-cordialifm and n are of differentparity.

Proof: Without loss of generality assumem<n.
LetK$_{m,n}$be (V$_1$,V$_2$)where
V$_1$={u$_1$,u$_2$, …,u$_m$}andV$_2$={v$_1$,v$_2$,…, v$_n$}.
Define $f(u_i)=$i, $f(v_i)=m+i$
Thenthevertexlabelsbef(u$_1$)=1;f(u$_2$)= 2;…; f(u$_m$)=mand
f(v$_1$)=m+1;f(v$_2$)=m+2;…; f(v$_n$)=m+n.

Lettheedgelabelsbe $f^*(u_i,v_j)=\begin{cases}1 \quad if \quad \left|f(v_j)-f(u_i)\right|\le\left[\frac{m+n}{2}\right]\\ 0 \qquad\qquad otherwise\end{cases}$

Case(i): Letmbeevenandn odd.
Thenumberofedgesincidentwithu$_1$labeled1= $\frac{n+1}{2}-\frac{m}{2}$

Thenumberofedgesincidentwithu$_2$labeled1= $\frac{n+1}{2}-\frac{m}{2}+1$

The number of edges incident with u$_3$ labeled 1 = $\frac{n+1}{2}-\frac{m}{2}+2$

…

Thenumberofedgesincidentwithu$_m$labeled1= $\frac{n+1}{2}-\frac{m}{2}+(m-1)$

∴Thetotalnumberofedgeswithlabel1

= $m(\frac{n+1}{2})-m(\frac{m}{2})+[1+2+3+...(m-1)]$

Thetotalnumberofedgeswithlabel1= $\frac{mn}{2}+\frac{m}{2}-\frac{m^2}{2}+\left(\frac{m-1}{2}\right)m = \frac{mn}{2}$

∴Thetotalnumberofedgeswithlabel0= $mn-\frac{mn}{2}=\frac{mn}{2}$

K$_{m,n}$is $\left[\frac{p}{2}\right]$- cordial ifm<nandm-even,n-odd.

Case (ii): Let m < n, m-odd, n-even.

Thenumberofedgesincidentwithu$_1$labeled1= $\frac{n}{2}-\frac{m-1}{2}$

Thenumberofedgesincidentwithu$_2$labeled1= $\frac{n}{2}-\frac{m-1}{2}+1$

Thenumberofedgesincidentwithu$_3$labeled1= $\frac{n}{2}-\frac{m-1}{2}+2$

…

Thenumberofedgesincidentwithu$_m$labeled1= $\frac{n}{2}-\frac{m-1}{2}+(m-1)$

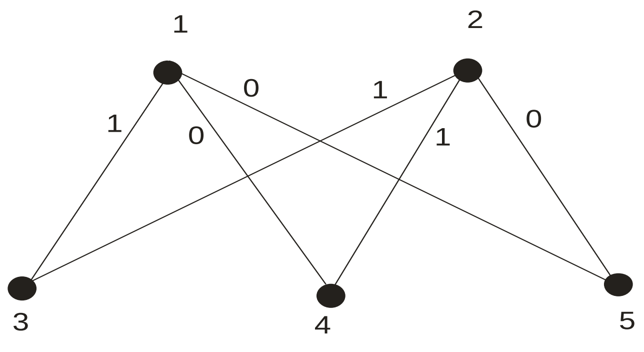
∴Thetotalnumberofedgeswithlabel1

= $m(\frac{n}{2})-m(\frac{m-1}{2})+[1+2+3+...+(m-1)]$

Thetotalnumberofedgeswithlabel1= $\frac{mn}{2}$

Thetotalnumberofedgeswithlabel0= $mn-\frac{mn}{2}=\frac{mn}{2}$

Thus K$_{m,n}$is $\left[\frac{p}{2}\right]$- cordial ifm<nandifm– odd,n-even.



The $\left[\frac{p}{2}\right]$ - cordial labeling of $K_{2,3}$.

THEOREM 2.2:$K_{m,n}$isnot $\left[\frac{p}{2}\right]$ - cordialifbothmandnareeveno r bothmand n are odd.

