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On Some New Linear Generating Relations Involving Multivariable H-Function

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Abstract: *The aim of this research paper is to establish some new linear generating relations involving multivariable H-function.*

I. Introduction

The multivariable H–function given in [1] is defined as follows:

$$H[z_1,\dots,z_r] = H^{0,n:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix}(a_j;\alpha_j',\dots,\alpha_j^{(r)})_{1,p}:(c_j';\gamma_j')_{1,p_1};\dots\dots;(c_j^{(r)};\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta_j',\dots,\beta_j^{(r)})_{1,q}:(d_j';\delta_j')_{1,q_1};\dots\dots;(d_j^{(r)};\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\dots\int_{L_r}\phi_1(\xi_1)\dots\phi_r(\xi_r)\psi(\xi_1,\dots,\xi_r)z_1^{\xi_1}\dots z_r^{\xi_r}d\xi_1\dots d\xi_r \qquad (1)$$

where $\omega = \sqrt{(-1)}$,

$$\psi(\xi_1,\dots,\xi_r) = \frac{\prod_{j=1}^{n}\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i)}{\prod_{j=n+1}^{p}\Gamma(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i)\prod_{j=1}^{q}\Gamma(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}\xi_i)}$$

$$\phi_i(\xi_i) = \frac{\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-\delta_j^{(i)}\xi_i)\prod_{j=1}^{n_i}\Gamma(1-c_j^{(i)}+\gamma_j^{(i)}\xi_i)}{\prod_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}+\delta_j^{(i)}\xi_i)\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-\gamma_j^{(i)}\xi_i)}$$

In (1), i in the superscript (i) stands for the number of primes, e.g., $b^{(1)} = b'$, $b^{(2)} = b''$, and so on; and an empty product is interpreted as unity.

Suppose, as usual, that the parameters

a_j, j = 1, …., p; $c_j^{(i)}$, j = 1,….,p_i;
b_j, j = 1, …., q; $d_j^{(i)}$, j = 1,….,q_i; $\forall$ i $\in$ {1,…..,r}

are complex numbers and the associated coefficients

$\alpha_j^{(i)}$, j = 1, …., p; $\gamma_j^{(i)}$, j = 1,….,p_i;
$\beta_j^{(i)}$, j = 1, …., q; $\delta_j^{(i)}$, j = 1,….,q_i; $\forall$ i $\in$ {1,…..,r}

positive real numbers such that the left of the contour. Also

$$V_i = \sum_{j=1}^{p}\alpha_j^{(i)} + \sum_{j=1}^{p_i}\gamma_j^{(i)} - \sum_{j=1}^{q}\beta_j^{(i)} - \sum_{j=1}^{q_i}\delta_j^{(i)} \le 0 \qquad (2)$$

$$\Omega_i = -\sum_{j=n+1}^{p}\alpha_j^{(i)} - \sum_{j=1}^{q}\beta_j^{(i)} + \sum_{j=1}^{m_i}\delta_j^{(i)} - \sum_{j=m_i+1}^{q_i}\delta_j^{(i)} + \sum_{j=1}^{n_i}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i}\gamma_j^{(i)} > 0 \qquad (3)$$

where the integral n, p, q, m_i, n_i, p_i and q_i are constrained by the inequalities $p \ge n \ge 0$, $q \ge 0$, $q_i \ge m_i \ge 0$ and $p_i \ge n_i \ge 0$ ($\forall$ i $\in$ {1, 2, …, r}) and the inequalities (2) hold for suitably restricted values of the complex variables z_1, …., z_r. The sequence of parameters in (1) are such that none of the poles of the integrand coincide, that is, the poles of the integrand in (1) are simple. The contour L_i in the complex ξ_i-plane is of the Mellin-Barnes type which runs from $-\omega\infty$ to $+\omega\infty$ with indentations, if necessary, to ensure that all the poles of $\Gamma(d_j^{(i)} - \delta_j^{(i)}\xi_i)$, j = 1,…, m_i are separated from those of $\Gamma(1 - c_j^{(i)} + \gamma_j^{(i)}\xi_i)$, i = 1, …., n_i.

In the present investigation we require the following formulae:
From Shrivastava and Manocha [2, p.37],

$$(\alpha)_n = (\alpha, n) = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}, \qquad (4)$$

$$(1-z)^{-a} = \sum_{n=0}^{\infty}(\alpha)_n\frac{z^n}{n!}, \qquad (5)$$

II. Linear Generating Relations:

In this section we establish the following linear Generating Relations:

$$\sum_{l=0}^{\infty}\frac{t^l}{l!}\,H^{0,\ n\ :(m_1,n_1);\ldots\ldots;(m_r,n_r)}_{p,q\ :(p_1+1,q_1);\ldots\ldots;(p_r,q_r)}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\,\middle|\,\begin{matrix}\ldots,\ldots:\ldots\ldots,(\lambda-l;\alpha):\ldots\ldots,\ldots\ldots\\ \ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots\end{matrix}\right]$$

$$=(1+t)^{(\square\square\square\square\square)}H^{0,\ n\ :(m_1,n_1);\ldots\ldots;(m_r,n_r)}_{p,q\ :(p_1+1,q_1);\ldots\ldots;(p_r,q_r)}\left[\begin{matrix}z_1(1+t)^{\alpha}\\ \vdots\\ z_r\end{matrix}\,\middle|\,\begin{matrix}\ldots,\ldots:\ldots\ldots,(\lambda,\alpha):\ldots\ldots,\ldots\ldots\\ \ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots\end{matrix}\right], \qquad (6)$$

$|\arg(z_k)| < \frac{1}{2}V_k\pi, \forall k \in [1,\ldots,r]$, where V_k is given in (2);

$$\sum_{l=0}^{\infty}\frac{(-t)^l}{l!}\,H^{0,\ n\ :(m_1,n_1);\ldots\ldots;(m_r,n_r)}_{p,q\ :(p_1+1,q_1);\ldots\ldots;(p_r,q_r)}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\,\middle|\,\begin{matrix}\ldots,\ldots:\ldots\ldots,(\lambda-l;\alpha):\ldots\ldots,\ldots\ldots\\ \ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots\end{matrix}\right]$$

$$=(1\ \square\ t)^{(\square\square\square\square\square)}H^{0,\ n\ :(m_1,n_1);\ldots\ldots;(m_r,n_r)}_{p,q\ :(p_1+1,q_1);\ldots\ldots;(p_r,q_r)}\left[\begin{matrix}z_1(1+t)^{\alpha}\\ \vdots\\ z_r\end{matrix}\,\middle|\,\begin{matrix}\ldots,\ldots:\ldots\ldots,(\lambda;\alpha):\ldots\ldots,\ldots\ldots\\ \ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots\end{matrix}\right], \qquad (7)$$

$|\arg(z_k)| < \frac{1}{2}V_k\pi, \forall k \in [1,\ldots,r]$, where V_k is given in (2).

Proof: To prove (6), consider

$$\Delta = \sum_{l=0}^{\infty}\frac{t^l}{l!}\,H^{0,\ n\ :(m_1,n_1);\ldots\ldots;(m_r,n_r)}_{p,q\ :(p_1+1,q_1);\ldots\ldots;(p_r,q_r)}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\,\middle|\,\begin{matrix}\ldots,\ldots:\ldots\ldots,(\lambda-l;\alpha):\ldots\ldots,\ldots\ldots\\ \ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots:\ldots\ldots,\ldots\ldots\end{matrix}\right]$$

On expressing multivariable H-function in contour integral form as given in (1), we get

$$\Delta = \sum_{l=0}^{\infty}\frac{t^l}{l!}\Big[\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi_1(\xi_1)\ldots\phi_r(\xi_r)\psi(\xi_1,\ldots,\xi_r)$$

$$\times\frac{1}{\Gamma\{\lambda-l-\alpha\xi_1\}}z_1^{\xi_1}\ldots z_r^{\xi_r}\,d\xi_1\ldots d\xi_r\Big]$$

$$=\sum_{l=0}^{\infty}\frac{(-t)^l}{l!}\Big[\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi_1(\xi_1)\ldots\phi_r(\xi_r)\psi(\xi_1,\ldots,\xi_r).$$

$$\times\frac{\{1-\lambda-\alpha\xi_1\}_l}{\Gamma\{\lambda-\alpha\xi_1\}}z_1^{\xi_1}\ldots z_r^{\xi_r}\,d\xi_1\ldots d\xi_r\Big].$$

On changing the order of summation and integration, we have

$$\Delta = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi_1(\xi_1)\ldots\phi_r(\xi_r)\psi(\xi_1,\ldots,\xi_r)z_1^{\xi_1}\ldots z_r^{\xi_r}$$

$$\times\frac{1}{\Gamma\{\lambda-\alpha\xi_1\}}\Big[\sum_{l=0}^{\infty}\frac{(-t)^l}{l!}\{1-\lambda-\alpha\xi_1\}_l\Big]\,d\xi_1\ldots d\xi_r$$

$$=(1+t)^{\lambda-1}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi_1(\xi_1)\ldots\phi_r(\xi_r)\psi(\xi_1,\ldots,\xi_r)z_1^{\xi_1}\ldots z_r^{\xi_r}$$

$$\times \frac{(1+t)^{\alpha\xi_1}}{\Gamma\{\lambda-\alpha\xi_1\}}\, d\xi_1 \ldots d\xi_r$$

which in view of (1), provides (6).
Proceeding on similar lines as above, the results (7) can be derived.

III. Particular Cases

On specializing the parameters, we get following generating relations in terms of H-function of one variable, which are the results given by Shrivastava & Shrivastava [3]:

$$\sum_{l=0}^{\infty} \frac{(t)^l}{l!} H^{m,n}_{p+1,q}\left[z\Big|^{(a_j,\alpha_j)_{1,p},(\lambda-l,\alpha)}_{(b_j,\beta_j)_{1,q}}\right]$$
$$= (1+t)^{(\square\square\square\square\square)} H^{m,n}_{p+1,q}\left[z(1+t)^{-\alpha}\Big|^{(a_j,\alpha_j)_{1,p},(\lambda,\alpha)}_{(b_j,\beta_j)_{1,q}}\right],$$

|argz| < ½ A□ , where A is givenby

$$\sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j \equiv A > 0;$$

$$\sum_{l=0}^{\infty} \frac{(-t)^l}{l!} H^{m,n}_{p+1,q}\left[z\Big|^{(a_j,\alpha_j)_{1,p},\,(\lambda-l,\alpha)}_{(b_j,\beta_j)_{1,q}}\right]$$
$$= (1 \,\square\, t)^{(\square\square\square\square\square)} H^{m,n}_{p+1,q}\left[z(1-t)^{-\alpha}\Big|^{(a_j,\alpha_j)_{1,p},\,(\lambda,\alpha)}_{(b_j,\beta_j)_{1,q}}\right],$$

|argz| < ½ A□ , where A is given by

$$\sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j \equiv A > 0.$$

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