Proof: Withoutlossofgeneralityletm <n
Let$K_{m,n}$be(V_1,V_2)whereV_1={u_1, u_2,….u_m}and
V_2={v_1,v_2,…, v_n}.
Case(i): m andnareeven

Thevertexlabelsare1,2,3,…, $\frac{m}{2}$, …, m, m + 1, …, (m + n).

Themaximumnumberofedgesthatcanbelabeled0= $\frac{mn}{2} - \frac{m}{2} < \frac{mn}{2}$

But$K_{m,n}$tobe $\left[\frac{p}{2}\right]$ - cordial,thenumberofedgeswithlabel0must be = $\frac{mn}{2}$

$\therefore$ $K_{m,n}$isnot $\left[\frac{p}{2}\right]$- cordial ifbothmandnareeven.

Case(ii): m andnareodd

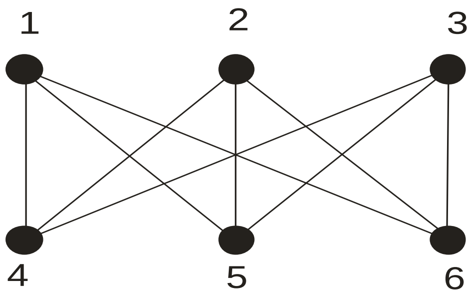
Thevertexlabelsare1,2,3,…, $\frac{m}{2},\ldots,m,m+1,\ldots(\frac{m+n}{2}),\ldots,(m+n)$

Thenumberofedges=mnwhichisodd.

Themaximumnumberofedgesthatcanbelabeled0= $\frac{mn-3}{2}$

But$K_{m,n}$tobe $\left[\frac{p}{2}\right]$ - cordial,thenumberofedgeswithlabel0mustbe= $\frac{mn-1}{2}$

$\therefore$ $K_{m,n}$isnot $\left[\frac{p}{2}\right]$- cordial ifbothmandnareodd.



$K_{3,3}$

Theorem2.3: Stargraphs$K_{1,n}$are $\left[\frac{p}{2}\right]$ -cordial.

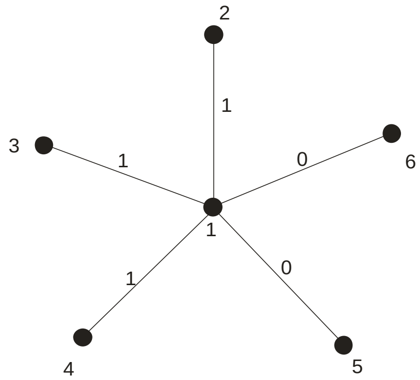
Proof: Let$K_{1,n}$be (V_1,V_2)whereV_1={u}andV_2={v_1,v_2,…, v_{n-1}}.
Letthevertexlabelsbef(u)= 1;f(v_1)= 2;f(v_2)= 3; ...,f(v_{n-1})= n.

$\therefore \left| f(v_i) - f(u_1) \right|$ are1,2,3,…,n-1for i =1,2,3,…,n-1

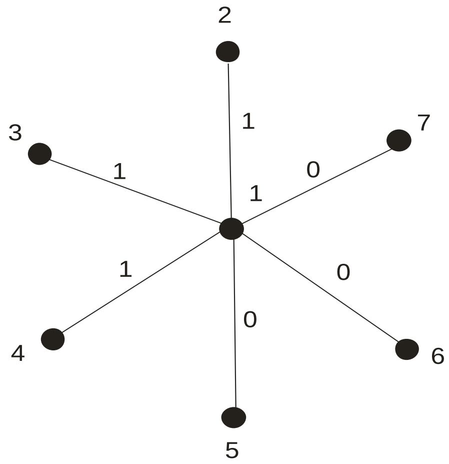
Thusif nisodd$e_f(0)$= $\frac{n-1}{2}$,$e_f(1)$= $\frac{n+1}{2}$

Andifnisevene$_f$(0)=e$_f$(1)= $\frac{n}{2}$

ThusStargraphsK$_{1,n}$are $\left[\frac{p}{2}\right]$- cordial.



The $\left[\frac{p}{2}\right]$–cordial labeling ofK$_{1,5}$



The $\left[\frac{p}{2}\right]$–cordial labeling ofK$_{1,6}$

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